

# Investigation of the Unbiased Characteristic of Doppler Frequency to Different Antenna Array Geometries

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**Abstract**—Array signal processing techniques have been recently developing in a variety application of the performance enhancement of receivers by refraining the power of jamming and interference signals. In this scenario, biases induced to the antenna array receiver degrade significantly the accurate estimation of the carrier phase. Owing to the integration of frequency becomes the carrier phase, we have obtained the unbiased doppler frequency for the high precision estimation of carrier phase. The unbiased characteristic of Doppler frequency to the power jamming and the other interference signals allows achieving the highly accurate estimation of phase carrier. In this study, we have rigorously investigated the unbiased characteristic of Doppler frequency to the variation of the antenna array geometries. The simulation results have efficiently verified that the Doppler frequency remains also unbiased and accurate to the variation of antenna array geometries.

**Keywords**—Array signal processing, unbiased Doppler frequency, GNSS, carrier phase, slowly fluctuating point target.

## I. INTRODUCTION

**C**ALIBRATION techniques for the antenna array receiver allow measuring the carrier phase with the high-accuracy precision. Of various interesting and exciting methods in the area of antenna array calibration [1]-[3], array signal processing techniques have been proposed in this work for the performance enhancement of receivers by refraining the power of jamming and interferences signals.

Since biases are induced in the antenna array receiver, the main practical challenge resides in degrading the accuracy of the carrier phase estimation and thereby in creating the phase ambiguities. Indeed, biases are induced in the antenna array receiver due to two of the underlying scenarios; (1) the carrier frequency characteristic of the transmitted signals, (2) distortions of the transmitted signals resulted from multipath and other interference signals. Hence, the distortion of the carrier phase has an unpredictable result and it is also highly sensitive to technological imperfections such as non-ideal antennas and the presence of interference signals [3].

Since the integration of frequency is carrier phase, the unbiased characteristic of Doppler frequency brings a merit for the accurate estimation of carrier phase for calibrating the antenna array receiver without employing extra apparatus or performing complex algorithms [1]-[8]. The Doppler frequency indeed remains unbiased and accurate to the power

of jamming and the other interference signals, as described in detail in [1]-[8], and thereby it allows moderating the biases as well as calibrating the antenna array receiver efficiently. On the one hand, this study has a major contribution to address the question of whether the Doppler frequency remains unbiased to the variation in antenna array geometries or not.

The results of this study have been obtained assuming the antenna array, as a receiver, is implemented for an active radar to transmit the signals at the frequency of interest of 300 MHz. Moreover, the numerical computations of the Doppler frequency estimation for the different antenna array geometries have been extensively fulfilled assuming the transmitted signals have a very low power, ensuring the robustness and accuracy of the Doppler frequency estimation even in a very low power circumstance.

## II. SIGNAL PROCESSING MODEL FOR THE DOPPLER FREQUENCY ESTIMATION

We have proposed the model of detection of a slowly fluctuating point target, described in details in [9], for the Doppler frequency estimation for the different antenna array geometries. The term of “slowly fluctuating” in this model refers to the target whose characteristics, including shape, motion, location and etc., are assumed to be fixed during illumination process, which is accompanied by the constant Doppler shift during time of illumination.

It is assumed that the monostatic active radar transmits a cosine wave continuously with the average power of  $P_t$  in [10]-[13]:

$$S_t(t) = \sqrt{2P_t} \cos \omega_c t = \sqrt{2} \operatorname{Re}[\sqrt{P_t} e^{j\omega_c t}] \quad -\infty < t < +\infty \quad (1)$$

The total received signals, including the reflected signals from the target, additive noise and some other return signals, as demonstrated in Fig. 1, can be expanded in:

$$r(t) = \sqrt{2} \operatorname{Re}[\tilde{r}(t) e^{-i\omega_c t}] \quad (2)$$

where

$$\tilde{r}(t) \triangleq \tilde{b} \sqrt{P_t} \tilde{f}(t - \lambda) e^{-i\omega_D t} + \tilde{n}(t) \quad (3)$$

Moreover,  $\tilde{b}$ , whose phase is uniformly distributed, refers to the complex Gaussian random variable. The envelope of  $|\tilde{b}|$  represents the Rayleigh random variable with a uniform phase.

The complex envelope of the transmitted signals,  $\tilde{f}(t)$ , in (3) can be normalized as:

$$\int_{-\infty}^{+\infty} |\tilde{f}(t)|^2 dt = 1 \quad (4)$$

The additive Gaussian noise of  $n(t)$  with a bandpass spectrum can be expanded as:

$$n(t) = \sqrt{2} \text{Re}[\tilde{n}(t)e^{-i\omega_c t}] \quad (5)$$

Consequently, as described in detail in [9], Doppler shift, which refers to the shift in the carrier frequency,  $\omega_c$ , is expressed in the following form,

$$\omega_D \triangleq \omega_c \left(\frac{2v}{c}\right) \quad (6)$$

where  $v$  refers to the velocity of the target in meters/second:

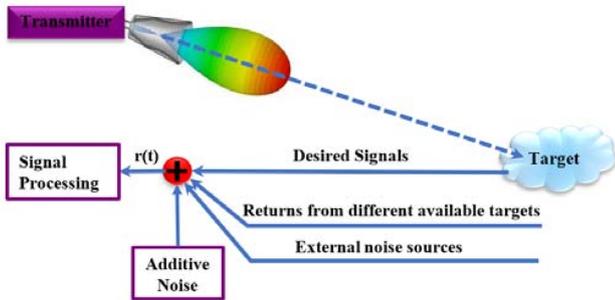


Fig. 1 Model for an active Radar or Sonar System

### III. VALIDATION AND VERIFICATION FOR THE DOPPLER FREQUENCY ESTIMATION

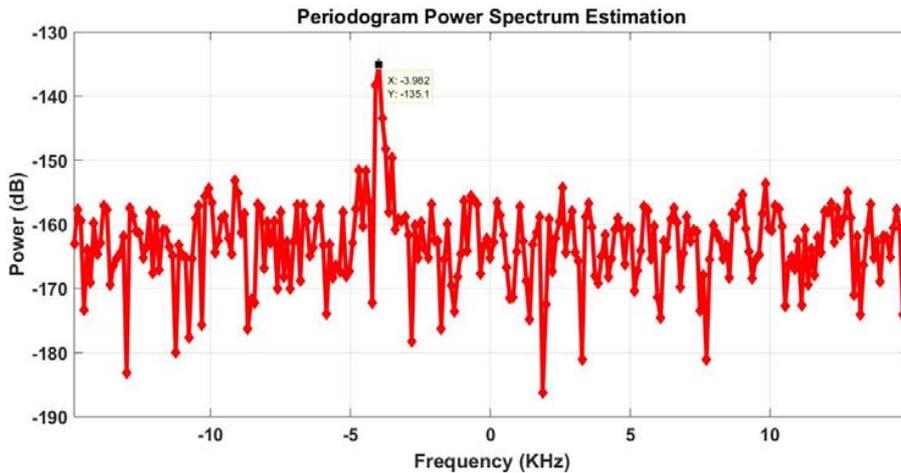
The discussion in the previous section focused on representing theoretical framework for the Doppler frequency estimation. In this section, we aim at investigating the unbiased and accurate characteristic of the Doppler frequency to the variation of the antenna array geometries. Furthermore, the main advantage of this technique over the different available techniques [1]-[8] consists of the accuracy and robustness in the Doppler frequency estimation even in a very low power that we aim at stressing in this section.

Figs. 2-5 and Table I represent analogous results of the Doppler frequency estimation for the different antenna array geometries for the two typical moving targets under the following initial circumstances;

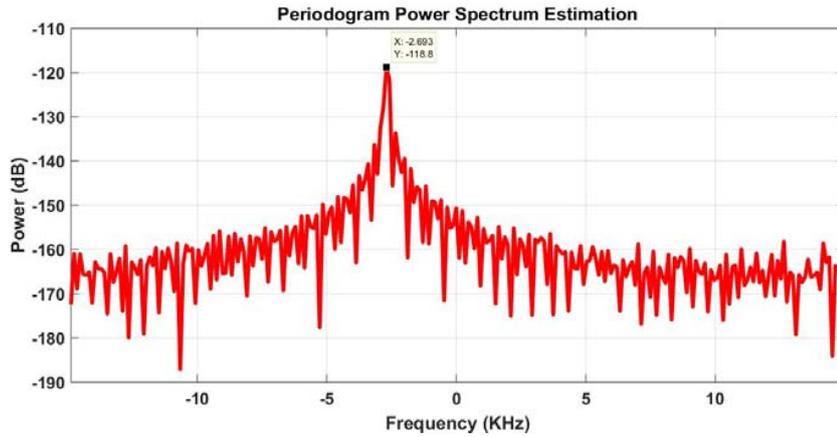
$$\text{Target 1: } \begin{cases} \text{Initial position: } [3500; 0; 0] \\ \text{Speed of the target: } [120; 100; 0] \text{ m/s} \end{cases}$$

$$\text{Target 2: } \begin{cases} \text{Initial position: } [2000; 0; 0] \\ \text{Speed of the target: } [80; 0; 40] \text{ m/s} \end{cases}$$

Three main features can be underlined from Figs. 2-5 and Table I; (1) since the simulation results have been obtained in the presence of a very low power, therefore, the proposed approach of the slowly fluctuating point target represents the extreme strength and robustness of the Doppler frequency estimation, (2) targets can be precisely detected by the different Doppler frequencies and spatial displacement due to they move with the two distinct speeds and initial positions, (3) the Doppler frequency is not very much affected by the antenna array geometries. In other words, we have rigorously verified the unbiased characteristic of Doppler frequency to the variation of the antenna array geometries.

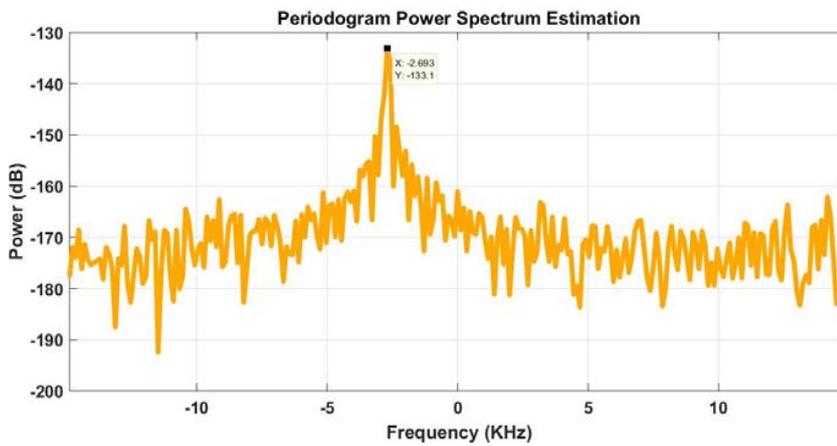


(a)

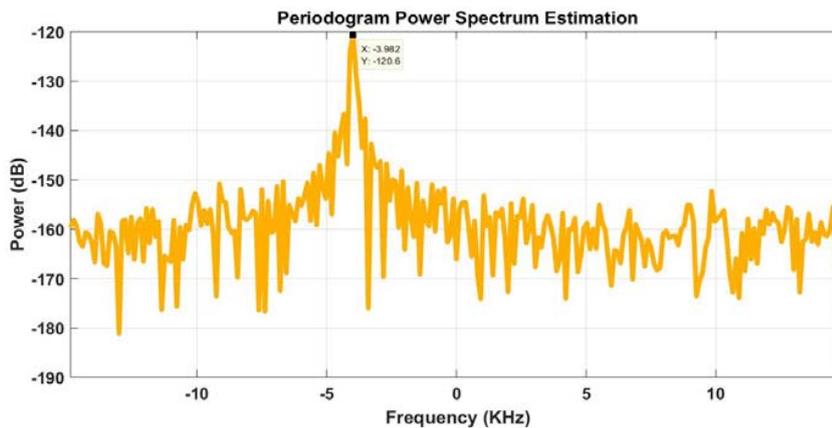


(b)

Fig. 2 Variation of Doppler frequency strength versus frequencies for the circular antenna array geometry [14], [15]. The carrier frequency is assumed to be  $5 \times 10^9$ , but the frequency of operation for the circular antenna array is 300 MHz. The circular array consists of 16 array elements with the radius of  $\lambda m$ . (a) for the target 1, (b) for the target 2

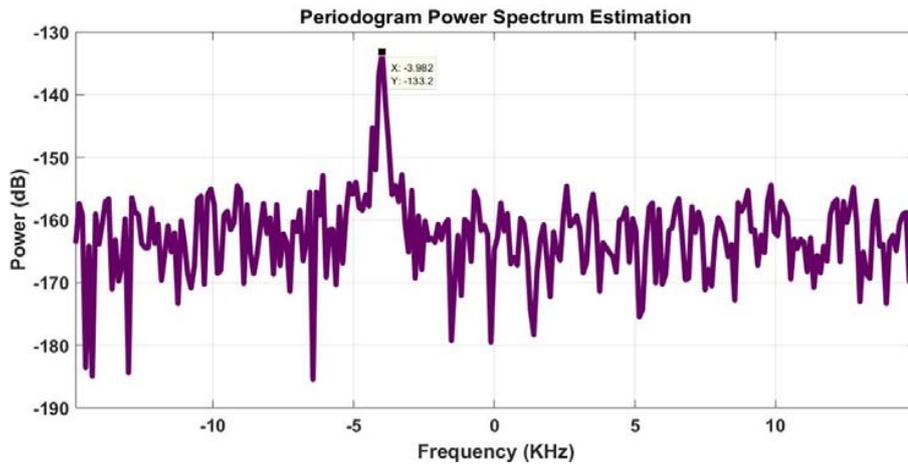


(a)

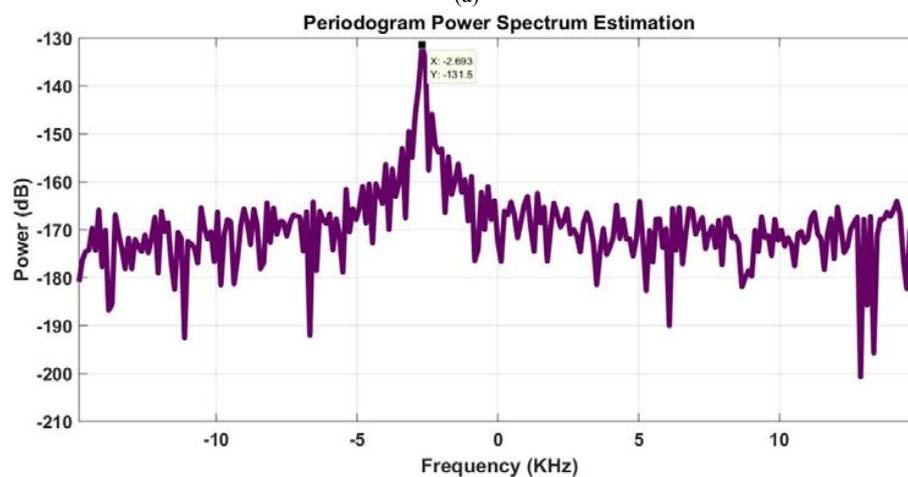


(b)

Fig. 3 Variation of Doppler frequency strength versus frequencies for the cylindrical antenna array geometry [16]. The carrier frequency is assumed to be  $5 \times 10^9$ , but the frequency of operation for the circular antenna array is 300 MHz. The cylindrical antenna array involves five concentric circular subarrays. Each circular subarray is composed of four array elements and has the radius of  $2\lambda m$ . (a) for the target 1, (b) for the target 2

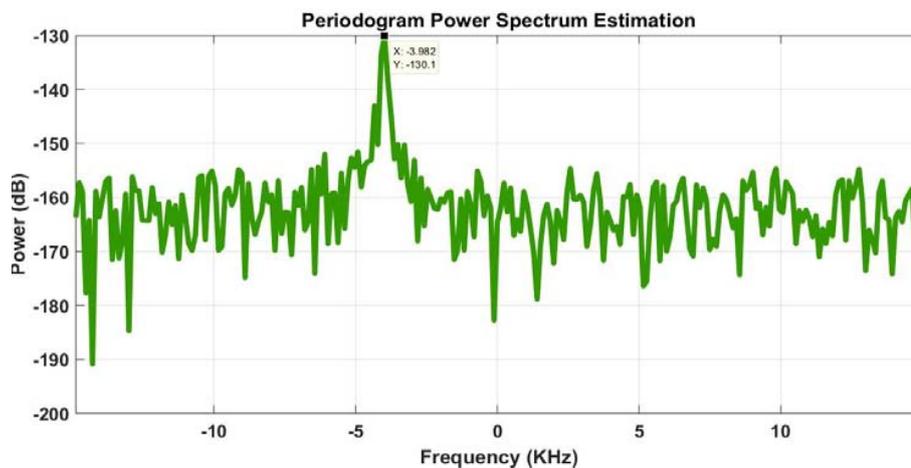


(a)



(b)

Fig. 4 Variation of Doppler frequency strength versus frequencies for the hybrid antenna array, [17]. The carrier frequency is assumed to be  $5 \times 10^9$ , but the frequency of operation for the circular antenna array is 300 MHz. The cylindrical antenna array is composed of four concentric circular subarrays, however with the different radii of  $\lambda$ ,  $0.75\lambda$ ,  $0.5\lambda$  and  $0.25\lambda$ m, respectively from the top to the bottom plane. Moreover, the bottom plane indeed consists of four concentric subarrays with radii of  $\lambda$ ,  $0.75\lambda$ ,  $0.5\lambda$  and  $0.25\lambda$ m. There are a number of 56 array elements in the cylindrical antenna array. (a) for the target 1, (b) for the target 2



(a)

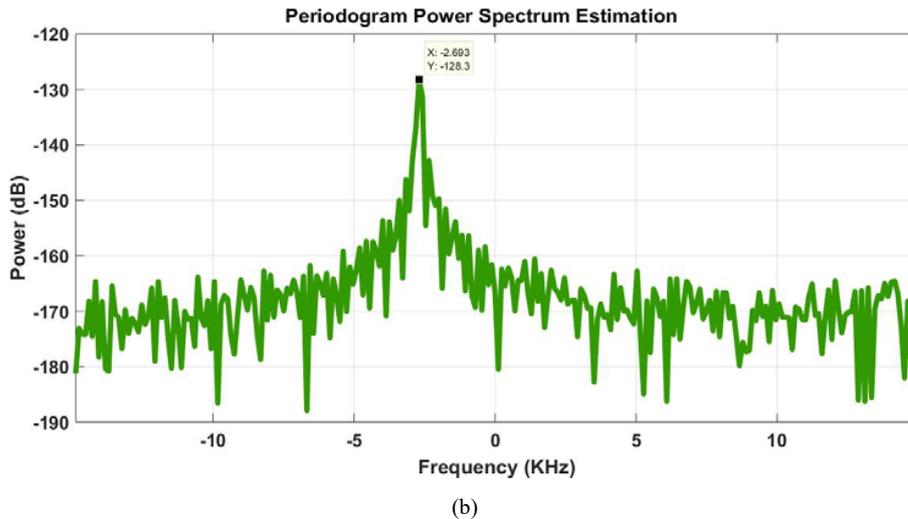


Fig. 5 Variation of Doppler frequency strength versus frequencies for the conical antenna array [18]. The carrier frequency is assumed to be  $5 \times 10^9$ , but the frequency of operation for the circular antenna array is 300 MHz. The conical array is composed of five concentric circular subarrays. Each circular subarray involves eight array elements. (a) for the target 1, (b) for the target 2

TABLE I  
QUANTITATIVE COMPARISON OF THE EFFECT OF THE DIFFERENT ANTENNA ARRAY GEOMETRIES ON THE DOPPLER FREQUENCY CONSISTENT WITH FIGS. 2-5

Antenna array Geometry	The Doppler frequency location for the target 1 on the Radar screen, (x, y):	The Doppler frequency location for the target 2 on the Radar screen, (x, y):
Circular antenna array	(-3.982, -135.1)	(-2.693, -118.8)
Cylindrical antenna array	(-3.982, -120.6)	(-2.693, -133.1)
Hybrid antenna array	(-3.982, -133.2)	(-2.693, -131.5)
Conical antenna array	(-2.693, -128.3)	(-3.982, -130.1)

#### IV. CONCLUSION

To conclude, in this study, we have concisely outlined and reviewed the slowly fluctuating point target model for the Doppler frequency estimation. We have presented the simulation results on the Doppler frequency estimation for the different antenna array geometries.

In general, it can be challenging to measure the phase carrier with high accuracy for calibrating antenna array receiver, since the distortion of the transmitted signals is quite unpredictable. The carrier phase is also very sensitive to the technological imperfections of antenna array receivers.

Since, the integration of carrier phase gets Doppler frequency; thereby we have estimated Doppler frequency rather than carrier phase due to unbiased characteristic of Doppler frequency to the imperfections, jamming, and other interferences. In this study, we have rigorously verified that the Doppler frequency stays unbiased and accurate to the variation of array geometries of receivers.

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