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Robust Stabilization of Rotational Motion of Underwater Robots against Parameter Uncertainties

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Abstract—This paper provides a robust stabilization method for rotational motion of underwater robots against parameter uncertainties. Underwater robots are expected to be used for various work assignments. The large variety of applications of underwater robots motivates researchers to develop control systems and technologies for underwater robots. Several control methods have been proposed so far for the stabilization of nominal system model of underwater robots with no parameter uncertainty. Parameter uncertainties are considered to be obstacles in implementation of the such nominal control methods for underwater robots. The objective of this study is to establish a robust stabilization method for rotational motion of underwater robots against parameter uncertainties. The effectiveness of the proposed method is verified by numerical simulations.

Keywords—Robust control, stabilization method, underwater robot, parameter uncertainty.

I. Introduction

In recent decades, the research interest in underwater robots has been increased because underwater robots can be used for various tasks in many fields such as rivers, lakes, ponds, seas. Underwater robots have become an important tool for various underwater tasks. For example, underwater robots can be used for investigating environmental condition under rivers, lakes, ponds, seas. Also, underwater robots are used for underwater inspection of sub-sea cables, oil and gas installations, and pipelines. Underwater robots are useful for various applications ranging from inspection to maintenance and cleaning of submerged surfaces and constructions. Moreover, underwater robots can be used for the exploration of the deep ocean. In the future, underwater robots play an important role to detect victims in flood disaster. The large variety of applications of underwater robots motivates researchers to develop control systems and technologies for autonomous underwater robots. Great efforts have been made in developing control methods of autonomous underwater robots to overcome challenging scientific and engineering problems caused by the underwater environment.

The control problem of autonomous underwater robots is very challenging due to the nonlinearity, time-variance, uncertain disturbances, such as the external forces generated by the fluid flow fluctuation, and the difficulty in accurately modeling the fluid phenomena. Some classical linear control systems were designed based on a simplified underwater robot model. However, they often resulted in poor performance because of the nonlinear, time-varying robot dynamics. Therefore, it is desirable to have a control system with the capability of adapting to the changes in the nonlinear and time-varying dynamics. Much efforts have been made in developing nonlinear control methods of autonomous underwater robots to overcome challenging nonlinear and time-varying problems.

Nonlinear model predictive control method has been proposed for remotely operated underwater robots in [1]. Model predictive control (MPC) [2]-[4], also known as receding horizon control [5]-[10], is a model-based control algorithm that solves a finite horizon optimal control problem, using the current state of the system as the initial state. As in traditional linear MPC, nonlinear MPC calculates control inputs at each control interval using a combination of model-based prediction and constrained optimization. The key differences between linear MPC and nonlinear MPC are as follows: The prediction model can be nonlinear and include time-varying parameters. The scalar cost function to be minimized can be a nonlinear function of the decision variables. Therefore, nonlinear MPC was well developed in [1] to deal with the nonlinear and time-varying properties of underwater robots. However, nonlinear MPC proposed in [1] uses a nominal system model of underwater robots without considering parameter uncertainties. The controller developed for a nominal system model may fail in satisfying performance requirements especially when changes in the system and environment occur during underwater robot movement.

The starting point of most studies of control of underwater robots is the system model that contains no parametric uncertainty. Parameter uncertainties are considered to be obstacles in implementation of control methods designed for a nominal system model of underwater robots. Therefore, it is desirable to have a control system with the robustness against parameter uncertainties. To this purpose, we consider the robust stabilization problem of rotational motion of underwater robots against parameter uncertainties. The objective of this study is to propose a robust control for the stabilization of rotational motion of underwater robots subject to parameter uncertainties.

This paper is organized as follows. In Section II, we define the system model and notations. In Section III, we consider the robust stabilization problem of rotational motion of underwater robots against parameter uncertainties. In Section IV, we provide the results of numerical simulations that verify the effectiveness of the proposed method. Finally, some concluding remarks are given in Section V.

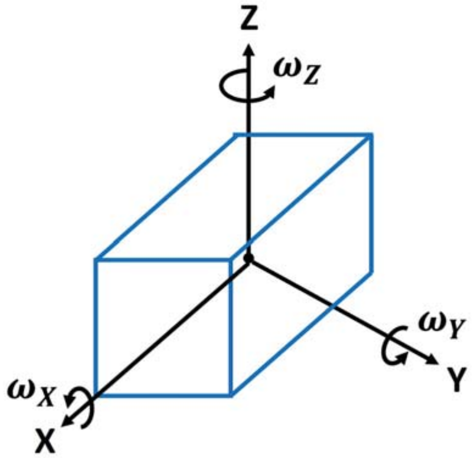

Fig. 1 A schematic view of underwater robot

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II. Notation and System Model

In this section, we introduce some notations that are adopted throughout this paper. Let the set of real numbers be denoted by $\mathbb{R}$. Let the set of non-negative real numbers be denoted by $\mathbb{R}_+$. Let $t \in \mathbb{R}_+$ denote the temporal variable.

A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to class $\mathbb{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to class $\mathbb{K}_\infty$ if $\alpha \in \mathbb{K}$ and $\lim_{s\to\infty} \alpha(s) = \infty$. Let the norm $\|x\|$ be defined by $\|x\| := \sqrt{x^{\mathrm{T}}x}$.

First, we introduce the system model of a rigid body. Let us consider a rigid body in an inertial reference frame and let $\omega_X(t)$, $\omega_Y(t)$, and $\omega_Z(t)$ denote the angular velocity components along a body fixed reference frame having the origin at the center of gravity and consisting of three principal axes. The Euler's equations for the rigid body with three independent controls aligned with three principal axes are

$$I_X\dot{\omega}_X(t) = (I_Y - I_Z)\omega_Y(t)\omega_Z(t) + v_1(t), \tag{1a}$$
$$I_Y\dot{\omega}_Y(t) = (I_Z - I_X)\omega_Z(t)\omega_X(t) + v_2(t), \tag{1b}$$
$$I_Z\dot{\omega}_Z(t) = (I_X - I_Y)\omega_X(t)\omega_Y(t) + v_3(t), \tag{1c}$$

where $I_X > 0$, $I_Y > 0$, and $I_Z > 0$ denote the principal moments of inertia and $v_1(t)$, $v_2(t)$, and $v_3(t)$ denote the control torques. The system model of underwater robots is different from the equations of (1) because the properties of fluid dynamics are not considered in (1). In this study, we assume that the flow velocity of fluids is sufficiently small so that we can neglect the influence of fluid momentum. When a rigid body is moving in a fluid, the additional inertia of the fluid surrounding the body, that is accelerated by the movement of the body, has to be taken into consideration [11]. This effect can be neglected in robotics on the ground since the density of the air is much lighter than the density of a moving mechanical system. In underwater applications, however, the density of the water is comparable with the density of the robots. The fluid surrounding the robot is accelerated with the robot itself, a force is then necessary to achieve this acceleration. The fluid exerts a reaction force which is equal in magnitude and opposite in direction. This reaction force is the added mass and inertia contribution. The added mass and inertia are not quantities of fluid to add to the system such that it has increased mass and inertia. Here, let $J_X > 0$, $J_Y > 0$, and $J_Z > 0$ denote the added moments of inertia.

Under the above assumptions, the system model of underwater robots is described by

$$(I_X + J_X)\dot{\omega}_X(t) = (I_Y + J_Y - I_Z - J_Z)\omega_Y(t)\omega_Z(t) + v_1(t), \tag{2a}$$
$$(I_Y + J_Y)\dot{\omega}_Y(t) = (I_Z + J_Z - I_X - J_X)\omega_Z(t)\omega_X(t) + v_2(t), \tag{2b}$$
$$(I_Z + J_Z)\dot{\omega}_Z(t) = (I_X + J_X - I_Y - J_Y)\omega_X(t)\omega_Y(t) + v_3(t). \tag{2c}$$

The system model (2) can be rewritten as

$$\dot{\omega}_X(t) = \frac{I_Y + J_Y - I_Z - J_Z}{I_X + J_X}\omega_Y(t)\omega_Z(t) + \frac{v_1(t)}{I_X + J_X}, \tag{3a}$$
$$\dot{\omega}_Y(t) = \frac{I_Z + J_Z - I_X - J_X}{I_Y + J_Y}\omega_Z(t)\omega_X(t) + \frac{v_2(t)}{I_Y + J_Y}, \tag{3b}$$
$$\dot{\omega}_Z(t) = \frac{I_X + J_X - I_Y - J_Y}{I_Z + J_Z}\omega_X(t)\omega_Y(t) + \frac{v_3(t)}{I_Z + J_Z}. \tag{3c}$$

For notational simplicity, we introduce the following notations:

$$a_1 := \frac{I_Y + J_Y - I_Z - J_Z}{I_X + J_X}, \tag{4a}$$
$$a_2 := \frac{I_Z + J_Z - I_X - J_X}{I_Y + J_Y}, \tag{4b}$$
$$a_3 : -\frac{I_X + J_X - I_Y - J_Y}{I_Z + J_Z}, \tag{4c}$$
$$b_1 := \frac{1}{I_X + J_X}, \tag{4d}$$
$$b_2 := \frac{1}{I_Y + J_Y}, \tag{4e}$$
$$b_3 := \frac{1}{I_Z + J_Z}. \tag{4f}$$

Furthermore, we introduce the state vector of system defined by

$$x(t) := \begin{bmatrix} \omega_X(t) \\ \omega_Y(t) \\ \omega_Z(t) \end{bmatrix}. \tag{5}$$

Using these notations in (4) and (5), the system model of underwater robots can be described by the following state equation:

$$\dot{x}(t) = f(x(t), v(t)), \tag{6}$$
$$f(x(t), v(t)) := \begin{bmatrix} a_1x_2(t)x_3(t) + b_1v_1(t) \\ a_2x_3(t)x_1(t) + b_2v_2(t) \\ a_3x_1(t)x_2(t) + b_3v_3(t) \end{bmatrix}.$$

Note that a_i and b_i for $i = 1, 2, 3$ are constant system parameters. In general, it is difficult to precisely identify the system parameters. From the practical point of view, certain amounts of model errors need to be considered. In this study, we assume that the system parameters a_i $(i = 1, 2, 3)$ are not exactly known and system parameters $\tilde{a}_i$ $(i = 1, 2, 3)$ that contain model uncertainties are only available to design the control system for stabilizing the rotational motion of underwater robot dynamics (6). Note that $\tilde{a}_i$ are given by

$$\tilde{a}_1 := a_1 + \Delta a_1 \tag{7a}$$
$$\tilde{a}_2 := a_2 + \Delta a_2 \tag{7b}$$
$$\tilde{a}_3 := a_3 + \Delta a_3 \tag{7c}$$

where Δa_i $(i = 1, 2, 3)$ are uncertain parameters. Parameter uncertainties such as model errors are considered to be obstacles in implementation of control methods designed for a nominal system model of underwater robots. Therefore, it is desirable to have a control system with the robustness against parameter uncertainties. To this purpose, we consider the robust stabilization problem of rotational motion of underwater robots against parameter uncertainties. The objective of this study is to propose a robust control for the stabilization of rotational motion of underwater robots subject to parameter uncertainties.

III. Robust Stabilization of Underwater Robots

In this section, we propose a robust stabilization method for system model (6). Under the assumption that the system parameters $\tilde{a}_i$ that contain parameter uncertainties are only available to design the control system for stabilizing the rotational motion of underwater robot dynamics (6), we propose the following control inputs:

$$v_1 = \frac{1}{b_1}(-\tilde{a}_1x_2(t)x_3(t) - k_1x_1), \tag{8a}$$
$$v_2 = \frac{1}{b_2}(-\tilde{a}_2x_3(t)x_1(t) - k_2x_2), \tag{8b}$$
$$v_3 = \frac{1}{b_3}(-\tilde{a}_3x_1(t)x_2(t) - k_3x_3), \tag{8c}$$

where k_i $(i = 1, 2, 3)$ are feedback gain to be designed so as to satisfy the robust stability condition.

Employing the control inputs (8) in (6), we see that the feedback closed-loop system is given by

$$\dot{x}(t) = \begin{bmatrix} -\Delta a_1 x_2(t)x_3(t) - k_1 x_1 \\ -\Delta a_2 x_3(t)x_1(t) - k_2 x_2 \\ -\Delta a_3 x_1(t)x_2(t) - k_3 x_3 \end{bmatrix}. \tag{9}$$

In the following, we provide the robust stability analysis of the system (9) which is given by substituting (7) and (8) into (6). The robust stability analysis in this study is based on the Lyapunov stability theory. The following statement is well known as Lyapunov stability theory.

Consider a system $\dot{x}(t) = f(x(t))$. Suppose that there exist a Lyapunov function $V(x) : \mathbb{R}^n \to \mathbb{R}_+$, class $\mathbb{K}_\infty$ functions α_1, α_2 and a positive definite function α_3 satisfying all the following conditions:

$$V(x) \geq \alpha_1(\|x\|),$$
$$V(x) \leq \alpha_2(\|x\|),$$
$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\dot{x} = \frac{\partial V(x)}{\partial x} f(x) \leq -\alpha_3(\|x\|).$$

Then, the origin $x = 0$ is asymptotically stable.

Here, let $V(x)$ be defined by

$$V(x) := \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \tag{10}$$

as a candidate of Lyapunov function. Then, the time derivative of $V(x)$ is given by

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\dot{x} = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \tag{11}$$

Substituting (9) into (11), we obtain the following equation:

$$\dot{V}(x) = -(\Delta a_1 + \Delta a_2 + \Delta a_3)x_1x_2x_3 - k_1x_1^2 - k_2x_2^2 - k_3x_3^2. \tag{12}$$

It is well known that the following relation between arithmetic mean and geometric mean holds true.

$$\frac{x_1^2 + x_2^2 + x_3^2}{3} \geq \sqrt[3]{x_1^2x_2^2x_3^2} \tag{13}$$

From (13), we have the following relation:

$$\frac{1}{3\sqrt{3}}\|x\|\|x\|^2 \geq x_1x_2x_3 \tag{14}$$

Applying (14) into (12), we have the following inequality:

$$\dot{V}(x) \leq \frac{|(\Delta a_1 + \Delta a_2 + \Delta a_3)|}{3\sqrt{3}}\|x\|\|x\|^2 - k_1x_1^2 - k_2x_2^2 - k_3x_3^2. \tag{15}$$

Let D be the constant defined so as to satisfy the following inequality:

$$\frac{|(\Delta a_1 + \Delta a_2 + \Delta a_3)|}{3\sqrt{3}}\|x\| \leq D. \tag{16}$$

Applying (16) into (15), we have the following inequality:

$$\begin{aligned}\dot{V}(x) &\leq D\|x\|^2 - k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \\ &= (D - k_1)x_1^2 + (D - k_2)x_2^2 + (D - k_3)x_3^2\end{aligned} \tag{17}$$

Suppose that $D < k_1$, $D < k_2$ and $D < k_3$, then we have the following:

$$\begin{aligned}\dot{V}(x) &\leq (D - k_1)x_1^2 + (D - k_2)x_2^2 + (D - k_3)x_3^2 \\ &\leq 0\end{aligned} \tag{18}$$

Consequently, we can state the following statement.

If the following condition is satisfied

$$D < k_i \quad \text{for all} \quad i = 1, 2, 3, \tag{19}$$

then, $x = 0$ of the system (9) is asymptotically stable.

It is seen that if we set the feedback gain so as to satisfy the inequality (19), then the closed-loop system (9) can be robustly stabilized using control input (8) against parameter uncertainties.

IV. Numerical Simulations

In this section, we provide numerical simulation results to verify the effectiveness of the proposed method. Here, we consider four cases for numerical simulations as shown in Table I.

TABLE I
SIMULATION CASES

	$[\Delta a_1, \Delta a_2, \Delta a_3]$	$[k_1, k_2, k_3]$	D
Case 1	[0.0662, -0.0061, -0.031]	[0.01, 0.01, 0.01]	0.161
Case 2	[0.0662, -0.0061, -0.031]	[5, 5, 5]	0.161
Case 3	[0.0071, -0.0007, -0.0033]	[0.01, 0.01, 0.01]	1.51
Case 4	[0.0071, -0.0007, -0.0033]	[10, 10, 10]	1.51

Other parameters employed in the numerical simulations are listed in Table II.

TABLE II
SIMULATION PARAMETERS

I_X	0.438
I_Y	0.833
I_Z	0.758
J_X	34.9
J_Y	101
J_Z	82.5
$x(0)$	$[10, 10, 10]^{\mathrm{T}}$

The results of numerical simulations by the proposed method are shown below. Fig. 2 shows the time responses of $x(t)$ in case 1. Fig. 3 shows the time responses of $x(t)$ in case 2. Fig. 4 shows the time responses of $x(t)$ in case 3. Fig. 5 shows the time responses of $x(t)$ in case 4. These figures reveal the effectiveness of the proposed method. It can be seen that the feedback gains satisfying (19) make the system robustly asymptotically stable.

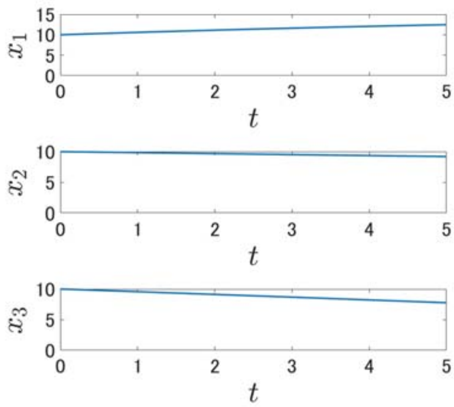

Fig. 2 Time responses of $x(t)$ in case 1

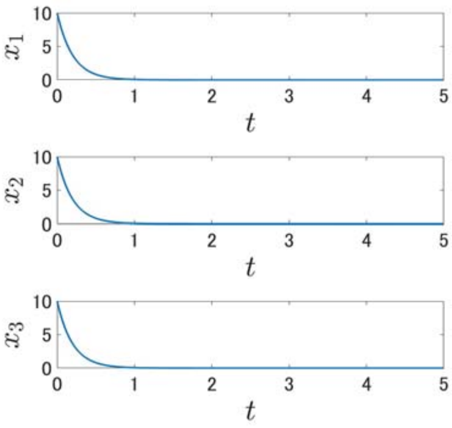

Fig. 3 Time responses of $x(t)$ in case 2

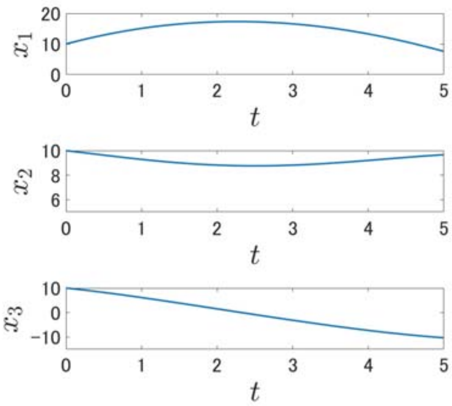

Fig. 4 Time responses of $x(t)$ in case 3

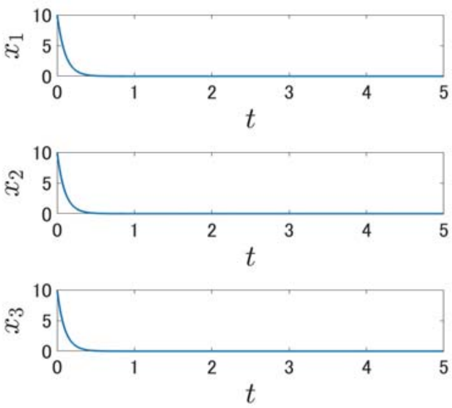

Fig. 5 Time responses of $x(t)$ in case 4

V. Conclusion

In this paper, we propose a robust stabilization method for rotational motion of underwater robots against parameter uncertainties. So far, most studies have proposed control systems for the nominal system model of underwater robots with no parameter uncertainty. However, parameter uncertainties are considered to be obstacles in implementation of such nominal control methods for underwater robots. Therefore, this study established a robust stabilization method for rotational motion of underwater robots against parameter uncertainties. The effectiveness of the proposed method was verified by numerical simulations.

It is also known that time delays may cause instabilities of the control systems of underwater robots [12]-[17]. The control problem of underwater robot systems with time delays is a possible future work.

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