

# Fatigue Life Prediction on Steel Beam Bridges under Variable Amplitude Loading

M. F. V. Montezuma, E. P. Deus, M. C. Carvalho

**Abstract**—Steel bridges are normally subjected to random loads with different traffic frequencies. They are structures with dynamic behavior and are subject to fatigue failure process, where the nucleation of a crack, growth and failure can occur. After locating and determining the size of an existing fault, it is important to predict the crack propagation and the convenient time for repair. Therefore, fracture mechanics and fatigue concepts are essential to the right approach to the problem. To study the fatigue crack growth, a computational code was developed by using the root mean square (RMS) and the cycle-by-cycle models. One observes the variable amplitude loading influence on the life structural prediction. Different loads histories and initial crack length were considered as input variables. Thus, it was evaluated the dispersion of results of the expected structural life choosing different initial parameters.

**Keywords**—Fatigue crack propagation, life prediction, variable loadings, steel bridges.

## I. INTRODUCTION

STEEL bridges usually are subjected to cyclical loads of variable amplitude with random type and frequency of traffic [1]. Due to defects such as cracks, porosities, inclusions, welding defects, and inadequate details inherent to the material, the manufacturing process, or the design, large concentrations of stresses can be produced in these places. Consequently, the fatigue process can begin even in a very early stage of the use of the structure.

The materials used to manufacture steel bridges and structural elements are high-strength steels with a low content of an alloying element (structural steels), which combine mechanical resistance properties with excellent toughness to fracture the material [2]. Steel bridge elements tend to be sensitive to the fatigue process because materials with high mechanical resistance have a low resistance to cracks, and consequently, the remaining resistance in the presence of cracks and defects is small. Thus, one observes that steel bridges, because they are dynamic behavior structures, are subject to failure due to fatigue. Furthermore, these structures can fracture catastrophically under tensions below the highest tensions for which they were designed, thus causing

significant economic damage and risk to human lives.

Several authors [3], [4] have used numerical methods to simulate and predict the behavior of cracks subjected to variable loads. Among the main crack propagation models most used in the literature, the RMS models and the Cycle-by-Cycle model stand out. The study of crack propagations under variable loads requires knowledge in the areas of fatigue and fracture mechanics. In this work, a numerical computational model was developed that considers the propagation of cracks in the specific case of steel bridge beams. Various loading histories of variable amplitude, obtained through numerical simulation, are added to the model as input data. Stationary and non-stationary variable amplitude loads are analyzed.

## II. CRACK PROPAGATION MODEL IN STEEL BRIDGE BEAMS

In this work, design code American Association of State Highway and Transportation Officials – AASHTO HB was used to define the steel bridge beam model. Subsequently, the crack propagation conditions were analyzed, considering different initial sizes of pre-existing defects.

Design of a non-cracked model bridge, the example of a bi-supported bridge with a span of 24,000 mm, was used. According to [5], project development is based on systematic assessments of causes of cracking and fracture in bridge members and their consequent failure. The design code [6] defines flat members of redundant and non-redundant bridges. Redundant fail does not cause the structure to collapse, as the loads are redistributed to adjacent members and alternative paths, which is not the case with non-redundant ones because if they fail in service, they cause the bridge to collapse. In the present work, the beam is non-redundant.

The beam model consists of a welded I-shaped profile, Fig. 1, where the upper flange has a dimension of 360 mm, the lower flange 750 mm, and the web has a total height of 1,400 mm.

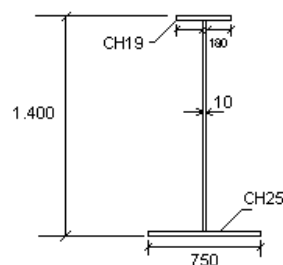


Fig. 1 Cross section of beam with profile I

The geometric properties of the beam are shown in Table I

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and illustrated in Fig. 2. All dimensions satisfy the design requirements of AASHTO HB, such as beam web height (hw), maximum web slenderness (tw) without longitudinal stiffening, thickness of the upper (tf1) and lower (tf2) flange, width of the upper (bf1) and lower (bf2) flange.

TABLE I  
GEOMETRIC PROPERTIES OF THE I-BEAM MODEL

	Lenght (mm)		Area (mm <sup>2</sup> )
Superior flange	tf1 = 19	bf1 = 360	6,840
Beam Web	tw = 10	hw = 1.356	13,560
Inferior flange	tf2 = 25	bf2 = 750	18,750

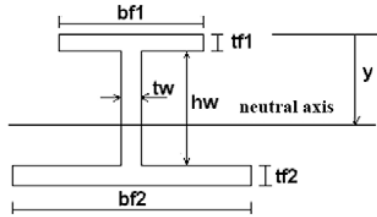


Fig. 2 Geometric definitions of the I-beam model

The design code [6] establishes the necessary limits for the verification of fatigue in redundant and non-redundant structures. Such structures, when subjected to repeated or reverse stress variations, must be designed according to the allowable stresses for cycles and categories, according to the construction details. The stress range  $F_r$  is defined as the algebraic difference between the maximum and the minimum stress ( $\sigma_{max} - \sigma_{min}$ ).

Table II shows the allowable stresses values and the respective number of cycles for non-redundant structures. The beam under study is classified in Category B and applies to a wide variety of details in welded beams, such as groove weld in top connection, located in the core and in the I beam flanges.

TABLE II  
PERMISSIBLE FATIGUE STRESS LIMITS FOR BEAM CATEGORY B

Category	Non-redundant structures - $F_r$ (MPa)			
	Up to 100,000 cycles	Up to 500,000 cycles	Up to 2,000,000 cycles	Above 2,000,000 cycles
B	269	159	110	110

In the model, a load of a class 45 train type [7] was used, in which the system base is a 450 kN vehicle with total weight and impact coefficient equal to 1.26. According to [8] the maximum stress in the lower flange ( $\sigma_r$ ) is calculated considering that two equal beams form the bridge, and the load can thus be divided into 225 kN for each beam. In this case, the bridge is class 45. The result is calculated by (1),  $M_{cm}$  is the moment from the moving load in kN.m, and  $W_m$  is the resistance module of the section in  $m^3$ . Therefore, the stress value found is less than 110 MPa, corresponding to a life prediction greater than 2 million cycles, according to [6]

$$\sigma_r = \frac{M_{cm}}{W_m} = \frac{1.701}{0.02419} = 70.32 \text{ MPa} < F_r \quad (1)$$

In this work, the beam material was A-36 steel, which has a ferritic-pearlitic microstructure, critical stress intensity factor and yield limit, respectively 55 MPa $\sqrt{m}$  and 276 MPa, modulus of elasticity equal to 200 GPa.

To estimate the maximum and minimum limits of the service stress on the lower flange, the minimum load applied was the own weight of the structure ( $P_{min}$ ), and the maximum load to be the own weight plus the weight of the vehicle-class 45 type with the impact factor for the dynamic load ( $P_{max}$ ). So  $P_{min} = 72.36$  kN and  $P_{max} = 355.86$  kN. Therefore, all loads applied to the beam model for crack propagation analysis must be within these limits. The FEM software was used as a tool for stress analysis on beam I. For rounding title, we consider  $P_{min} = 70$  kN and  $P_{max} = 360$  kN. According to the modeling results, it was observed that the stress limits for a beam without crack were:

- $\sigma_{pr min} = 12.1$  MPa (where  $\sigma_{pr min}$  is the minimum design stress);
- $\sigma_{pr max} = 62.3$  MPa ( $\sigma_{pr max}$  is the maximum design stress);
- $\Delta\sigma_{pr} = 50.12$  MPa ( $\Delta\sigma_{pr}$  is the maximum design stress variation).

It was verified that the fatigue design requirements are met according to the standard for bridge design code [6], but it should be noted that this applies to the design of new structures and does not consider the presence of defects found later.

In this model, initially, a crack of length ( $a_0$ ) in Mode I of opening in the beam web is assumed, crossing the lower flange, and positioned in the half-length of the span, as shown in Fig. 3. The fatigue and crack growth process will develop in the beam due to a load ( $P$ ) of variable amplitude. In opening Mode I, there is tensile loading, with the displacement of the crack surfaces perpendicular to themselves.

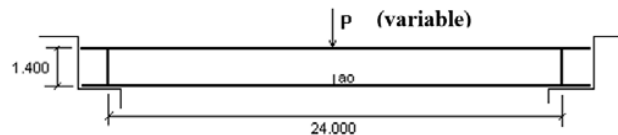


Fig. 3 Bi-supported beam model

The present work uses the Paris-Erdogan rule as a crack propagation rule. This rule correlates the crack growth rate ( $da/dN$ ) with the variation of the stress intensity factor ( $\Delta K$ ), where  $A$  and  $m$  are material constants (2):

$$\frac{da}{dN} = A \Delta K^m \quad (2)$$

Estimates of actual data cited by [3] indicate that fatigue crack growth rates for ferritic-pearlitic steels can be calculated by (3):

$$\frac{da}{dN} = 6.9 \times 10^{-12} (\Delta K)^3 \quad (3)$$

Growth methods using the RMS and cycle-by-cycle method estimate fatigue life under variable amplitude loading. The interaction problems between cycles predicted by [9], which may cause delays or stops in the crack growth, were not included in the model and the effect of wind loads in the fatigue design criterion, as mentioned in [10]. We must note that this model also does not consider the residual stresses in the weld region; this is because it is assumed that the crack originated as a small crack or defect in the weld region and is at an advanced stage where it already has a considerable size in the beam web and outside the influence of residual stresses. It is then desired to predict the lifetime of the structure for a convenient repair.

The RMS method addresses the problem of fatigue life prediction of a part subjected to a load of variable amplitude, replacing it with a load of constant amplitude that is equivalent to it, in the sense of causing the same crack growth. Then the value of the variation of the stress intensity factor ( $\Delta K_{rms}$ ) can be calculated from the RMS values, or the mean square root of the peaks and valleys of the stresses acting on the part. Considering that the negative part of the shipments must be disregarded, the expression:

$$\sigma_{maxrms} = \sqrt{\frac{\sum_{i=1}^p (\sigma_{maxi})^2}{p}} \quad \text{and} \quad \sigma_{minrms} = \sqrt{\frac{\sum_{i=1}^q (\sigma_{mini})^2}{q}} \quad (4)$$

with  $(\sigma_{max}, \sigma_{min} \geq 0)$  p and q are the numbers of peaks and valleys of the loading, respectively; the variation of the stress intensity factor ( $\Delta K_{rms}$ ) is given by (5):

$$(\Delta K_{rms}) = [\Delta \sigma_{rms}] \left[ \sqrt{\pi a} \right] [f(a/w)] \quad (5)$$

Therefore, the forecast of the number of cycles that the crack takes to grow from the initial length ( $a_0$ ) to the end ( $a_f$ ) is given by (6):

$$N = \int_{a_0}^{a_f} \frac{da}{f(\Delta K_{rms})} \quad (6)$$

In this work, the elaboration of the algorithm for the RMS method and life prediction solution is solved using the 4th order numerical Runge-Kutta method for calculating fatigue life.

The Crack Growth Model Using the Cycle-by-Cycle Method associates each reversal of the load with the growth that the crack would have if only that half-cycle acted on the part.

Since the crack propagation rate  $da/dN = f(\Delta K_i)$  and in the i-th 1/2 half load cycle the crack length is  $a_i$ , then the active stress range is  $\Delta \sigma_i$  and the crack increment  $\delta a_i$  can be given by (7) and the variation of the stress intensity factor ( $\Delta K_i$ ) by (8):

$$\delta a_i = (1/2) \cdot f(\Delta K_i) \quad (7)$$

$$\Delta K_i = f(a_i/W) \Delta \sigma_i \sqrt{\pi a_i} \quad (8)$$

where,  $f(a_i/W)$  is the factor that depends on the geometry of the part.

Crack growth is quantified by  $\sum (\delta a_i)$ ; however, before quantifying the crack growth, reducing the loading history to a sequence of events can be estimated as compatible with constant amplitude fatigue data. The methods that make such reductions possible are known as cycle counting methods.

In this work, we will use the cycle counting method called "rain-flow". This method uses a specific cycle counting scheme to estimate the order of effective stresses and to identify stress cycles related to closed hysteresis loops in the stress-strain response of the material when subjected to cyclic loading.

Implementing an algorithm of the cycle-by-cycle method is not numerically difficult, but it requires much computational effort. Nevertheless, it has the advantages of ensuring crack inactivity when in a  $\Delta K_i < \Delta K_{th}$  cycle and predicting the sudden fracture caused by a prominent peak during variable loading when  $K_{max} = K_c$ . In this work, an algorithm was developed in Fortran language to quantify the crack growth by the cycle-by-cycle method.

To find an expression for calculating the stress intensity factor KI. It is considered that the crack propagates in the core of beam I, where a height ( $hw$ ) is observed that is well above its thickness ( $tw$ ) and about 136 times. For this geometry, the web is in a plane stress condition.

For beams with single edge crack under bending, the stress intensity factor can be deduced from (9):

$$KI = (Mc/I) \cdot (\pi a)^{1/2} \cdot f(a/hw) \quad (9)$$

where M is the bending moment, c is the distance from the crack tip to the neutral axis, and I is the moment of inertia of the cross-section, and  $f(a/hw)$  is the geometry factor. This expression can be developed for the case of a T-shaped cross-section seen in Fig. 4 since, in the present model, the initial crack is assumed initially in the beam web.

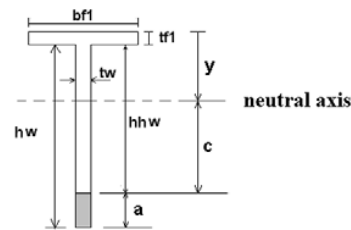


Fig. 4 Geometry of the Web cracked beam

### III. RESIDUAL LIFE SIMULATIONS OF THE STRUCTURE

For purposes of comparison, results of all life calculations were made assuming the structure of initial cracks equal to sizes 10 mm, 20 mm, 30 mm, 40 mm, and 50 mm in length from the lower flange. The structure's life was related to the number of cycles that the crack takes to grow from the initial size to the critical final size  $a_c = 95$  mm. Two types of variable loads were applied to the cracked beam model, defined by [11] as stationary (Stationary Variable Amplitude

Loading - CAVE) and random (Random Variable Amplitude Loading - CAVR), where for each type 10 increasing load levels were applied ( $\Delta P = P_{\max} - P_{\min}$ ). Finally, the growth behavior was observed by two methods described previously, RMS and cycle-by-cycle.

In the generation of Stationary Variable Amplitude Loads (CAVE), (10) is used, which is a solution to the beat phenomenon, Fig. 5. The repetition of loading cycles characterizes them over time. Although they are not typical loads to be found in bridge structures, their analysis is opportune to predict the growth of cracks under such atypical everyday conditions and compare them with other loads.

$$P = P_m + P_{\text{ampl}} \sin(\omega_1 N) \sin(\omega_2 N) \quad (10)$$

P is the load;  $P_m$  is the average request load;  $P_{\text{ampl}}$  is the loading range, N is the number of cycles, and  $\omega_1$  and  $\omega_2$  are variable. Thus, the movement is an oscillation with an amplitude that varies according to (11):

$$\text{Amplitude} = P_{\text{ampl}} \sin(\omega_1 N) \quad (11)$$

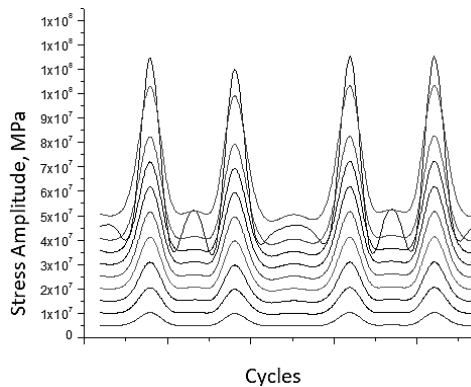


Fig. 5 Variation of stress amplitude (MPa) x cycles (CAVE)

A random variable amplitude loading (CAVR) can be considered a type of loading where there is no repetition in the loading cycles and the amplitude varies randomly with time. In the simulation of these types of loads, (12) is used, whose loading amplitude is randomized in the presence of random numbers generated by the program after each cycle, Fig. 6:

$$P = P_p + \text{rand}(P_{\text{ampl}}) \quad (12)$$

P is loading;  $P_p$  is its weight.  $P_{\text{ampl}}$  is the loading range, "rand" is a random number generated by the program for each cycle, and  $0 \leq \text{rand} \leq +1$ .

The generation of this random sequence depends on three source numbers, and for each different sequence of these numbers, a different random sequence with a certain number of cycles is also generated.

#### IV. RESULTS

The growth curves of the initial size cracks ( $a_0$ ) up to the critical value ( $a_c$ ) for each of the loading blocks used are

presented. An advantage was observed in the RMS model for constructing the graphs, Figs. 7 and 8, due to the smaller number of data needed to plot the curves. With the use of the Cycle-by-Cycle model in the crack growth simulations, Fig. 9, a great computational effort was observed to calculate the life of the structure; however, the model has the advantages of guaranteeing the inactivity of the crack when in a cycle  $\Delta K_i < \Delta K_{th}$  and predicts the sudden fracture caused by a significant peak during variable loading when  $K_{\max} = K_c$ . On the other hand, an increase in the accuracy of the results does not justify using the model compared to the RMS model.

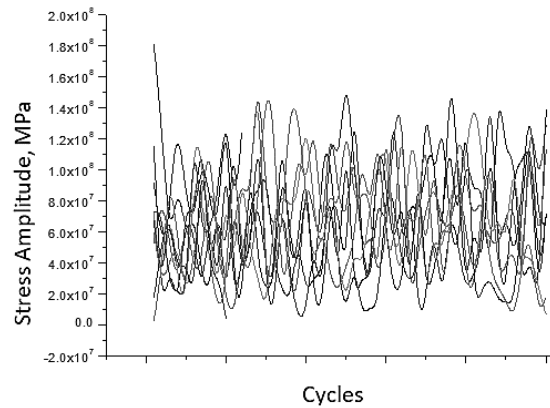


Fig. 6 Variation of stress amplitude (MPa) x cycles (CAVR)

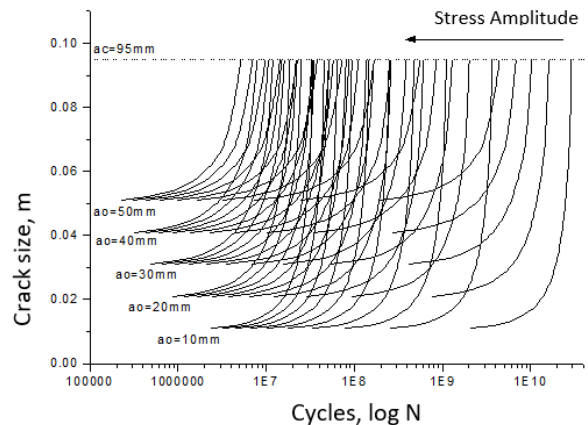


Fig. 7 Crack size x number of cycles for different initial crack lengths when using the RMS method for stationary variable amplitude loading (CAVE)

Other plotted graphs were those of the structure's life versus stress amplitude, Figs. 10 and 11. Through these graphs, it is observed that the greater the stress amplitude, the smaller the dispersion of the structure's life results for different initial crack lengths.

#### V. CONCLUSIONS

In this work, one observes crack propagation behavior in steel bridge beams when subjected to variable amplitude loading. Through the theory of Fatigue and Fracture

Mechanics, it was observed that the problem of crack propagation requires correct analysis of stresses at the critical point of the structure. Then, a computational model of crack propagation was developed, using the theory of the RMS and Cycle-by-Cycle.

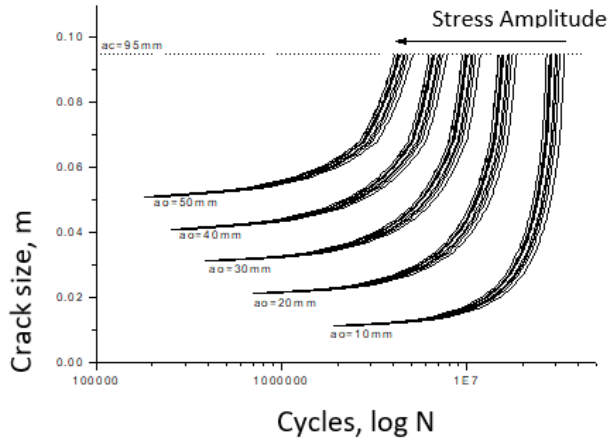


Fig. 8 Crack size x number of cycles for different initial crack lengths when using the RMS method for random variable amplitude loading (CAVR)

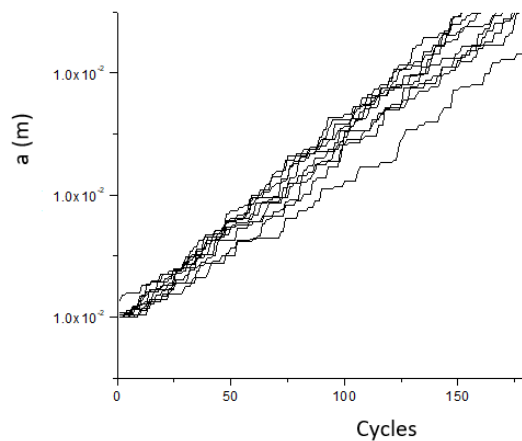


Fig. 9 crack growth with  $a_o = 10$  mm subjected to CAVR by the cycle-by-cycle method during the first 170 cycles

It was observed that to carry out a good simulation; the stress intensity factor must be chosen appropriately for the specific type of geometric detail and type of effort required.

In the results of the simulations, it was observed that the loadings of variable amplitudes with different spectra could have the same influence on the crack propagation when evaluated by the RMS model because they have similar variations in stress amplitude  $\Delta\sigma_i$  over time. It was also found that the magnitude of the variation in the stress amplitude  $\Delta\sigma_i$  has a significant influence on the crack propagation results, which may remain inactive for certain stress levels.

There was a natural tendency for the structure's life to decrease with the increase in the size of the initial cracks and the increase in the RMS stress amplitude ( $\Delta\sigma_{rms}$ ). It has been

found that cracks of different initial sizes can take the same time to grow to a critical  $a_c$  value if they are subject to different levels of stress amplitude.

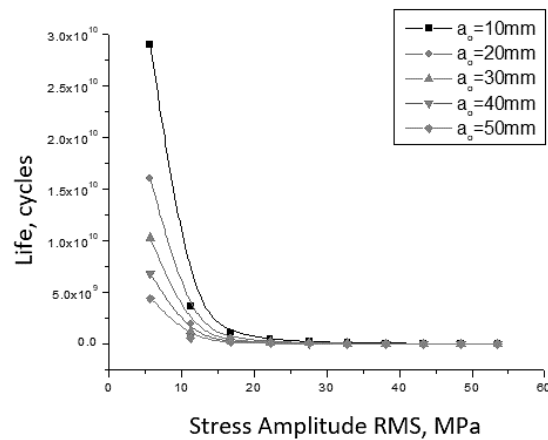


Fig. 10 Structure life x RMS stress amplitude for different initial crack sizes (CAVE)

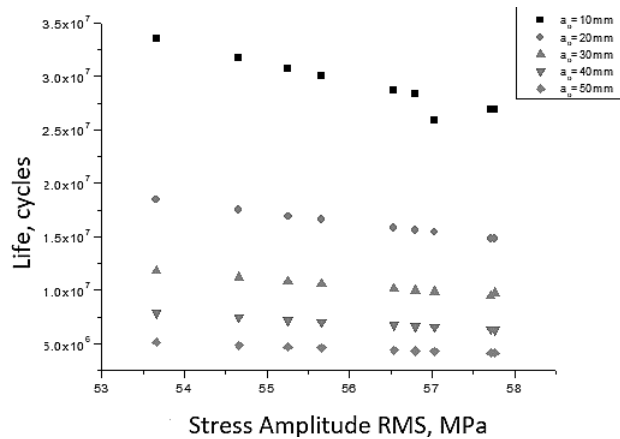


Fig. 11 Structure life x RMS stress amplitude for different initial crack sizes (CAVR)

Analyzes of the life prediction in the simulations showed that the greater the stress amplitude, the smaller the dispersion of the expected life results of the structure for different initial crack sizes. Above a certain level of the stress amplitude, there was a tendency for the dispersion of the structure's life results to remain constant for different sizes of initial cracks.

During the simulations, it was verified that the RMS model has the advantage of being computationally more efficient about the Cycle-by-Cycle; however, this does not predict the crack inactivity for low values of the stress intensity factor or sudden rupture due to overload.

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