Data Centers' Temperature Profile Simulation Optimized by Finite Elements and Discretization Methods

José Alberto García Fernández, Zhimin Du, Xinqiao Jin

***Abstract*—**Nowadays, data center industry faces strong challenges for increasing the speed and data processing capacities while at the same time is trying to keep their devices a suitable working temperature without penalizing that capacity. Consequently, the cooling systems of this kind of facilities use a large amount of energy to dissipate the heat generated inside the servers, and developing new cooling techniques or perfecting those already existing would be a great advance in this type of industry. The installation of a temperature sensor matrix distributed in the structure of each server would provide the necessary information for collecting the required data for obtaining a temperature profile instantly inside them. However, the number of temperature probes required to obtain the temperature profiles with sufficient accuracy is very high and expensive. Therefore, other less intrusive techniques are employed where each point that characterizes the server temperature profile is obtained by solving differential equations through simulation methods, simplifying data collection techniques but increasing the time to obtain results. In order to reduce these calculation times, complicated and slow computational fluid dynamics simulations are replaced by simpler and faster finite element method simulations which solve the Burgers' equations by backward, forward and central discretization techniques after simplifying the energy and enthalpy conservation differential equations. The discretization methods employed for solving the first and second order derivatives of the obtained Burgers' equation after these simplifications are the key for obtaining results with greater or lesser accuracy regardless of the characteristic truncation error.

***Keywords*—**Burgers' equations, CFD simulation, data center, discretization methods, FEM simulation, temperature profile.

I. Introduction

THE capacity of servers in the data center industry is continually increasing due to the growing global demand for information processing, which implies a greater consumption in the refrigeration equipments responsible for maintaining adequate temperature conditions inside the servers.

The temperature knowledge inside the servers would be a great advance to develop and improve new refrigeration techniques and thus, reduce the energy demand. The installation of a temperature sensors network could facilitate the acquisition of these kinds of data. However, this procedure is expensive and complicated, reason why CFD simulations [1], [2] are normally employed which often delay obtaining results solving the differential equations that define the thermal flow processes. In order to obtain instant temperature profiles with enough accuracy the energy and enthalpy differential equations are simplified to others more simple assuming less conservative flow and heat transfer conditions. On the other hand, CFD simulations are replaced by simple and fast FEM simulations [3], [4] discretizing the obtained differential equations by the three discretization methods most usually employed [5]: forward, backward and central differentiation approximation methods; FDAM, BDAM and CDAM respectively.

A forward differentiation is appropriate for solving differential equations by single-step predictor-corrector methods if data are available ahead; instead, a backward differentiation is more useful if the data are not. In the other hand, a central differentiation is useful in solving differential equations if data values are available both ahead and behind.

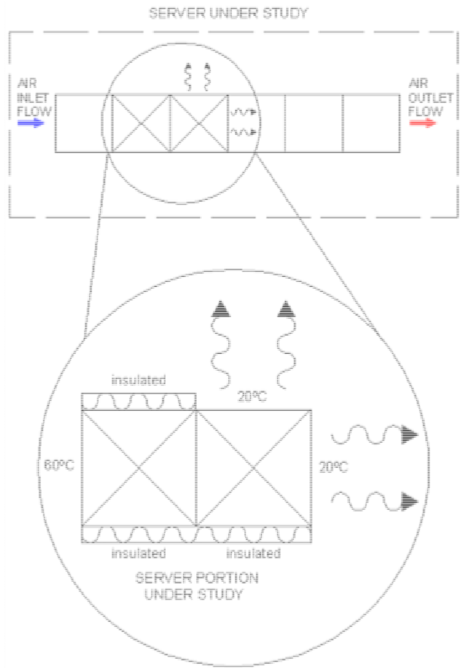

Fig. 1 Imposed initial and boundary conditions over the studied server portion

In Fig. 1 initial and boundary conditions are imposed according to the selected portions of the server under study. CPU temperature extreme conditions of 60 ºC and a standard

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air cooling temperature of 20 ºC are considered. Air flow conditions to consider are the next: constant heat air capacity Cp = 1005 Ws/Kg k, air density ρ = 1.205 kg/m^3 and constant air thermal conductivity k = 0.026 W/mK.

The FEM algorithms and the discretized differential equations have been implemented with the robust Python programming language [6], [7] which provides an easy usage of code lines and debugging, allowing the elaboration of programming sequences with a reduced number of instructions. However other programming languages can be used for obtaining the same results. In the same way statistical and algorithm models based on machine learning [8] disciplines and deep learning [9], [10] in agreement with random processes based on Markov chain models [11] could be applied for improving the obtained results optimizing the accuracy and reducing time of the calculations.

II. Methodology

A. Energy and Enthalpy Equation

The general energy conservation equation in differential form characterizing the heat flux under study is shown as:

$$\frac{d(\rho e_t)}{dt} + \nabla(\rho e_t \bar{u}) = \dot{q}_v - \nabla q_s + \rho \bar{f}\bar{u} + \nabla(\bar{\bar{\tau}}\bar{u}) \quad (1)$$

After decomposing the temporal derivative and gradient terms, (1) becomes:

$$\rho \frac{de_t}{dt} + e_t\left(\frac{d\rho}{dt} + \rho\nabla\bar{u} + \bar{u}\nabla\rho\right) + \rho\bar{u}\,\nabla e_t = \dot{q}_v - \nabla q_s + \bar{u}\left(\rho\bar{f} + \nabla\bar{\bar{\tau}}\right) + \bar{\bar{\tau}}\nabla\bar{u} \quad (2)$$

Equation (2) becomes simplified next as (3) considering steady incompressible conditions and the flow stress tensor as the different between the static pressure and the viscous stress tensor of the flow.

$$\rho\,\bar{u}\,\nabla e_t = \dot{q}_v - \nabla q_s + \bar{u}\left(\rho\bar{f} + \nabla\bar{\bar{\tau}}\right) \quad (3)$$

More simplifications on (3) are taken into in account finally to obtain the simplified enthalpy conservation equation (4) known as the Burgers ‘equation: thermodynamic definition of the enthalpy e_t = h_t – Pv-U is considered taking in consideration a non-potential energy (U = 0), the most important heat transfer process involved is by conduction with a constant thermal conductivity and the superficial forces are considered present, while the only force on the body acting at distance over the fluid volume is the gravitational force that is neglected.

$$\rho\,\bar{u}\,\nabla h_t = \dot{q}_v + k\nabla^2 T \quad (4)$$

The Burgers’ equation (4) is represented as (5) after decomposing its gradient components in derivative terms for each dimension.

$$\rho\left(u\cdot\frac{\partial h_t}{\partial x} + v\cdot\frac{\partial h_t}{\partial y} + w\cdot\frac{\partial h_t}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}_v \quad (5)$$

Analyzing the obtained equation (5) it is observed that the intensive properties of density, speed and thermal conductivity are known unlike the temperature dT variation and the total specific enthalpy dht. However, considering ideal gas conditions these two properties are linearly related to the specific heat Cp assuming the next thermodynamical relation dht = Cp dT. Therefore, after considering a constant specific heat capacity Cp and imposing the thermal diffusivity α definition, (5) becomes (6) and the temperature variation is the only value to be determined.

$$\left(u\cdot\frac{\partial T}{\partial x} + v\cdot\frac{\partial T}{\partial y} + w\cdot\frac{\partial T}{\partial z}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}_v}{\rho\cdot Cp} \quad (6)$$

B. Discretization Method

The temperature derivative dT can be calculated numerically by estimative methods employing close known data forward or backward, with respect to the temperature value to be calculated.

Since the simplest way to approximate the temperature derivatives is to consider the slope of the secant line that connects two temperature points, depending on which data with respect the initial value are employed for the approximation: previous value, next value or both, the temperature derivatives are approximated by backward difference approximation method (BDAM) taking the previous value of the temperature, forward difference approximation method (FDAM) employing the value of the temperature ahead or central difference approximation method (CDAM) adopting both.

Independently of the employed method for obtaining the temperature values there is a truncation error with infinite numbers of terms between the approximated value calculated and the real value, defined by a Taylor series expansion [12] . Therefore, it is taken in consideration this truncation error [5] describing the asymptotic behavior of the function. It is represented by the Landau’s symbol ’*O*’ and for the temperature truncation error is denoted as *O*(T).

The existing error between the real and approximate values of temperature calculated employing BDAM and FDAM is linear and the truncation error *O*(T) is first order. On the other hand, considering CDAM for the approximation, the existing error is quadratic and not lineal and is represented as $O(T)^2$. This truncation error is second order, and its value is more accurate because it has been obtained as a second approximation.

The second temperature derivative d^2T from (6) is calculated in the same way as the first derivative with similar conclusions.

III. Results and Discussions

A. Simulation by Difference Approximation Methods and FEM

The two-dimensional temperature profile under study is simulated by FEM after discretizing the differential equation employing the mentioned three discretization approximated methods from the previous section, avoiding iterative simulations based on CFD methods. As consequence, the three dimensional Burgers 'equation (6) from the last section is simplified as (7) in two dimensions as:

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{q_v}{\rho\cdot Cp} \tag{7}$$

First order differential part of the equation is on the left-side where the velocity components are known, the first right-side term corresponds to the second order differential part multiplied by a known thermal diffusivity α value, and the last terms are constant values determined in the initial and contour conditions of the case under study.

It is shown next in Fig. 2 the discretization of the first-order differential term corresponding to the left side of (7) according to BDAM, FDAM and CDAM.

$$\left(u\frac{dT}{dx}+v\frac{dT}{dy}\right)\begin{cases} FDAM & u_{i,j}\left(\frac{T^n_{i,j}-T^n_{i-1,j}}{\Delta x}\right)+v_{i,j}\left(\frac{T^n_{i,j}-T^n_{i,j-1}}{\Delta y}\right) \\ BDAM & u_{i,j}\left(\frac{T_{i+1,j}-T_{i,j}}{\Delta x}\right)+v_{i,j}\left(\frac{T_{i,j+1}-T_{i,j}}{\Delta y}\right) \\ CDAM & u_{i,j}\left(\frac{T_{i+1,j}-T_{i-1,j}}{\Delta x}\right)+v_{i,j}\left(\frac{T_{i,j+1}-T_{i,j-1}}{\Delta y}\right) \end{cases}$$

Fig. 2 discretized left side of the Burgers 'equation in two dimensions by the three discretization methods. The same is performed in Fig. 3 on the right side of (7) where a second order discretization over the second differential derivative is done by BDAM, FDAM and CDAM.

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)\begin{cases} FDAM & \alpha\left(\frac{T_{i-2,j}-2T_{i-1,j}+T_{i,j}}{\Delta x^2}+\frac{T_{i,j-2}-2T_{i,j-1}+T_{i,j}}{\Delta y^2}\right) \\ BDAM & \alpha\left(\frac{T_{i+2,j}-2T_{i+1,j}+T_{i,j}}{\Delta x^2}+\frac{T_{i,j+2}-2T_{i,j+1}+T_{i,j}}{\Delta y^2}\right) \\ CDAM & \alpha\left(\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{\Delta x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\Delta y^2}\right) \end{cases}$$

Fig. 3 discretized right side by the three discretization methods of the Burgers 'equation in two dimensions.

For discretizing the left-side first order differential part of (7), CDAM will be employed, because as it was mentioned above, it is more precise, since the second order truncation error obtained is lower than the corresponding first order truncation error to other discretization methods. Thus, the BDAM and FDAM will not be considered for the discretization of this equation part. On the other hand, the three discretization methods will be considered for discretizing the right-side second order differential part of the equation, because the truncation error of them is lower according to the second order derivative.

The differential equation (7) is solved by FEM after its discretization considering the established initial and boundary conditions. According to the CDAM discretization of the first order derivative term of (7) and the three discretization methods employed in the second order derivative, three scenarios are considered in Figs. 4-6 respectively. In each scenario two cases are studied, in the first case (A) is considered a null flow velocity, while in the second case (B) the velocity is unitary.

Respective equations (8)-(10) from Figs. 4-6 represent the temperature values Ti,j obtained after rearranging the discretized equations terms in accordance with the discretization method considered in each scenario and case.

$$T_{ij}=\frac{\Delta y^2\Delta x u_{ij}T_{i-1,j}+\Delta x^2\Delta y v_{ij}T_{i,j-1}}{\Delta y^2\Delta x u_{ij}+\Delta x^2\Delta y v_{ij}+2\alpha\left(\Delta y^2+\Delta x^2\right)}+\frac{\alpha\left[\Delta y^2\left(T_{i+1,j}+T_{i-1,j}\right)+\Delta x^2\left(T_{i,j+1}+T_{i,j-1}\right)\right]+\frac{\dot{q}_v}{\rho Cp}\Delta x^2\Delta y^2}{\Delta y^2\Delta x u_{ij}+\Delta x^2\Delta y v_{ij}+2\alpha\left(\Delta y^2+\Delta x^2\right)} \tag{8}$$

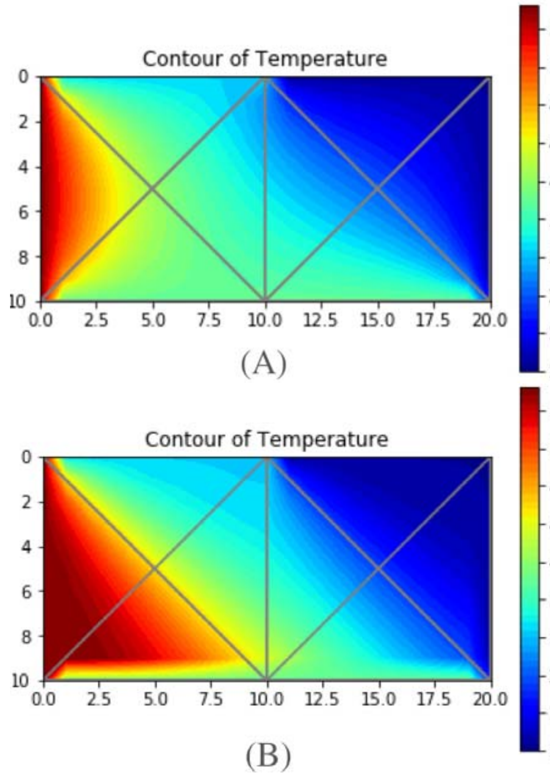

Fig. 4 Right-side BDAM and left-side CDAM Burgers 'equation discretization for a case (A) considering a null velocity u = v = 0 m/s and for a case (B) unitary velocity values u = v = 1 m/s respectively

$$T_{ij}=\frac{-\Delta y^2\Delta x u_{ij}T_{i+1,j}-\Delta x^2\Delta y v_{ij}T_{i,j+1}}{-\Delta y^2\Delta x u_{ij}-\Delta x^2\Delta y v_{ij}+2\alpha\left(\Delta y^2+\Delta x^2\right)}+\frac{\alpha\left[\Delta y^2\left(T_{i+1,j}+T_{i-1,j}\right)+\Delta x^2\left(T_{i,j+1}+T_{i,j-1}\right)\right]+\frac{\dot{q}_v}{\rho Cp}\Delta x^2\Delta y^2}{-\Delta y^2\Delta x u_{ij}-\Delta x^2\Delta y v_{ij}+2\alpha\left(\Delta y^2+\Delta x^2\right)} \tag{9}$$

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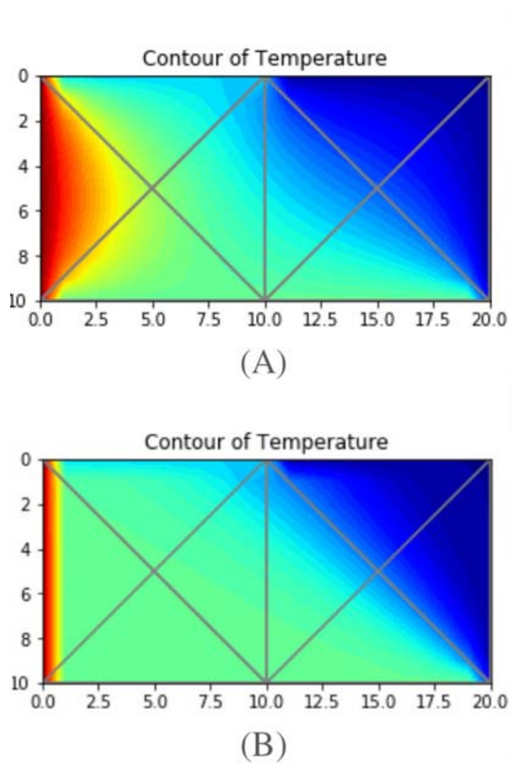

Fig. 5 Right-side FDAM and left-side CDAM Burgers´equation discretization for a case (A) considering a null velocity u = v = 0 m/s and for a case (B) unitary velocity values u = v = 1 m/s respectively

As it was mentioned previously, CDAM is more accurate than other discretization methods. However, considering case (A) in Figs 4-6, where the velocity flow is considered null, similar temperature profiles are obtained in the three scenarios in addition to obtain consistent results. The reason of these similar results is due to the fact that zero velocity removes first order values from the equation keeping only second order values, where the existing second derivatives make the grade one truncation error generated in the BDAM and FDAM too small to notice the difference from the even smaller grade two truncation error generated with a CDAM.

Considering case (B) with unit flow velocity, the first order values of the equation are not canceled and play an important role in the differentiation, therefore, it is expected that employing CDAM, better results will be obtained than with other methods employed, because in this case the truncation error from the BDAM and FDAM is bigger and not small enough to be compared to truncation error of degree two obtained in the CDAM. However, as seen in case (B) of Fig. 6, the results are not as expected, presenting inconsistent values after employing CDAM for solving the second order derivative terms of (7). On the other hand, BDAM and FDAM show more consistent values as it can be observed in case (B) of Figs. 4 and 5 respectively.

$$T_{ij} = \frac{-\Delta y^2 \Delta x u_{ij}\left(T_{i+1,j}+T_{i-1,j}\right) - \Delta x^2 \Delta y v_{ij}\left(T_{i,j+1}+T_{i,j-1}\right)}{4\alpha\left(\Delta y^2+\Delta x^2\right)} + \frac{\alpha\left[2\Delta y^2\left(T_{i+1,j}+T_{i-1,j}\right)+2\Delta x^2\left(T_{i,j+1}+T_{i,j-1}\right)\right]+\frac{\dot{q}_v}{\rho Cp}2\Delta x^2\Delta y^2}{4\alpha\left(\Delta y^2+\Delta x^2\right)} \quad (10)$$

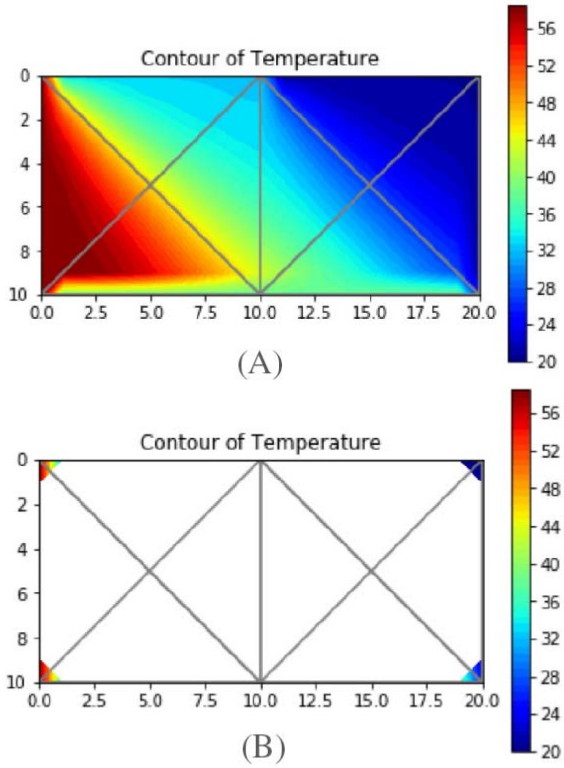

Fig. 6 Right-side CDAM and left-side CDAM Burgers equation discretization for a case (A) considering a null velocity u = v = 0 m/s and for a case (B) unitary velocity values u = v = 1 m/s respectively

Comparing these last two methods, a greater coherence is observed in BDAM results than in FDAM results, providing the first, most real data in Fig. 4 according to the initial and boundary conditions. The reason why more consistent values are obtained in this scenario is because the discretized terms by BDAM from the first order derivatives employ temperature values already known located behind the calculated temperature values. On the contrary, FDAM employs temperature values ahead still not available and which must be predicted in the calculations from the calculated temperature values, unlike in BDAM where the values behind, have been calculated already. On the other hand, the terms discretized by CDAM of the first-order derivatives use both temperature values ahead and temperature values behind, so the uncertainty of the error produced is greater than in other cases.

IV. Conclusion

Energy and enthalpy conservation differential equation becomes Burgers' equation after several simplifications and considerations: steady and incompressible flow conditions are assumed, it is considered the irrational flow behavior with zero velocity curl according to mass conservation state, the main superficial forces considered in the flow are the pressure and the viscous forces, while the only force acting at distance on the flow body is the gravitational force which is not considered, no potential energy is assumed and conductive processes are the main involved heat transfer process. After these simplifications, the derivatives of the first and second order Burgers 'equation obtained are solved discretizing its differential terms and employing FEM techniques, thereby obtaining temperature profiles with enough accuracy to predict temperatures within a server in real time and avoid CFD calculations with complex simulations based on unnecessary time consuming iterations. A good choice of the discretization method involves obtaining better or worse results on the calculations. Several temperature profiles obtained by discretization employing BDAM, FDAM and CDAM demonstrate that the second order Burgers' equation terms can be solved by the three discretization methods indistinctly, even though the latter involves a minor truncation error. On the other hand, for solving the first order Burgers' equation terms, more accurate results are obtained employing BDAM despite the truncation error is greater compared to other discretization methods as it has been shown.

NOMENCLATURE

CFD	Computational fluid dynamics
Cp	isobaric heat capacity of the fluid (J/k)
e_t	specific total energy exchanged by the fluid (J/kg)
$\bar{f}$	specific force for each fluid particle (N/kg)
FEM	finite element methods
h_t	total specific enthalpy of the fluid (J)
k	thermal conductivity of the fluid (w/mk)
q_s	specific heat exchanges by fluid surfaces (J/kg)
$\dot{q_v}$	fluid volume temporal specific heat exchanged (J/kgs)
t	time (s)
T	temperature (k)
Ti,j	temperature (k). i,j = index grid in x,y
$\bar{u}$	velocity of the flow $\mid \bar{u}_c \mid = (m/s)$
$u_{i,j}$	horizontal velocity (m/s). i,j = index of the grid in x,y
u, v, w	components of the flow velocity (m/s) $\bar{u}$ in x,y,z
α	thermal diffusivity of the fluid (m^2/s)
ρ	density of the fluid (kg/m^3)
$\bar{\bar{\tau}}$	stress tensor of the flow (N/m^2)
∇	vector differential operator of the gradient

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