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State Estimation Method Based on Unscented Kalman Filter for Vehicle Nonlinear Dynamics

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Abstract—This paper provides a state estimation method for automatic control systems of nonlinear vehicle dynamics. A nonlinear tire model is employed to represent the realistic behavior of a vehicle. In general, all the state variables of control systems are not precisedly known, because those variables are observed through output sensors and limited parts of them might be only measurable. Hence, automatic control systems must incorporate some type of state estimation. It is needed to establish a state estimation method for nonlinear vehicle dynamics with restricted measurable state variables. For this purpose, unscented Kalman filter method is applied in this study for estimating the state variables of nonlinear vehicle dynamics. The objective of this paper is to propose a state estimation method using unscented Kalman filter for nonlinear vehicle dynamics. The effectiveness of the proposed method is verified by numerical simulations.

Keywords—State estimation, control systems, observer systems, unscented Kalman filter, nonlinear vehicle dynamics.

I. Introduction

In recent years, several control problems of vehicle dynamics such as collision avoidance [1], rollover prevention [2], wheel slip control [3], driver assistance control [4] have been investigated. In particular, model predictive control (MPC) method is applied to solving the stabilization problem of vehicle nonlinear dynamics to avoid the second collision accident [5]. MPC is a well-established control method in which the current control input is obtained by solving a finite horizon open-loop optimal control problem using the current state of the system as the initial state [6]–[8]. However, the control methods proposed in the above papers are inapplicable to systems whose all state variables are not exactly known.

In general, it is usual that the state variables of systems are measured through output sensors. Thus, only limited parts of them can be used for designing control inputs. In other words, it is usual that all the state variables of control systems are not exactly known, because those variables are observed through output sensors and limited parts of them might be only observable. Therefore, automatic control systems must incorporate some type of state estimation. In order to apply the MPC method to the automatic control systems for nonlinear vehicle dynamics, we need to establish a state estimation method for nonlinear vehicle dynamics with limited measurable state variables.

The objective of this study is to establish a state estimation method for nonlinear vehicle dynamics. For this purpose, we introduce an observer system for estimating the state variables of nonlinear vehicle dynamics. Kalman filter is a well-known optimal estimation method which enables us to minimize estimation errors with taking the process noise and sensor noise into account. The application of the Kalman filter to nonlinear systems has been well investigated in recent decades. The simple approach is to use the Extended Kalman Filter (EKF) [9] which simply linearizes nonlinear models so that the traditional linear Kalman filter can be applied. However, the EKF is only reliable for systems which are almost linear on the time scale of the update intervals. On the other hand, the different method is to use the Unscented Kalman Filter (UKF) [10] that uses a set of appropriately chosen weighted points to parameterize the means and covariances of probability distributions. The state estimator using UKF yields better performance than the one using EKF for control systems with high nonlinearities. Consequently, the objective of this study is to propose a state estimation method based on the UKF for nonlinear vehicle dynamics.

This paper is organized as follows. In Section II, we define the system model and notations. In Section III, we consider the state estimation problem of nonlinear vehicle dynamics. In Section IV, we provide the results of numerical simulations that verify the effectiveness of the proposed method. Finally, some concluding remarks are given in Section V.

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II. Notation and System Model

In this section, we introduce a vehicle system model under the following assumptions. First, we assume that the difference between the vertical loads of the left and right wheels is negligible. Second, we assume that rolling and pitching motions are negligible. Finally, we assume that the rear tires are not steered. Under those assumptions, the cornering forces of the left and right wheels are equal each other. Thus, a four-wheeled vehicle model can be regarded as a two-wheeled vehicle model. In this study, we consider a two-wheeled vehicle model as shown in Fig. 1, which is equivalent to a four-wheeled vehicle model. The system parameters used in this model are listed in Table I.

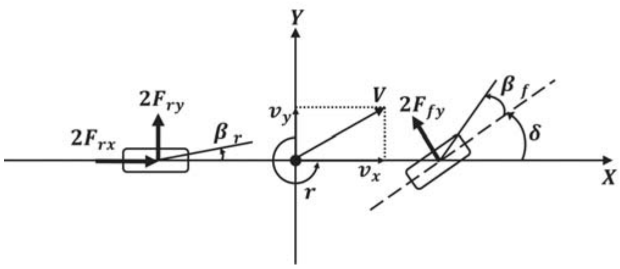

Fig. 1 System Model

First, the equation of motion in the longitudinal direction is described as

$$m(\dot{v_x} - rv_y) = 2F_{rx} - 2F_{fy}\sin(\delta) - C_D A_D v_x^2, \tag{1}$$

and the equation of motion in the lateral direction is described as

$$m(\dot{v_y} + rv_x) = 2F_{ry} + 2F_{fy}\cos(\delta). \tag{2}$$

Furthermore, the equation of rotational motion is described as

$$I_{zz}\dot{r} = 2F_{fy}l_f\cos(\delta) - 2F_{ry}l_r. \tag{3}$$

TABLE I
SYSTEM PARAMETERS

m	vehicle mass
g	gravitational acceleration
μ	frictional coefficient
I_{zz}	vehicle moment of inertia around z axis
l_f	center of mass distance to the front axle
l_r	center of mass distance to the rear axle
C_D	aerodynamic drag coefficient
A_D	effective aerodynamic drag area
F_{rx}	driving and braking force
δ	steering angle of front wheel

The angle between the directions of movement and rotation of the tires is called the slip angle of the tires. The slip angles of the front and rear tires β_f and β_r are given by

$$\beta_f = \tan^{-1}\left(\frac{v_y + l_f r}{v_x}\right) - \delta, \tag{4}$$

$$\beta_r = \tan^{-1}\left(\frac{v_y - l_r r}{v_x}\right). \tag{5}$$

In the range where the slip angle is sufficiently small, the lateral force increases in portion to the slip angle. However, the lateral force will saturate and decrease from the maximum value when the slip angle increases beyond a certain value. In other words, the lateral force increases approximately linearly for the first few degrees of slip angle, and then increases non-linearly to a maximum before beginning to decrease. In order to take more realistic tire model into account, we introduce a nonlinear tire model called Magic Formula [11] as follows:

$$F_{yf} = -\frac{1}{2}\mu m g \sin(H \tan^{-1}(\beta_f)), \tag{6}$$

$$F_{yr} = -\frac{1}{2}\mu m g \sin(H \tan^{-1}(\beta_r)). \tag{7}$$

Magic Formula is an empirical formula obtained from experimental data. It is difficult to interpret the formula physically. However, it is more accurate than linear tire model. H is a constant determined to represent the experimental data [11].

Consequently, the equation of motion for nonlinear vehicle dynamics can be described by

$$\dot{v_x} = \frac{2F_{rx} - 2F_{fy}\sin(\delta) - C_D A_D x_1^2 + m r v_y}{m}, \tag{8a}$$

$$\dot{v_y} = \frac{2F_{ry} + 2F_{fy}\cos(\delta) - m r v_x}{m}, \tag{8b}$$

$$\dot{r} = \frac{2F_{fy} l_f \cos(\delta) - 2F_{ry} l_r}{I_{zz}}. \tag{8c}$$

For notational simplicity, we introduce the state and input vectors as follows:

$$x(t) = [v_x\ ,\ v_y\ ,\ r]^{\mathrm{T}}, \tag{9}$$

$$u(t) = [F_{rx}\ ,\ \delta]^{\mathrm{T}}. \tag{10}$$

Using these notations, the equations of vehicle motion (8) are described by the state equation as follows:

$$\dot{x}(t) = f(x(t), u(t)), \tag{11}$$

$$f(x(t),\ u(t)) := \begin{bmatrix} \frac{2u_1 - 2F_{fy}\sin(u_2) - C_D A_D x_1^2 + m x_3 x_2}{m} \\ \frac{2F_{ry} + 2F_{fy}\cos(u_2) - m x_1 x_3}{m} \\ \frac{2F_{fy} l_f \cos(u_2) - 2F_{ry} l_r}{I_{zz}} \end{bmatrix}.$$

Hereafter, we consider the discretized model shown below for system model (11).

$$\begin{aligned} &x(t+1) = F(x(t), u(t)), \\ &F(x(t), u(t)) := x(t) + \Delta t f(x(t), u(t)), \\ &y(t) = Cx(t). \end{aligned} \tag{12}$$

The objective of this study is to propose a state estimation method for system model (12).

III. ESTIMATION BASED ON UNSCENTED KALMAN FILTER

In this section, we propose a state estimation method based on the UKF for system model (12). First, we introduce the following observer system:

$$\hat{x}(t+1) = F(\hat{x}(t), u(t)) + z(t), \tag{13a}$$

$$\hat{y}(t) = C\hat{x}(t) + w(t), \tag{13b}$$

where $\hat{x}$ and $\hat{y}$ denote the estimated state and output of x and y, respectively. Moreover, z and w denote the process noise and the observation noise, respectively, which can be caused by disturbances.

In the minimum mean-squared error sense, the optimal state estimate is given by the conditional mean. Let $\hat{x}(i|j)$ be the mean of $\hat{x}(i)$ conditioned on all of the observations up to and including time j, i.e., $\hat{x}(i|j) = \mathbb{E}\left[\hat{x}(i)|Y^j\right]$, where $Y^j := \{\bar{y}(1), \bar{y}(2), \cdots, \bar{y}(j)\}$.

It is assumed that the means of $z(t)$ and $w(t)$ are zero for all time t. Let $Q^z(t)$ and $Q^w(t)$ be the covariances of $z(t)$ and $w(t)$, respectively.

The UKF [10] first predicts the mean and covariance of a future state using the process model and weighted sigma points as follows:

$$\chi^i(t+1|t) = F(\chi^i(t), u), \tag{14}$$

$$\hat{x}(t+1|t) = \sum_{i=0}^{2n} W^i \chi^i(t+1|t), \tag{15}$$

$$\begin{aligned} &Q^{\hat{x}}(t+1|t) = Q^z(t+1) \\ &+ \sum_{i=0}^{2n} W^i \left(\chi^i(t+1|t) - \hat{x}(t+1|t)\right)\left(\chi^i(t+1|t) - \hat{x}(t+1|t)\right)^{\mathrm{T}}, \end{aligned} \tag{16}$$

where W^i and χ^i denote the weight and sigma point, respectively. The definitions of W^i and χ^i can be found in [10].

$\chi^i(t+1|t)$ can be determined from (14). Then, $\hat{x}(t+1|t)$ and $Q^{\hat{x}}(t+1|t)$ are determined form (15) and (16), respectively.

After we redraw a new set of sigma points $\bar{\chi}^i$ to incorporate the effect of the additive process noise, the predicted observation is calculated by

$$\hat{y}(t+1|t) = \sum_{i=0}^{2n} W^i C(\bar{\chi}^i(t+1|t)). \tag{17}$$

Moreover, the cross covariance P and innovation covariance R are determined by

$$\begin{aligned} P(t+1|t) = &\sum_{i=0}^{2n} W^i \left(\chi^i(t+1|t) - \hat{x}(t+1|t)\right) \\ &\times \left(C(\bar{\chi}^i(t+1|t)) - \hat{y}(t+1|t)\right)^{\mathrm{T}}, \end{aligned} \tag{18}$$

$$\begin{aligned} R(t+1|t) = &\sum_{i=0}^{2n} W^i \left(C(\bar{\chi}^i(t+1|t)) - \hat{y}(t+1|t)\right) \\ &\times \left(C(\bar{\chi}^i(t+1|t)) - \hat{y}(t+1|t)\right)^{\mathrm{T}} \\ &+ Q^w(t+1). \end{aligned} \tag{19}$$

Consequently, the state estimate at time $t+1$ is obtained by updating the prediction by the linear update rule:

$$K(t+1) = P(t+1|t)R^{-1}(t+1|t), \tag{20a}$$

$$\hat{x}(t+1|t+1) = \hat{x}(t+1|t) + K(t+1)\left(\bar{y}(t+1) - \hat{y}(t+1|t)\right), \tag{20b}$$

$$Q^{\hat{x}}(t+1|t+1) = Q^{\hat{x}}(t+1|t) - K(t+1)R(t+1|t)K^{\mathrm{T}}(t+1). \tag{20c}$$

Note that the state estimator using UKF yields better performance than the one using EKF for control systems with high nonlinearities. Fig. 2 shows that the EKF simply linearizes nonlinear models so that the traditional linear Kalman filter can be applied. On the other hand, Fig. 3 shows that the UKF uses a set of appropriately chosen weighted points to parameterize the means and covariances of probability distributions without linearization.

IV. Numerical Simulations

In this section, we provide numerical simulation results to verify the effectiveness of the proposed method. Here, we consider three cases for numerical simulations as shown in Table II. In case 1, all the state variables of the system are measurable. In case 2, x_2 is not measurable. In case 3, only x_1 is measurable.

TABLE II
Simulation Cases

Case 1	$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Case 2	$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Case 3	$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then, we set the initial state and the initial estimated state as follows:

$$x(0) = \begin{bmatrix} 25 \\ 1.5 \\ 0.1 \end{bmatrix}, \quad (21)$$

$$\hat{x}(0) = \begin{bmatrix} 20 \\ 0.5 \\ 0.05 \end{bmatrix}. \quad (22)$$

Moreover, the initial covariance matrices are set as

$$P = R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (23)$$

The covariances of $z(t)$ and $w(t)$ are given by

$$Q^z = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \quad (24)$$

$$Q^w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.17 \end{bmatrix}. \quad (25)$$

Other parameters employed in the numerical simulations are as follows: $H = 1$, $\Delta t = 0.1$, $u = [0, 0]^{\mathrm{T}}$.

The results of numerical simulations by the proposed method are shown below. In Figs. 2–4, the solid and dashed lines show the time histories of the real state x and the estimated state $\hat{x}$, respectively. Fig. 2 shows the time histories of the state in case 1. Fig. 3 shows the time histories of the state in case 2. Fig. 4 shows the time histories of the state in case 3. We can see that the estimated state $\hat{x}$ converges to the real state x in all the cases. Fig. 5 shows the time histories of the norm of the estimate error e defined by $e = x - \hat{x}$ in each case. These figures reveal the effectiveness of the proposed method.

V. Conclusion

The model predictive control method proposed in [5] for nonlinear vehicle dynamics is inapplicable when all the state variables are not exactly known. In general, the state variables of systems are measured through output sensors, hence, only limited parts of them can be directly known. Thus, it is unrealistic that all the state variables of nonlinear vehicle system are exactly known for every time. Hence, it should be assumed that the limited state variables can be only known.

To apply the MPC method proposed in [5] to the nonlinear vehicle control systems, we need to establish a state estimation method for nonlinear vehicle dynamics with limited measurable state variables. In this study, we established a state estimation method for nonlinear vehicle dynamics. We proposed a state observer system using the unscented Kalman filter for estimating the state of nonlinear vehicle system. The effectiveness of the proposed method was verified by numerical simulations. To incorporate the model predictive control with the state estimation method proposed here is a possible future work.

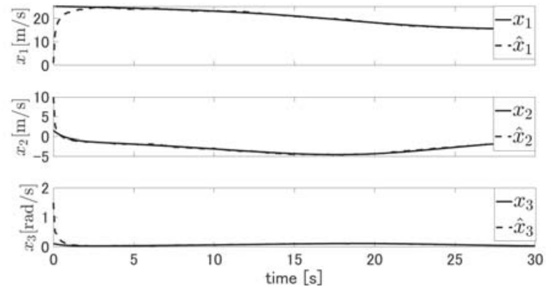

Fig. 2 Time histories of x and xˆ in case 1

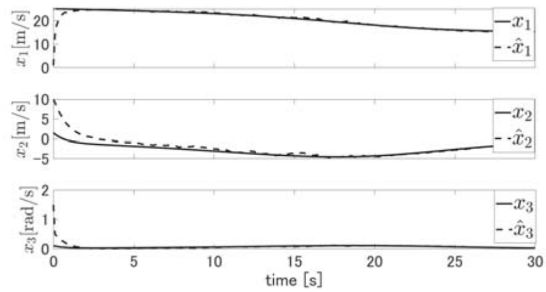

Fig. 3 Time histories of x and xˆ in case 2

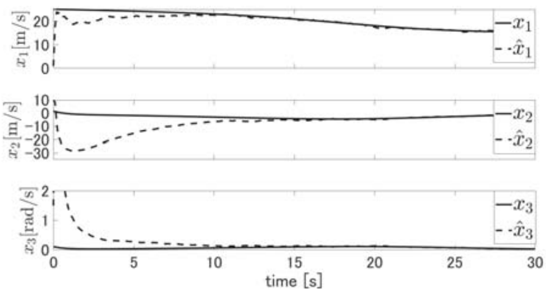

Fig. 4 Time histories of x and xˆ in case 3

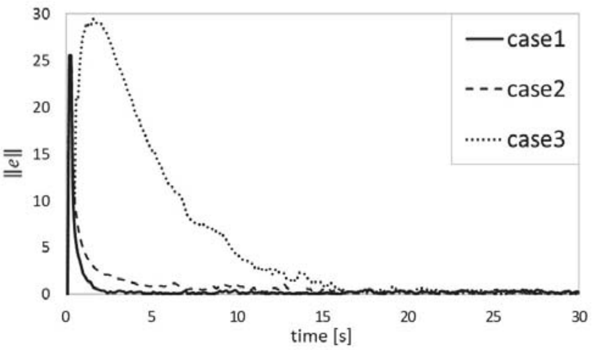

Fig. 5 Time histories of $||e||$

REFERENCES

[1] J. Liu, P. Jayakumar, J.L. Stein and T. Ersal, A nonlinear model predictive control formulation for obstacle avoidance in high-speed autonomous ground vehicles in unstructured environments, *Vehicle System Dynamics*, Vol. 56, Issue 6, pp. 853-882, 2018.

[2] M. Ataei, A. Khajepour and S. Jeon, Model predictive control for integrated lateral stability, traction/braking control, and rollover prevention of electric vehicles, *Vehicle System Dynamics*, Vol. 58, Issue 1, pp. 49-73, 2019.

[3] M.S. Basrah, E. Siampis, E. Velenis, D. Cao and S. Longo Wheel slip control with torque blending using linear and nonlinear model predictive control, *Vehicle System Dynamics*, Vol. 55, Issue 11, pp. 1665-1685, 2017.

[4] S.A. Sajadi-Alamdari, H. Voos and M. Darouach, Nonlinear Model Predictive Control for Ecological Driver Assistance Systems in Electric Vehicles, *Robotics and Autonomous Systems*, Vol. 112, No. 2, pp. 291-303, 2019.

[5] T. Baba, T. Hashimoto and Liang-Kuang Chen, Model Predictive Control for Stabilization of Vehicle Nonlinear Dynamics to Avoid the Second Collision Accident, *Proceedings of the 22nd International Conference on Advances in Materials and Processing Technology*, TA1-6, 2019.

[6] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control With Numerical Solution for Nonlinear Parabolic Partial Differential Equations, *IEEE Transactions on Automatic Control*, Vol. 58, No. 3, pp. 725-730, 2013.

[7] T. Hashimoto, R. Satoh and T. Ohtsuka, Receding Horizon Control for Spatiotemporal Dynamic Systems, *Mechanical Engineering Journal*, Vol. 3, No. 2, 15-00345, 2016.

[8] T, Shimizu and T. Hashimoto, Model Predictive Control with Unscented Kalman Filter for Nonlinear Implicit Systems, *International Journal of Mathematical and Computational Sciences*, Vol. 12, No. 7, pp. 147-151, 2018.

[9] H.W. Sorenson, Ed., Kalman Filtering: Theory and Application, Piscataway, NJ: IEEE, 1985.

[10] S. Julier, J. Uhlmann and H.F. Durrant-Whyte, A New Method for the Nonlinear Transformation of Means and Covariances in Filters and Estimators, *IEEE Transactions on Automatic Control*, Vol. 45, 2000, pp. 477-482.

[11] H.B. Pacejka, A New Tire Model with an Application in Vehicle Dynamics Studies, *SAE Technical Paper*, 890087, 1989.