

# 2-DOF Observer Based Controller for First Order with Dead Time Systems

Ashu Ahuja, Shiv Narayan, Jagdish Kumar

**Abstract**—This paper realized the 2-DOF controller structure for first order with time delay systems. The co-prime factorization is used to design observer based controller  $K(s)$ , representing one degree of freedom. The problem is based on  $H_\infty$  norm of mixed sensitivity and aims to achieve stability, robustness and disturbance rejection. Then, the other degree of freedom, prefilter  $F(s)$ , is formulated as fixed structure polynomial controller to meet open loop processing of reference model. This model matching problem is solved by minimizing integral square error between reference model and proposed model. The feedback controller and prefilter designs are posed as optimization problem and solved using Particle Swarm Optimization (PSO). To show the efficiency of the designed approach different variety of processes are taken and compared for analysis.

**Keywords**—2-DOF, integral square error, mixed sensitivity function, observer based controller, particle swarm optimization, prefilter.

## I. INTRODUCTION

THE dynamics of any industrial process can be described by first order with time delay model. When low order with delay is used to represent higher order system, delay time is used to compensate model reduction [1]. But this dead time complicates both the design of the controller and the implementation of the controller to obtain the control objectives - disturbance rejection and set point tracking simultaneously. To avoid the tradeoff between disturbance rejection and set point tracking two degrees of freedom control was introduced in 1985 [2], [3]. To separate feedforward and feedback characteristics, the observer based feedback controller was introduced from doubly coprime factorization of the plant and bezout components [4]-[6]. When the same parameterization is applied to dead time systems, the generalized smith predictor is formed [7]-[9]. Many authors described various modifications in the smith predictor [10]-[13] but parameterization based on coprime factorization is more advantageous as (i) finite dimensional feedforward part is obtained and (ii) it can be applied to any general dead time process.

But as said earlier, any higher order system model can be reduced to first order with time delay, so, proposed work is applied to only first order with dead time.

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Many authors have reported their work in literature [14]-[17] for these kind of systems. The proposed work shows its efficiency by comparative analysis.

This paper is organized in 5 sections:

In Section II, observer based feedback controller scheme is described to obtain robustness. This is achieved by formulating an optimization problem based on mixed sensitivity and solving it using particle swarm optimization.

In Section III, feed forward controller is designed using model matching problem. Reference model is assumed by the desired set point response. The integral square error (ISE) is minimized using particle swarm optimization.

Section IV takes examples of various kinds of processes. Feed forward and feedback controllers are designed and the response of overall close loop system is compared with [13].

Section V concludes the whole proposed work.

## II. FEEDBACK CONTROLLER

Feedback controller is designed as observer based controller, which is described further:

### A. Observer Feedback Controller

Observer feedback controller based on doubly coprime factorization is used to obtain disturbance rejection and robustness. Consider a single input single output first order plus time delay system

$$G(s) = P(s)e^{-hs} \quad (1)$$

where,  $P(s)$  is delay free first order system and  $h$  is the delay

$$P(s) = \frac{K_p}{Ts+1} = \frac{K_p/T}{s+1/T} = \begin{bmatrix} -1/T & 1 \\ K_p/T & 0 \end{bmatrix} \quad (2)$$

If  $(A,B,C,D)$  are the state space representation of  $P(s)$  and  $F$  and  $L$  are chosen such that stable  $A+BF$  and  $A+LC$  are obtained, then right coprime factors of  $P(s)=NM^l$  can be found out by the relation

$$N = \begin{pmatrix} K_p/T \\ s+a \end{pmatrix} \text{ and } M = \begin{pmatrix} s+1/T \\ s+a \end{pmatrix} \quad (3)$$

where  $-a$  ( $a>0$ ) be the location of  $A+BF$  and  $A+LC$ .

Introducing a rational transfer matrix  $P_0(s) = N_0M^l$  which has no delay

$$P_0(s) = \left[ \begin{array}{c|c} -1/T & 1 \\ \hline K_p/T e^{h/T} & 0 \end{array} \right] = \frac{K_p/T e^{h/T}}{s+1/T}$$

The coprime factorization of  $P_0(s)$  is given by

$$\left[ \begin{array}{cc} M & -Y \\ N_0 & X_0 \end{array} \right] = \left[ \begin{array}{cc|cc} A+BF & B & -e^{Ah}L & \\ \hline F & I & 0 & \\ Ce^{-Ah} & 0 & I & \end{array} \right] \quad (5)$$

Therefore,

$$M = \frac{s+1/T}{s+a}$$

$$N_0 = \frac{K_p/T e^{h/T}}{s+a}$$

$$Y = \frac{(-aT+1)^2 e^{-h/T}}{K_p T (s+a)}$$

$$\text{and } X_0 = \frac{Ts+2aT-1}{T(s+a)} \quad (6)$$

A predictor  $Z(s) \in H^\infty$  is defined such that it gives a finite impulse response by chosen  $P_0(s)$

$$\begin{aligned} Z(s) &= P_0(s) - P(s)e^{-hs} \\ &= N_0 M^{-1} - (N e^{-hs}) M^{-1} \\ &= \frac{K_p/T}{s+1/T} (e^{h/T} - e^{-hs}) \end{aligned} \quad (7)$$

Considering the above parameters and  $F(s)$  as prefilter, 2-DOF controller structure is designed in Fig. 1. The feedback controller is defined as

$$K(s) = (1 - \tilde{K}_0(s)Z)^{-1} \tilde{K}_0(s) \quad (8)$$

where  $\tilde{K}_0(s) = (\tilde{X}_0 + Q\tilde{N}_0)^{-1}(-\tilde{Y} + Q\tilde{M})$  is the set of controllers for  $P_0(s)$  (delay free process).  $\tilde{X}_0, \tilde{Y}$  and  $\tilde{N}_0, \tilde{M}$  are left coprime factorization components of the Bezout identity and process  $P_0(s)$ . Left coprime factorization components are found to be same as right coprime factorization components for first order with time delay systems.

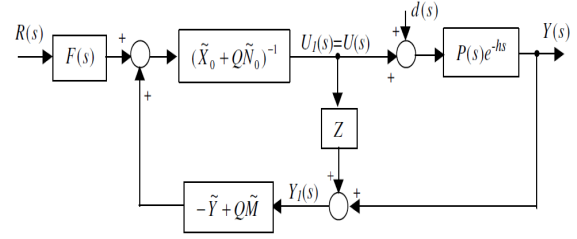


Fig. 1 Controller Parameterization with 2DOF for time delay systems

Another delay free rational transfer matrix  $P_1(s)$  is introduced to obtain a stable finite impulse response block  $Z_1(s)$

$$P_1(s) = \left[ \begin{array}{c|c} A & B \\ \hline Ce^{Ah} & 0 \end{array} \right] = \left[ \begin{array}{c|c} -1/T & 1 \\ \hline K_p/T e^{-h/T} & 0 \end{array} \right] = \frac{K_p/T e^{-h/T}}{s+1/T} \quad (9)$$

$$Z_1(s) = P(s) - P_1(s)e^{-hs} = \frac{K_p/T}{s+1/T} [1 - e^{-(hs+h/T)}] \quad (10)$$

According to Fig. 1, disturbance response is

$$\begin{aligned} G_{yd}(s) &= P e^{-hs} (I - \tilde{K}_0 P_0)^{-1} (I - \tilde{K} Z) \\ &= N (\tilde{X}_0 + \tilde{Y}Z + Q\tilde{N}e^{-hs}) e^{-hs} \\ &= (Z_1 + (P_1 - N\tilde{Y}P + NQ\tilde{N})e^{-hs}) e^{-hs} \end{aligned} \quad (11)$$

$G_{yd}(s)$  has two parts: one is  $Z_1$  (FIR part which acts for  $0 < t < h$ ) and other is  $P_1 - N\tilde{Y}P + NQ\tilde{N}$  (IIR part which acts for  $t > h$ ). The impact of  $Z_1$  on the disturbance response at time  $t > h$  is computed by

$$Z_1(0) = K_p [1 - e^{-h/T}] \quad (12)$$

To make disturbance response zero at  $t > h$ ,

$$Z_1(0) + (P_1 - N\tilde{Y}P + NQ\tilde{N}) = 0 \quad (13)$$

which gives

$$\begin{aligned} Q_{opt}(s) &= -N^{-1} (Z_1(0) + P_1 - N\tilde{Y}P) \tilde{N}^{-1} \\ &= -\frac{1}{K_p} [(1 - e^{-h/T}) T^2 (s+a)^2 + e^{-h/T} (Ts - 1 + 2aT)] \end{aligned}$$

But, ideal disturbance response is not achieved by this  $Q_{opt}(s)$ . To obtain proper  $Q(s)$ , a low pass filter is introduced to  $Q_{opt}(s)$

$$Q(s) = \frac{Q_{opt}(s)}{(\lambda s + 1)^2} \quad (14)$$

where  $\lambda > 0$  is tuned by optimization technique to find the controller and the various responses of the system.

Open loop transfer matrix

$$L(s) = K(s) * G(s)$$

Sensitivity function

$$S(s) = (1 - L(s))^{-1} \quad (15)$$

Complementary transfer function

$$T(s) = 1 - S(s) \quad (16)$$

### B. Fitness Function for Designing Feedback Controller

The fitness function is based on the concept of robust mixed-sensitivity control which is the infinity norm of weighted sensitivity and complimentary sensitivity function. In this control method, the multiplicative weights are taken to formulate the uncertainty which is generated due to changes in the parameters of the plant. In this paper,  $W_1$  is the performance weighting function, which is specified for the disturbance rejection of the system to limit the magnitude of the sensitivity function, and  $W_2$  is the robustness weighting function and is specified for the uncertainty in the plant to limit the magnitude of the complementary sensitivity function. This technique, called loop shaping technique, is widely used for selecting the weight functions for the synthesis of the controller. The cost function can be written in terms of infinity norm as:

$$J = \left\| \begin{matrix} W_1 S \\ W_2 T \end{matrix} \right\|_{\infty} < 1 \quad (17)$$

### III. FEEDFORWARD CONTROLLER

Feedforward controller is designed as fixed structure polynomial controller.

#### A. Polynomial Controller

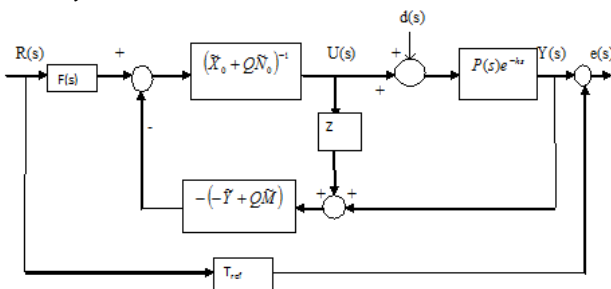


Fig. 2 Model Matching for designing of prefilter F(s)

When  $d(s)=0$ , the transfer function between  $U(s)$  and  $R(s)$  is given by

$$G_{ur} = (I - \tilde{K}_0 P_0)^{-1} (\tilde{X}_0 + Q\tilde{N}_0)^{-1} F \quad (18)$$

Therefore, transfer function between  $Y(s)$  and  $R(s)$  is given by

$$\begin{aligned} G_{yr} &= P(s)e^{-hs} G_{ur}(s) \\ &= NM^{-1}e^{-hs} (I - \tilde{K}_0 P_0)^{-1} (\tilde{X}_0 + Q\tilde{N}_0)^{-1} F \\ &= N(s)F(s)e^{-hs} \end{aligned} \quad (19)$$

In Fig. 2,  $T_{ref}$  is the desired reference model of the closed loop system. The idea is to obtain the feedforward controller  $F(s)$  such that error  $e(t)$  between step response of reference model and step response of  $G_{yr}$  is minimum. For this,  $F(s)$  is chosen to be fixed structure polynomial controller and its coefficients are found out by optimization technique to minimize the cost function defined in Section III B.

$$F(s) = \frac{F(1)s + F(2)}{s + F(3)} \quad (20)$$

### B. Fitness Function for Designing Feedforward Controller

$$\text{Integral square error (ISE)} = \int_0^{\infty} e(t) dt \quad (21)$$

ISE is proposed as cost function for design of  $F(s)$ . Particle Swarm Optimization is used to find out the minimum value of fitness function.

### IV. EXAMPLES

By the various examples, the performance of proposed technique is compared with Nemati and Bagheri (NB) [15] in this section.

#### A. Three Equal Poles

$$G_{p1}(s) = \frac{1}{(s+1)^3}$$

Water control of three tank system is a common example of multiple poles. In order to tune the parameters of the proposed controller  $G_{p1}(s)$  is approximated to first order plus delay time (FOPDT). The methods of reduction are discussed by Sigurd Skogestad [1]. The approximated system of  $G_{p1}(s)$  is given as

$$G_{m1}(s) = \frac{e^{-1.039s}}{2.448s + 1}$$

The closed loop step response with disturbance applied at  $t=25$  sec by the proposed method and NB is shown in Fig. 3.

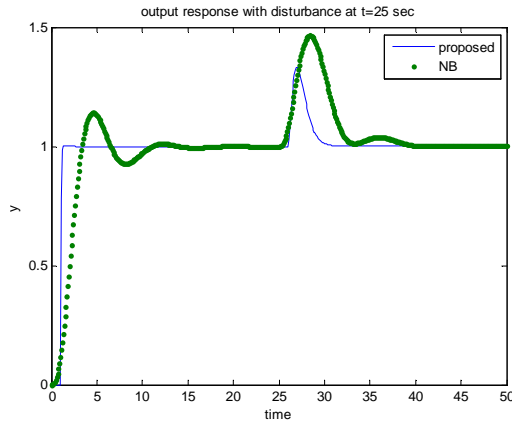


Fig. 3 Output response of proposed and NB method for example 1

The values of various properties of the proposed technique and comparative values are shown in Table I. It is shown that all results are better by the proposed technique.

TABLE I  
COMPARISON OF THE PROPERTIES OF THE PROPOSED AND NB TECHNIQUE  
APPLIED TO  $G_{p1}(s)$

Properties	Proposed	NB
Maximum Sensitivity ( $M_s$ )	0.7324	1.7301
Maximum Complimentary Sensitivity ( $M_T$ )	0.7613	1.1262
Settling Time ( $T_s$ )	1.25	18.7
Overshoot (O)	0	14%
Rise Time ( $T_r$ )	0.576	3
Integral of Absolute Error (IAE)	11.0936	25.4365

*B. Oscillating System*

The disturbances occurring in power system because of changes in load include electro mechanical oscillations of electrical generators. These oscillations are also called power swings and these must be effectively damped to maintain the system stability. Torsional oscillation in electrical drive system with elastic shaft is one another well known problem of oscillatory system. Oscillatory systems have transfer function with complex poles.

$$G_{p2}(s) = \frac{9}{(s+1)(s^2+2s+9)}$$

The approximated FOPDT model of  $G_{p2}(s)$  is

$$G_{m2}(s) = \frac{e^{-0.43s}}{0.825s+1}$$

The closed loop step response with disturbance applied at t=15 sec by the proposed method and NB is shown in Fig. 4.

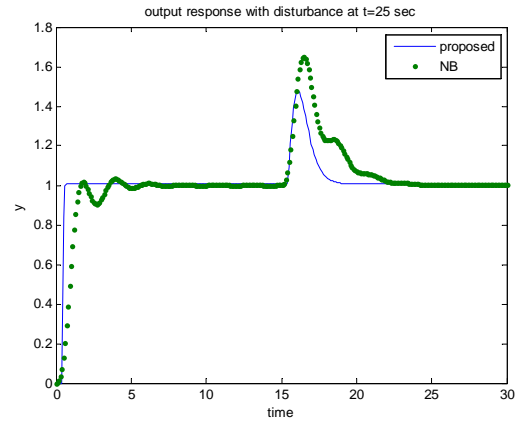


Fig. 4 Output response of proposed and NB method for example 2

The values of various properties of the proposed technique and comparative values are shown in Table II. It is shown that the values of all properties are better in case of proposed method.

TABLE II  
COMPARISON OF THE PROPERTIES OF THE PROPOSED AND NB TECHNIQUE  
APPLIED TO  $G_{p2}(s)$

Properties	Proposed	NB
Maximum Sensitivity ( $M_s$ )	0.1399	1.5456
Maximum Complimentary Sensitivity ( $M_T$ )	0.9975	0.9989
Settling Time ( $T_s$ )	0.708	5.47
Overshoot (O)	0.756%	3.5%
Rise Time ( $T_r$ )	0.307	1.375
Integral Absolute Error (IAE)	2.1980	10.9384

*C. Non Minimum Phase System*

Tank boiler in power plants or heat stations is typical example of non-minimum phase system. To observe the proposed controller effect on non-minimum phase system, the transfer function considered is  $G_{p3}(s)$

$$G_{p3}(s) = \frac{-2s+1}{(s+1)^3}$$

The approximated FOPDT model of  $G_{p3}(s)$  is

$$G_{m3} = \frac{e^{-3.5s}}{1.5s+1}$$

The closed loop step response with disturbance applied at t=25 sec by the proposed method and NB is shown in Fig. 5.

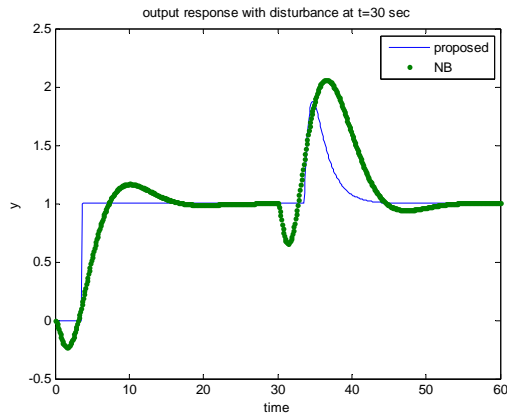


Fig. 5 Output response of proposed and NB method for example 3

The values of various properties of the proposed technique and comparative values are shown in Table III. It is clear that for non-minimum phase systems also proposed technique gives better results than NB.

TABLE III  
COMPARISON OF THE PROPERTIES OF THE PROPOSED AND NB TECHNIQUE  
APPLIED TO  $G_{p3}(s)$

Properties	Proposed	NB
Maximum Sensitivity ( $M_s$ )	0.3959	1.8513
Maximum Complimentary Sensitivity ( $M_r$ )	0.2816	1.0728
Settling Time ( $T_s$ )	3.75	26.7
Overshoot (O)	0.08%	17%
Rise Time ( $T_r$ )	0.2	6.63
Integral Absolute Error (IAE)	15.0035	64.1413

## V. CONCLUSION

This paper presents 2-DOF realization which decouples the disturbance rejection response and set point tracking response. Observer controller is designed as feedback controller and polynomial controller as feedforward controller for first order with time delay systems. The coprime factorization of all stabilizing controllers is presented for various kinds of processes. Results show that the proposed technique is better than the Nemati Bagheri technique.

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