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Flow Duration Curves and Recession Curves Connection through a Mathematical Link

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Abstract**—**This study helps Public Water Bureaus in giving reliable answers to water concession requests. Rapidly increasing water requests can be supported provided that further uses of a river course are not totally compromised, and environmental features are protected as well. Strictly speaking, a water concession can be considered a continuous drawing from the source and causes a mean annual streamflow reduction. Therefore, deciding if a water concession is appropriate or inappropriate seems to be easily solved by comparing the generic demand to the mean annual streamflow value at disposal. Still, the immediate shortcoming for such a comparison is that streamflow data are information available only for few catchments and, most often, limited to specific sites. Subsequently, comparing the generic water demand to mean daily discharge is indeed far from being completely satisfactory since the mean daily streamflow is greater than the water withdrawal for a long period of a year. Consequently, such a comparison appears to be of little significance in order to preserve the quality and the quantity of the river. In order to overcome such a limit, this study aims to complete the information provided by flow duration curves introducing a link between Flow Duration Curves (FDCs) and recession curves and aims to show the chronological sequence of flows with a particular focus on low flow data. The analysis is carried out on 25 catchments located in North-Eastern Italy for which daily data are provided. The results identify groups of catchments as hydrologically homogeneous, having the lower part of the FDCs (corresponding streamflow interval is streamflow Q between 300 and 335, namely: Q(300), Q(335)) smoothly reproduced by a common recession curve. In conclusion, the results are useful to provide more reliable answers to water request, especially for those catchments which show similar hydrological response and can be used for a focused regionalization approach on low flow data. A mathematical link between streamflow duration curves and recession curves is herein provided, thus furnishing streamflow duration curves information upon a temporal sequence of data. In such a way, by introducing assumptions on recession curves, the chronological sequence upon low flow data can also be attributed to FDCs, which are known to lack this information by nature.

Keywords**—**Chronological sequence of discharges, recession curves, streamflow duration curves, water concession.

I. INTRODUCTION

KNOWING a watercourse regime with a especial focus on low flows and droughts is of crucial importance for designing storage and capture works, managing regulation devices, and maintaining appropriate qualitative standards for the ecosystem, etc. [23]. This information would be better obtained from hydrometrical gauges directly but, in most cases, is gained by regionalization procedures which extend the few data available at specific sites in a catchment to ungauged sites of the same catchment or to other non-monitored catchments. In particular, when dealing with water concessions, the FDCs have demonstrated to be suitable in many hydrological problems such as: hydropower field [29], [32], water quality assessment [38], water distribution and habitat suitability [1], [2], [11]-[14], [27]. A forerunner introduction to their utility has been given by Searcy [28]. His Manual of Hydrology highlighted the influence of geology on the low part of the curve and gave a prime suggestion on how to extend curves based on few data to represent long-term conditions. The author considers FDCs, hydrographs and double mass curves, the major tools of investigation for hydrologists. Focusing on FDCs, Searcy pointed out that, when a sufficiently long record and reliable sample are adopted, the curves represent the cumulative frequency and entail probabilistic information useful to defining future water scenarios.

Other authors like [34], [35], and [16] provided a brief history of the widespread use of FDC in hydrology while recent efforts tend to focus on the problem of curve regionalization. In this last context [8]-[10] investigated 51 ungauged river stations situated in eastern Italy, thus exploring the effectiveness and reliability of different regional approaches. Other regionalization studies have been previously carried out by [14], [30], [33], [37], [24], [22] and [29]. Bevera [6] tried to link the central-low index streamflow values: Q182, Q274, Q355 to some geomorphological variables able to synthesize the different characteristics of the catchments considered. In particular, the author found a high correlation (equal to 0.90) between the streamflow datum and the catchment size, but very weak correlations between other parameters and low flow values.

Vogel and Fennessey [34], [35] introduced the idea of annual FDCs which have shown to be quite useful for drawing probabilistic considerations on wet, dry years and for computing confidence intervals associated with the curves themselves. Following these conclusions, [8] modeled the relationship between FDCs and AFDCs (AFDC: annual FDCs) built on daily streamflow series, thus enabling the generation of time series of daily streamflows for ungauged sites. The procedure has been tested over three catchments, varying from 400 to 3000 km^2 of size. LeBoutillier and Waylen [20] tried to relate FDC to AFDC introducing a five parameter stochastic model of daily streamflows. However, in the widespread application of FDCs, curves based upon daily discharges seem yet to be of predominant use.

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II. PROCEDURE'S DESCRIPTION

A. Preamble: Analysis on Data

Daily duration curves are closely related to the cumulative distribution functions of the mean daily discharges, expressed in their marginal form, i.e. in such a form that disregards the temporal sequence of flow. Herein, a log-normal function for the marginal distribution of mean daily streamflows is adopted and inserted in:

$$D(Q) = 365\left[1 - F_q(Q)\right] \qquad (1)$$

D(Q) and Fq(Q) stand for duration of FDC and for the cumulative streamflow marginal distribution of the given value Q. Most of the regionalization techniques proposed in literature introduce log-normal distribution to approximate FDCs. Beard [5] is probably the forerunner suggester of such a use. The author declares that, besides the practical use, such a distribution provides a good approximation on the lower half of daily FDCs whereas our attention is mostly placed. The parameter estimation is carried out using the least squares error method applied to the difference of target and experimental streamflow values varying their common durations. The procedure has been tested over the Tyrrhenic versants of Liguria region, in north western Italy. Fig. 1 maps the region under study.

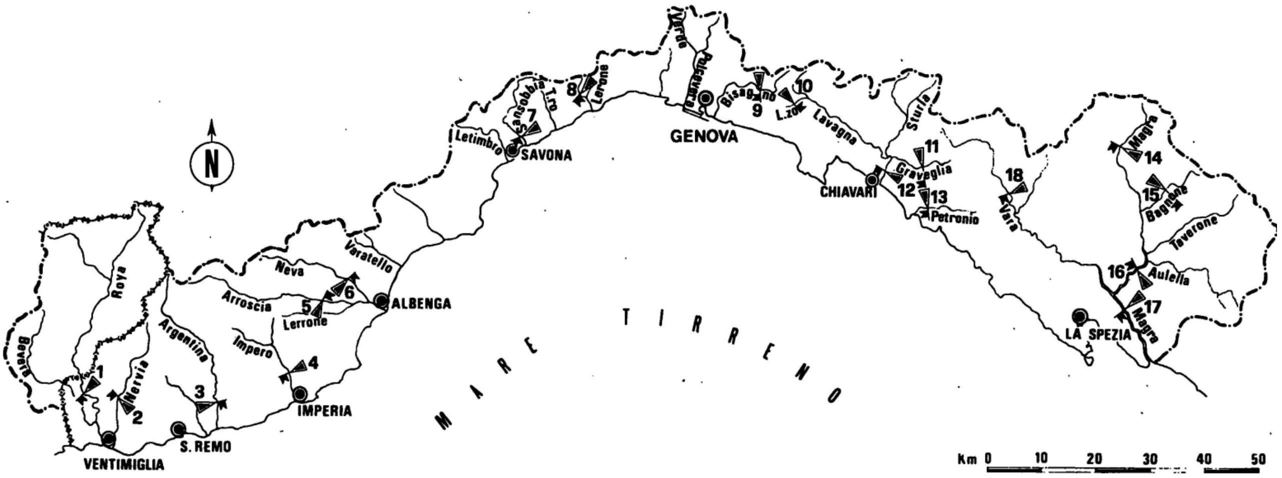

Fig. 1 Map of the major hydrometric stations of Ligurian region

Table I summarizes the 25 catchments selected (for which, at least, 10 years of continuous daily observations are collected from [3] 1951-1971 source) and reports S as the catchment's size, H as the corresponding mean rainfall annual height and μ_q and σ_q parameters.

B. Regionalization of the Streamflow Duration Curve

The relation can be expressed as:

$$\mu_Q = a_1 \cdot S^{b_1} \cdot H^{c1} \qquad (2)$$

$$\sigma_Q = a_2 \cdot S^{b_2} \cdot H^{c2} \qquad (3)$$

Parameters *a, b, c* can be obtained minimizing the mean square error defined below:

$$sqm = \sqrt{\frac{1}{N_t}\sum_{j=1}^{Ns}\sum_{D=1}^{365} N_j\left\{\frac{\left[Q_{t,j}(D) - Q_{o,j}(D)\right]}{\min\left[Q_{t,j}(D), Q_{o,j}(D)\right]}\right\}} \qquad (4)$$

where the $Q_{t,j}(D)$ and $Q_{o,j}(D)$ are, respectively, the theoretical and the observed streamflow for the *j-th* section and for the *D-th* duration considered. In more detail: Q_t is obtained through the quantile estimation of the lognormal distribution while Q_o is from the historical sequence of streamflow series ranked in a descendent order. N_j represents the total number of days for the generic section while N_t stands for the total number of daily streamflows considered, *Ns* being the number of sections listed in Table I.

Beard [5] is the forerunner suggester of the use of a lognormal probability density function to approximate FDCs. The author suggests that, in general, the two parameter lognormal distribution provides a good approximation on the lower half of daily FDCs, whereas our attention is mostly placed.

Table II summarizes the assessment of *a, b, c* parameters for all the 25 catchments considered.

It has to be noticed that both *a1* and *a2*, contrary to the other coefficients, are dimensional numbers; whose dimensions still depend on the other parameters' dimensions. The peculiarities of each basin are condensed into two contributes, the catchment's surface and mean annual rainfall height, while other geomorphological characteristics have been totally disregarded due to their hard quantification. However, a weak attempt to limit an excessive generalization in results has been carried out repeating the same procedure but dividing the catchments into two groups, belonging respectively to: the western and eastern side of Genova. Results are summarized in Tables III and IV.

TABLE I
List of the River Stations and the years Considered.

River station	Number of years considered	S (km^2)	H (mm)	μ_q (m^3/s)	σ_q (m^3/s)
Bevera	19	155	1100	2.518	3.742
Nervia	35	123	1103	2.297	4.665
Argentina	47	192	1188	4.435	8.574
Impero	32	69	1066	1.366	2.730
Arroscia	42	202	1158	4.107	7.480
Lerrone	14	47	1208	0.977	3.373
Neva	38	124	1113	2.419	3.829
Varatella	9	17.5	1202	0.463	0.653
Letimbro	14	33	1282	0.702	1.783
Ellera	10	41	1409	1.104	2.093
Sansobbia	38	32	1321	1.027	3.483
Lerone	10	14.9	1455	0.441	1.416
Bisagno	40	34	1679	1.306	2.941
Lavagna	36	163	1786	7.055	25.76
Sturla	19	102	1887	4.835	7.130
Graveglia	28	41	1801	1.606	2.691
Entella	39	364	1741	15.99	46.61
Petronio	14	57	1429	2.579	15.95
Vara	34	206	1850	8.150	15.48
Padivarma	11	454	1867	18.87	32.56
Piccatello	28	77	2037	2.890	4.059
Bagnone	44	51	1908	2.209	2.730
Taverone	12	79	1883	3.305	4.298
Aulella	18	208	1668	8.581	12.33
Calamzza	44	939	1789	38.370	53.030

Means and standard deviation values are obtained using the least mean squared method on duration curves.

TABLE II
Evaluation of the Regionalization Coefficients

a1	b1	c1	a2	b2	c2	Sqm
0.02	0.990	1.43	0.0720	0.92	1.148	0.847

TABLE III
Evaluation of the Regionalization Coefficients for the 12 Western Basins

a1	b1	c1	a2	b2	c2	Sqm
0.01	1.046	2.29	0.0151	1.09	4.760	0.597

TABLE IV
Evaluation of the Regionalization Coefficients for the 13 Eastern Basins

a1	b1	c1	a2	b2	c2	Sqm
0.01	0.984	2.42	0.0118	0.92	0.000	0.586

Some parameters like: *a1, b1, c1, b2* show a modest variation in respect to the corresponding values listed in Table II. Specifically, we can notice that *a2* and *c2* entail the different behavior of the two groups of catchments: having the eastern basins dependent on the annual cumulative rainfall heights. Table V reports the new parameter estimations for each percentage P of the total period considered. For instance, P equal to zero represents interpolation extended along all durations and P = 0.3 excludes the first 30% of ranked data, etc.

TABLE V
Percentage Estimation of Regionalization Coefficients for the Total 25 Basins Considered.

P %	a 1	b 1	c 1	a 2	b 2	c 2	Sqm
90	0.148	1.123	-2.45	1.294	1.138	-4.59	2.397
80	0.084	0.981	-0.30	0.578	0.925	-1.41	1.774
70	0.051	0.972	0.526	0.278	0.908	-0.18	1.472
60	0.040	0.957	0.952	0.198	0.884	0.445	1.291
50	0.033	0.954	1.198	0.153	0.877	0.811	1.162
40	0.028	0.957	1.335	0.123	0.880	1.015	1.069
30	0.024	0.966	1.431	0.099	0.891	1.160	0.995
20	0.021	0.978	1.478	0.082	0.907	1.229	0.937
10	0.019	0.986	1.503	0.072	0.918	1.266	0.886
0	0.019	0.990	1.430	0.072	0.924	1.148	0.847

In order to guarantee a better fitting of theoretical curves to observed data [15], [25], the entire procedure has been repeated focusing each time, on a shorter tail of the curve, where low flows lay and our attention is mostly placed. Tables V and VI summarize some obtained results, respectively for western and eastern basins.

TABLE VI
Percentage Estimation of Regionalization Coefficients for the 12 Western Basins

P %	a 1	b 1	c 1	a 2	b 2	c 2	Sqm
30	0.0057	1.152	4.06	0.004	1.25	7.63	0.67
20	0.0064	1.135	3.57	0.005	1.22	6.85	0.64
10	0.0075	1.111	3.135	0.007	1.187	6.146	0.60
0	0.0125	1.04	2.29	0.015	1.09	4.76	0.59

For the eastern catchments, Table VII seems dominated by negative values of H exponents both in the mean and in the variance estimations.

TABLE VII
Percentage Estimations of Regionalization Coefficients for the 13 Eastern Basins

P %	a 1	b 1	c 1	a 2	b 2	c 2	Sqm
30	0.598	0.887	-3.5	83.61	0.78	-9.9	0.627
20	0.309	0.917	-2.7	29.56	0.82	-8.6	0.592
10	0.155	0.944	-1.8	9.81	0.86	-7.1	0.564
0	0.0585	0.980	-0.4	2.00	0.92	-4.8	0.551

Following these last results, it seems appropriate to adopt, from now on, a unique regionalization procedure for the whole area under study; committing only to rainfall variability the difference response of catchments of the same size.

III. Comparison between Duration Curves and Decay Curves

As stated by Searcy [28]: “A simpler concept of the flow duration curve is that it is another means of representing streamflow data combining in one curve the flow characteristics of stream throughout the range of discharge. Still the flow duration curve does not show the chronological sequence of flows”.

Duration curves express the cumulative probability function of mean daily streamflows represented in their marginal form; in other words, in such a form that leaves out of the link between following values of a selected streamflow datum. However, the author believes that the total absence of chronological sequence does not represent a strong shortcoming for high streamflow values: high discharges are separated by interarrival times such that the former value of each interval may be considered an independent random variable [21]. Conversely, the same cannot be said for low streamflows [19], [26], [31] because of the strong dependence between following data.

In this study, we aim to find an interpretation of the FDC expressed in their marginal form, focusing on the time evaluation that low discharge may occur.

Let us consider a total series of *T* observed mean daily streamflows, belonging to a hypothetic never-ending sample. We denote with d_i the length of the daily streamflow sequence which overtakes a certain threshold Q. Let *N* be the total number of data considered. It can be said:

$$1-F_Q(Q)=\lim_{T\longrightarrow\infty}\frac{1}{T}\sum_{i=1}^{N(Q,T)}d_i \tag{5}$$

$$D(Q)=\lim_{T\longrightarrow\infty}\frac{365}{T}\sum_{j=1}^{N(Q,T)}d_j=365(1-F_q(Q)) \tag{6}$$

It follows:

$$D(Q+\Delta Q)=\lim_{T\longrightarrow\infty}\frac{365}{T}\sum_{j=1}^{N(Q,T)}d_j=365(1-F_q(Q+\Delta Q)) \tag{7}$$

The derivate function of *D(Q)* has, therefore, the following expression:

$$-\frac{dD}{dQ}=\lim_{\Delta Q\to 0}\left[\lim_{T\to\infty}\frac{365}{T}\left\{\sum_{j=1}^{N(Q,T)}d_j-\sum_{j=1}^{N(Q+\Delta Q,T)}d_j\right\}\right]=365\frac{dF_Q(Q)}{dQ} \tag{8}$$

Let us introduce the following assumptions for threshold *Q* considered as:

- crossed, in the ascending slope by a vertical line of the hydrograph;
- crossed, in the descendent slope by a part of the decay curve;

in order to express the following appossimate relationship:

$$\frac{dD}{dQ}=\frac{1}{\gamma\cdot Q}\lim_{T\to\infty}\frac{365N(Q,T)}{T} \tag{9}$$

where γ represents the coefficient of the decay exponential curve, such as in the simplified relation: $Q=Q_0e^{-\gamma t}$. Let us introduce the mean annual *Na* of the overtaken intervals of *Q*, *Na* is defined as:

$$Na(Q)=\lim_{T\to\infty}\frac{365N(Q,T)}{T} \tag{10}$$

We obtain:

$$\frac{dD}{dQ}=-\frac{N_a(Q)}{\gamma\cdot Q} \tag{11}$$

The range of validity of the hypotheses stated is tested by calculating, as *D* changes, the given ratio:

$$\gamma=-\frac{Na}{Q}\frac{dQ}{dD} \tag{12}$$

Furthermore, if we assign for the given duration *D* the log-normal formula, the previous link gives for *Na* the corresponding expression:

$$Na(Q)=365\gamma Qf_Q(Q)=k\exp\left[-\frac{(\ln(Q)-\mu_y)^2}{2\sigma^2 y}\right] \tag{13}$$

where $k=\frac{365}{\sigma_y\sqrt{2\pi}}\gamma$. Parameters μ_y and σ_y known functions of mean and variance values of mean daily streamflows. The model is completely defined having set values: μ_y,σ_y and γ.

Once again, by introducing several optimization techniques we can obtain the results reported in Table VIII. Herein, estimated values of the three parameters (μ_y,σ_y,γ) are compared to the ones obtained previously by interpolating each duration curve section by section. In order to minimize the *FF* cost function calculated over modeled and target data, the following criteria are adopted:

$$\varepsilon_1=D(Q)-365[1-F_Q(Q)] \tag{14}$$

$$\varepsilon_2=N_a(Q)-365bQf_Q(Q) \tag{15}$$

$$FF=(\varepsilon_1\varepsilon_2)^2 \tag{16}$$

In Table VIII, the values of the three parameters are obtained by extending the interpolation curve to all the different durations considered (included in the percentage interval between *n0* and *n1*).

The comparison introduced in Table VIII seems appropriate since old mean and variance values (μ_q and σ_q) appear to confirm the new corresponding ones (μ_q and σ_q) for almost all 25 catchments considered. The reader is sent back to detailed studies of decay curves provided by: [4], [7], [17], [18], [36]. Further improvements, however, can be obtained by reducing the field of potential durations considered. For instance, Table IX considers results limited to the period between 37 days and 365 days. These results correspond to a 10% reduced duration period of exploration, and to the elimination of original samples which mostly diverge from given assumptions stated in Chapter III.

TABLE VIII
COMPARISONS BETWEEN MEAN AND STANDARD DEVIATION VALUES OBTAINED USING THE LEAST MINIMUM SQUARES TECHNIQUE ON STREAMFLOW DURATION CURVES OF LOGNORMAL DISTRIBUTION (FOR N0 = 1 AND N1 = 365 DAYS)

Station	μ_Y m^3/s	σ_Y m^3/s	γ1/days
1	2.307	3.719	0.039
2	2.132	4.335	0.081
3	4.110	7.908	0.066
4	1.276	2.502	0.060
5	3.807	6.703	0.063
6	0.597	0.995	0.059
7	2.449	4.169	0.060
8	0.365	0.472	0.045
9	0.717	2.106	0.088
10	1.131	4.488	0.089
11	0.631	1.572	0.105
12	1.298	3.069	0.085
13	6.274	19.062	0.086
14	4.802	6.844	0.119
15	1.586	2.669	0.068
16	15.712	44.073	0.111
17	1.458	3.201	0.078
18	8.202	16.415	0.071
19	2.889	4.325	0.062
20	2.172	2.821	0.076
21	3.504	5.309	0.069
22	7.759	10.288	0.077
23	38.608	62.834	0.048

TABLE IX
COMPARISONS BETWEEN MEAN AND STANDARD DEVIATION VALUES OBTAINED USING THE LEAST MINIMUM SQUARES TECHNIQUE ON STREAMFLOW DURATION CURVES OF LOGNORMAL DISTRIBUTION (FOR N0 = 37 AND N1 = 365 DAYS)

Station	μ_Y (m^3/s)	σ_Y (m^3/s)	γ(1/days)
1	2.323	3.825	0.0422
2	2.157	5.134	0.0740
3	4.152	8.524	0.0659
4	1.279	2.541	0.0625
5	3.831	7.065	0.0672
6	0.598	0.999	0.0591
7	2.462	4.359	0.0614
8	0.365	0.520	0.0481
9	0.721	2.150	0.0903
10	1.132	4.504	0.0898
11	0.630	1.564	0.1056
12	1.316	3.252	0.0896
13	6.255	18.867	0.0854
14	4.802	6.844	0.1196
15	1.587	2.669	0.0682
16	15.698	44.010	0.1106
17	1.449	3.111	0.0757
18	8.210	16.488	0.0735
19	2.892	4.351	0.0652
20	2.183	2.947	0.0794
21	3.524	5.475	0.0746
22	7.769	10.397	0.0795
23	38.855	64.464	0.0595

IV. CONCLUSIONS

A valid procedure linking streamflow duration curves to recession curves has been herein adopted. In such a way, by introducing appropriate assumptions, the chronological sequence upon low flow data can be attributed also for FDCs which are known to lack of this information by nature.

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