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2D Structured Non-Cyclic Fuzzy Graphs

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***Abstract*—**Fuzzy graphs incorporate concepts from graph theory with fuzzy principles. In this paper, we make a study on the properties of fuzzy graphs which are non-cyclic and are of two-dimensional in structure. In particular, this paper presents 2D structure or the structure of double layer for a non-cyclic fuzzy graph whose underlying crisp graph is non-cyclic. In any graph structure, introducing 2D structure may lead to an inherent cycle. We propose relevant conditions for 2D structured non-cyclic fuzzy graphs. These conditions are extended even to fuzzy graphs of the 3D structure. General theoretical properties that are studied for any fuzzy graph are verified to 2D structured or double layered fuzzy graphs. Concepts like Order, Degree, Strong and Size for a fuzzy graph are studied for 2D structured or double layered non-cyclic fuzzy graphs. Using different types of fuzzy graphs, the proposed concepts relating to 2D structured fuzzy graphs are verified.

***Keywords*—**Double layered fuzzy graph, double layered non-cyclic fuzzy graph, strong, order, degree and size.

I. Introduction

To understand and accommodate uncertainties and vagueness, the concepts in crisp set theory were not sufficient. Zadeh proposed the theory of fuzzy sets in 1965 that deals with uncertainties [1]. Fuzzy graph theory was introduced by Rosenfeld in 1975, which has now advanced into large number of branches [2]. Yeh and Bang studied various connectedness concepts on fuzzy graph such as connectivity matrix, degree of connectivity, edge connectivity, vertex connectivity [3]. Complement of a fuzzy graph was studied by Sunita and Kumar [4]. Nagoorgani and Malarvizhi had discussed the properties of μ- complement of a fuzzy graph [5]. Nagoorgani and Radha discussed the degree of vertex in some fuzzy graphs [6]. Double layered fuzzy graph was introduced by Pathinathan and Roseline in 2014 [7]. In the same year, they further studied the matrix representation of double layered fuzzy graph [8] and vertex degree of Cartesian product of intuitionistic fuzzy graph [9].

As a generalization of fuzzy sets, Attanasov introduced intuitionistic fuzzy set in 1986 [10]. Pathinathan and Roseline developed intuitionistic double layered fuzzy graph in 2015 and studied its properties [11], further they verified its Cartesian product [12]. In the same year, they introduced structural core graph of double layered fuzzy graph [13] and later introduced triple layered fuzzy graphs [14]. In double layered fuzzy graphs, they had dealt with those graphs whose crisp graph is cyclic. In this paper, we have introduced the concept of double layered non-cyclic fuzzy graph. Pathinathan and Peter have also studied balanced double layered fuzzy graphs [15]. Peter studied the co-normal product of fuzzy graphs [16].

Along with the introduction of double layered non-cyclic fuzzy graphs, we have also defined the conditions to frame the 3D structure for a fuzzy graph whose crisp graph is non-cyclic and also, we have defined its order, degree and size.

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II. Preliminaries

We recall some definitions.

A. Double Layered Fuzzy Graph

Double Layered Fuzzy Graph (Crisp graph G^* is a cycle)

Let $G:(\sigma,\ \mu)$ be a fuzzy graph with the underlying crisp graph $G^*:(\sigma^*,\ \mu^*)$. The pair $DL(G):(\sigma_{DL},\ \mu_{DL})$ is defined as follows. The node set of $DL(G)$ be $\sigma^* \cup \mu^*$. The fuzzy subsets σ_{DL} on $\sigma^* \cup \mu^*$ are defined as $\sigma_{DL} = \begin{cases} \sigma(u) \text{ if } u \in \sigma^*. \\ \mu(uv) \text{ if } uv \in \mu *. \end{cases}$

$$\mu_{DL} = \begin{cases} \mu(uv) & \text{if } u,v \in \sigma^*. \\ \mu(e_i) \wedge \mu(e_j) & \text{if the edge } e_i \text{ and } e_j \\ & \text{have a common node between them.} \\ \sigma(u_i) \wedge \mu(e_j) & \text{if } u_i \in \sigma^* \textit{ and } e_i \in \mu^* \\ & \text{and each } e_i \text{is incident withsingle } u_i \\ & \text{either clockwise or anticlockwise.} \\ 0 & \text{otherwise.} \end{cases}$$

B. Order of Fuzzy Graph

Given a fuzzy graph $G:(\sigma,\ \mu)$, with the underlying set S, the order of G is defined as $\sum_{x\in S}\sigma(x)$ and is denoted as O (G).

C. Size of Fuzzy Graph

Let $G:(\sigma,\ \mu)$ be a fuzzy graph, then the size of G is defined as $S(G)=\sum_{x,y\in V}\mu(x,y)$.

D. Vertex Degree

Let $G:(\sigma,\ \mu)$ be a fuzzy graph, the degree of a vertex u is defined as $\text{d(u)}=\sum_{x,y\in E}\mu(x,y)$.

E. Strong Fuzzy Graph

A fuzzy graph $G:(\sigma,\ \mu)$ is said to be a strong fuzzy graph if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all (u,v) in μ^*.

F. μ-Complement

Let G be a fuzzy graph, the μ- complement of G is denoted as $G^{\mu}:(\sigma^{\mu},\mu^{\mu})$ where $\sigma^{*}\cup\mu^{*}$ and

$$G^{\mu}:(\sigma^{\mu},\mu^{\mu}(u,v)=\begin{cases}\sigma(u)\wedge\sigma(v)-\mu(u,v)\ if\ \mu(u,v)>0.\\ 0\ if\ \mu(u,v)=0.\end{cases}$$

III. Double Layered Non-Cyclic Fuzzy Graph

A. Double Layered Non-Cyclic Fuzzy Graph

Let $G:(\sigma,\mu)$ be a fuzzy graph with the underlying crisp graph $G^{*}:(\sigma^{*},\ \mu^{*})$. The pair $NCDL(G):(\sigma_{NCDL},\mu_{NCDL})$ is defined as follows. The vertex set of $NCDL(G)$ be $\sigma^{*}\cup\mu^{*}$. The fuzzy subset σ_{NCDL} is defined as:

$$\sigma_{NCDL}=\sigma(u)\ if\ u\in\sigma^{*}.$$
$$\sigma_{NCDL}=\mu(uv)\ if\ uv\in\mu^{*}.$$

and the fuzzy relation μ_{NCDL} on $\sigma^{*}\cup\mu^{*}$ is defined as:

(i) $\mu_{NCDL}=\mu(u_iu_j)=e_i$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$, i.e. there is only one edge incident with the corresponding vertex if $u_i,u_j\in\sigma^{*}$ *and* $e_i\in\mu^{*}$.

(ii) $\mu_{NCDL}=\mu(u_iu_j)\wedge\mu(u_iu_k)=e_i\wedge e_j$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$, e_j is incident with $\sigma(u_i)$ and $\sigma(u_k)$ i.e. there exist more than one edge incident with the corresponding vertex and if $u_i,u_j\in\sigma^{*}$ and $e_i,e_j\in\mu^{*}$.

(iii) $\mu_{NCDL}=\mu(e_i)\wedge\mu(e_j)$ if the edge e_i and e_j have a node in common between them.

(iv) $\mu_{NCDL}=\sigma(u_i)\wedge\mu(e_i)$ if $u_i\in\sigma^{*}$ and $e_i\in\mu^{*}$, each e_i is incident with single u_i either clockwise or anticlockwise.

(v) $\mu_{NCDL}=0$ otherwise.

By definition $\mu_{NCDL}(u,v)\leq\sigma_{NCDL}(u)\wedge\sigma_{NCDL}(v)$ for all u, v in $\sigma^{*}\cup\mu^{*}$. Here μ_{NCDL} is a fuzzy relation on the fuzzy subset σ_{NCDL}. Hence the pair $NCDL(G):(\sigma_{NCDL},\mu_{NCDL})$ is defined as the double layered non-cyclic fuzzy graph (NCDLFG) or 3-D Non-Cyclic Fuzzy Graph.

B. Example for Double Layered Non-Cyclic Fuzzy Graph with 3 Vertices

Consider the fuzzy graph G, whose crisp graph G^{*} is a non-cycle with n = 3 vertices.

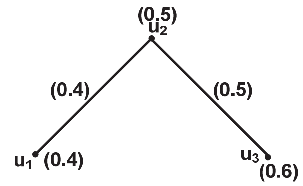

Fig. 1 Fuzzy Graph $G:(\sigma,\mu)$ whose crisp graph $G^{*}:(\sigma^{*},\ \mu^{*})$ is a non-cycle

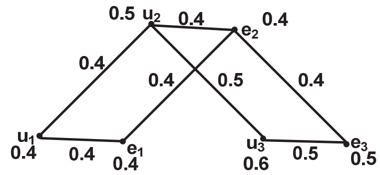

Fig. 2 Double Layered Non-Cyclic Fuzzy Graph NCDL (G) $=(\sigma_{NCDL},\mu_{NCDL})$

C. Example for Double Layered Non-Cyclic Fuzzy Graph with 4 Vertices

Consider the fuzzy graph G, whose crisp graph G^{*} is a non-cyclic with n = 3 vertices.

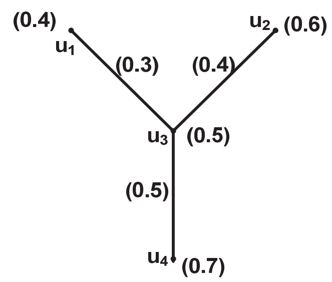

Fig. 3 Fuzzy Graph $G:(\sigma,\mu)$ whose crisp graph $G^{*}:(\sigma^{*},\ \mu^{*})$ is a non-cycle

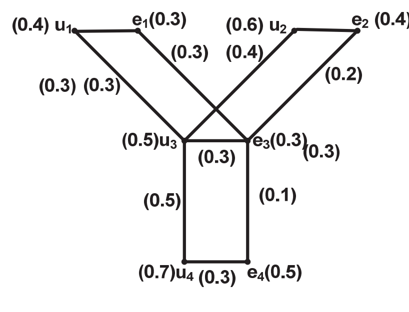

Fig. 4 Double Layered Non-Cyclic Fuzzy Graph NCDL(G)= $(\sigma_{NCDL},\mu_{NCDL})$

D. Example for Double Layered Non-Cyclic Fuzzy Graph with 5 Vertices

Consider the fuzzy graph G, whose crisp graph G^{*} is a non-cyclic with n = 5 vertices.

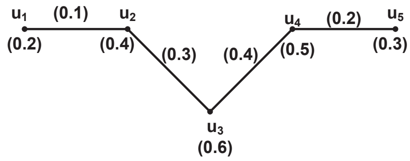

Fig. 5 Fuzzy Graph $G:(\sigma,\mu)$ whose crisp graph $G^*:(\sigma^*, \mu^*)$ is a non-cycle

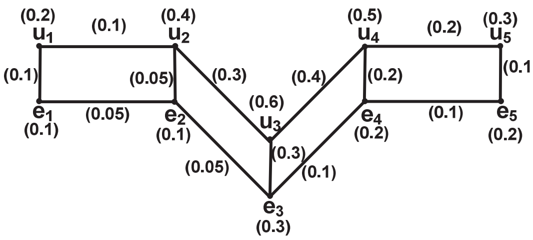

Fig. 6 Double Layered Non-Cyclic Fuzzy Graph NCDL (G) $=(\sigma_{NCDL}, \mu_{NCDL})$

We can obtain different double layered non-cyclic fuzzy graphs for a given fuzzy graph G, whose crisp graph is non-cyclic.

IV. Theoretical Concepts

Theorem: If G is a strong fuzzy graph then NCDL (G) is also a strong fuzzy graph.

Proof: Given G is a strong fuzzy graph, then by definition of strong fuzzy graph we have $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all (u,v) in μ^*.

Now we have to prove that the NCDL (G) is also a strong fuzzy graph. By the definition of NCDL (G) as

(i) $\mu_{NCDL}=\mu(u_i u_j)=e_i$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$, i.e. there is only one edge incident with the corresponding vertex, if $u_i,u_j\in\sigma^*$ and $e_i\in\mu^*$.

(ii) $\mu_{NCDL}=\mu(u_i u_j)\wedge\mu(u_i u_k)=e_i\wedge e_j$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$, e_j is incident with $\sigma(u_i)$ and $\sigma(u_k)$ i.e. there exist more than one edge incident with the corresponding vertex and if $u_i,u_j\in\sigma^*$ *and* $e_i,e_j\in\mu^*$.

(iii) $\mu_{NCDL}=\mu(e_i)\wedge\mu(e_j)$ if the edge e_i and e_j have a node in common between them.

(iv) $\mu_{NCDL}=\sigma(u_i)\wedge\mu(e_i)$ if $u_i\in\sigma^*$ and $e_i\in\mu^*$, each e_i is incident with single u_i either clockwise or anticlockwise.

(v) $\mu_{NCDL}=0$ otherwise.

Case i. $\mu_{NCDL}=\sigma(e_i)$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$. i.e. there is only one edge incident with the corresponding vertex. Since the parental graph is a strong fuzzy graph then it is trivial that $\mu(u,v)=\sigma(u)\wedge\sigma(v)=\mu(uv)=\mu(e_i)$ for all (u,v) in μ_{NCDL}.

Case ii. $\mu_{NCDL}=\sigma(e_i)\wedge\sigma(e_j)$ if e_i is incident with $\sigma(u_i)$ and $\sigma(u_j)$ and e_j is incident with $\sigma(u_i)$ and $\sigma(u_k)$ i.e. there exist more than one edge incident with the corresponding vertex.

Here $\sigma(e_i)=\sigma(u_i)\wedge\sigma(u_j)$ and $\sigma(e_j)=\sigma(u_i)\wedge\sigma(u_k)$, where given the fuzzy graph G is strong we have $\mu(u_i,u_j)=\sigma(u_i)\wedge\sigma(u_j)=\mu(e_i)$. Similarly, $\mu(u_i,u_k)=\sigma(u_i)\wedge\sigma(u_k)=\mu(e_j)$ and hence $\mu_{NCDL}=\mu(e_i)\wedge\mu(e_j)=\mu(e_i e_j)$.

Case iii. $\mu_{NCDL}=\mu(e_i)\wedge\mu(e_j)$ if the edge e_i and e_j have a node in common between them. If $u=e_i, v=e_j$ where $u,v\in\mu^*$ are adjacent in G^* then $\mu_{NCDL}(u,v)=\sigma_{NCDL}(e_i)\wedge\sigma_{NCDL}(e_j)$.

Case iv. $\mu_{NCDL}=\sigma(u_i)\wedge\mu(e_i)$ if $u_i\in\sigma^*$ and $e_i\in\mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise in G^*. Then $\mu_{NCDL}=\sigma_{NCDL}(u_i)\wedge\sigma_{NCDL}(e_j)$ (by the definition of σ_{NCDL}). Hence if G is a strong fuzzy graph, then by Case i, ii, iii and iv we have for $\mu_{NCDL}=\sigma_{NCDL}(u)\wedge\sigma_{NCDL}(v)$ all (u, v) in μ^*_{NCDL}.

1. Order of Non-Cyclic Double Layered Fuzzy Graph

Order NCDL(G)=Order(G)+ $\sum_{e_i\in E}\mu_{NCDL}(e_i)$, where $G=(V,E)$ is a non-cyclic fuzzy graph.

Theorem: Let $G=(V,E)$ be non-cyclic fuzzy graph then

$$\text{Size NCDL(G)=Size(G)} + \sum_{e_i,e_j\in E}\mu(e_i)\wedge\mu(e_i) + \sum_{u_i\in V,e_i\in E}\sigma(u_i)\wedge\mu(e_i).$$

Proof. We know by the definition that size of a fuzzy graph $S(G)=\sum_{u,v\in V}\mu(u,v)$.

$$Size\ NCDL(G)=\sum_{u,v\in V}\sigma_{NCDL}(u)\wedge\sigma_{NCDL}(v) + \sum_{e_i,e_j\in E}\mu_{NCDL}(e_i)\wedge\mu_{NCDL}(e_i) + \sum_{u_i\in V,e_i\in E}\sigma_{NCDL}(u_i)\wedge\mu_{NCDL}(e_i).$$

$$Size\ NCDL(G)=\sum_{u,v\in V}\sigma(u)\wedge\sigma(v) + \sum_{e_i,e_j\in E}\mu_{NCDL}(e_i)\wedge\mu_{NCDL}(e_i) + \sum_{u_i\in V,e_i\in E}\sigma_{NCDL}(u_i)\wedge\mu_{NCDL}(e_i).$$

$$Size\ NCDL(G)=Size(G) + \sum_{e_i,e_j\in E}\mu(e_i)\wedge\mu(e_j) + \sum_{u_i\in V,e_i\in E}\sigma(u_i)\wedge\mu(e_i).$$

Theorem: Let $G = (\mathrm{V}, \mathrm{E})$ be a non-cyclic fuzzy graph then:

$$d_{NCDL(\mathrm{G})} = \begin{cases} d_G(\mathrm{u}) + (\sigma(\mathrm{u}_i) \wedge \mu(\mathrm{e}_i)) \ \text{ if } \mathrm{u} \in \sigma^*. \\ \sum_{e_i \in \mu^*} \mu(\mathrm{e}_i) \wedge \mu(\mathrm{e}_j) + \sigma(\mathrm{u}_i) \wedge \mu(\mathrm{e}_i) \ \text{if } \mathrm{u} \in \mu^*. \end{cases}$$

Proof: We know that the degree of a vertex in a fuzzy graph is given by $d_G(u) = \sum_{\substack{v \neq u \\ v \in V}} \mu(u, v)$ ·

Case i. Let $u \in \sigma^*$, then:

$$d_{NCDL(\mathrm{G})}(\mathrm{u}) = \sum_{v \in \sigma^*} \mu_{NCDL}(\mathrm{u}, \mathrm{v}) + \mu_{NCDL}(\mathrm{u}_1, \mathrm{e}_1)$$
$$= \sum_{v \in \sigma^*} \sigma_{NCDL}(u, v) + \sigma_{NCDL}(u_1) \wedge \mu_{NCDL}(e_1)$$
$$= d_G(u) + \sigma(u_1) \wedge \mu(e_1).$$

Case ii. Let $u \in \mu^*$, then:

$$d_{NCDL(G)}(u) = \sum_{e_i, e_j \in \mu^*} \mu_{NCDL}(e_i, e_j) + \mu_{NCDL}(u_i, e_i)$$
$$= \sum_{e_i, e_j \in \mu^*} \mu(e_i) \wedge \mu(e_j) + \sigma(u_i) \wedge \mu(e_i) \cdot$$

The μ-complement of NCDL (G) is the fuzzy graph G with its edge membership value is always less than G.

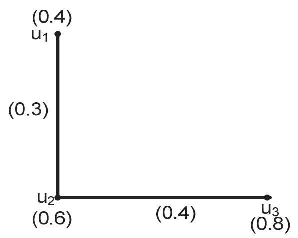

Fig. 7 Fuzzy Graph $G : (\sigma, \mu)$ whose crisp graph $G^* : (\sigma^*, \ \mu^*)$ is a non-cycle

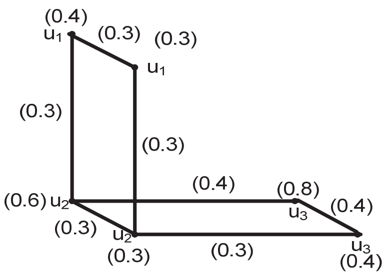

Fig. 8 Double Layered Non-Cyclic Fuzzy Graph NCDL (G) $= (\sigma_{NCDL}, \mu_{NCDL})$

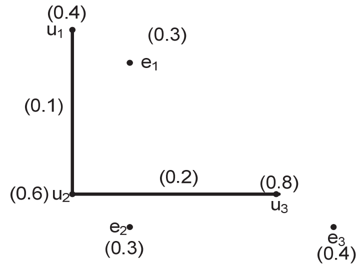

Fig. 9 Complement of Double Layered Non-Cyclic Fuzzy Graph NCDL (G^μ) $= (\sigma^{\mu}{}_{NCDL}, \mu^{\mu}{}_{NCDL})$

If G is a strong fuzzy graph, then the μ-complement of NCDL (G) is the fuzzy graph G with its edge membership value is always less than G.

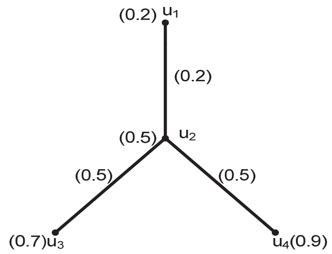

Fig. 10 Fuzzy Graph $G : (\sigma, \mu)$ whose crisp graph $G^* : (\sigma^*, \ \mu^*)$ is a non-cycle

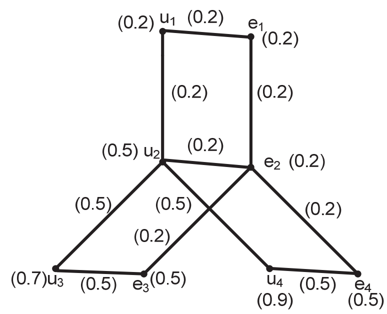

Fig. 11 Double Layered Non-Cyclic Fuzzy Graph NCDL (G) $= (\sigma_{NCDL}, \mu_{NCDL})$

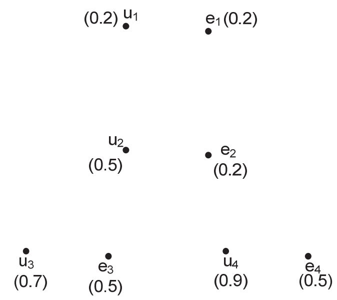

Fig. 12 Complement of Double Layered Non-Cyclic Fuzzy Graph NCDL (G^μ) $= (\sigma^{\mu}{}_{NCDL}, \mu^{\mu}{}_{NCDL})$

V. Conclusion

The 3D-structure for non-cyclic double layered fuzzy graph was obtained and defined in this paper. Some of its properties were derived. Further this can be extended to study its degree of non-membership values. Similar studies can be done for triple layered fuzzy graphs [7]. Its application can be verified in networking structures in future.

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