Vibration Analysis of Magnetostrictive Nano-Plate by Using Modified Couple Stress and Nonlocal Elasticity Theories

Hamed Khani Arani, Mohammad Shariyat, Armaghan Mohammadian

***Abstract*—**In the present study, the free vibration of magnetostrictive nano-plate (MsNP) resting on the Pasternak foundation is investigated. Firstly, the modified couple stress (MCS) and nonlocal elasticity theories are compared together and taken into account to consider the small scale effects; in this paper not only two theories are analyzed but also it improves the MCS theory is more accurate than nonlocal elasticity theory in such problems. A feedback control system is utilized to investigate the effects of a magnetic field. First-order shear deformation theory (FSDT), Hamilton's principle and energy method are utilized in order to drive the equations of motion and these equations are solved by differential quadrature method (DQM) for simply supported boundary conditions. The MsNP undergoes in-plane forces in x and y directions. In this regard, the dimensionless frequency is plotted to study the effects of small scale parameter, magnetic field, aspect ratio, thickness ratio and compression and tension loads. Results indicate that these parameters play a key role on the natural frequency. According to the above results, MsNP can be used in the communications equipment, smart control vibration of nanostructure especially in sensor and actuators such as wireless linear micro motor and smart nano valves in injectors.

***Keywords*—**Feedback control system, magnetostrictive nano-plate, modified couple stress theory, nonlocal elasticity theory, vibration analysis.

I. Introduction

NANOSTRUCTURES have increased considerable attention among the experimental and theoretical research communities and in recent years, mechanical, electrical, and chemical properties of nanostructures have drawn the attention of various researchers. One of the typical structures of nano-systems is nano-plates, which are two-dimensional and feature superior mechanical characteristics compared to conventional engineering materials. A vast area of novel applications of these nanostructures is foreseen in the coming years. These include aerospace, biomedical, bioelectrical, superfast microelectronics, etc. Understanding the accurate mechanical and physical properties of these nanostructures and their impacts on its performance and reliability are thus necessary for its production applications. Also, nano-plates have applications in various fields of nanotechnology, for example in nano-electromechanical devices, they can be potentially exploited as bio and mechanical sensors, electro-catalysts, DNA detectors, drug deliverer, and energy storage systems [1], [2].

Magnetostriction is the change of ferromagnetic materials shape by elongating or contracting in the direction of the magnetic field. Magnetostrictive materials (MsMs) such as iron, ferrite, nickel, cobalt and their alloys such as Terfenol-D can provide large strain and quick response; also, these materials are appropriate in providing giant forces, strains, high-energy densities, noise, and vibration control, and have applications in the development of fusion reactors, communications equipment, and computers [3]-[5]. Therefore, MsNP can improve the properties of plates and it has different applications at various means in leading years. The following papers are a small part of done works in this field.

Hua et al. [6] first introduced and described magnetostriction and the history of MsM. After that, they reviewed the recent developments of both rare earth and non-rare earth MsM and presented the tendency of their development. An application of MsM, analysis of thermal vibration and transient response of them by using the generalized DQM was investigated by Hong [7], [8]. He examined some parametric effects on the Terfenol-D functionally graded material plates such as shear correction coefficient values, the thickness of a mounted magnetostrictive layer, control gain values, temperature of the environment, and the effect of different mechanical boundary conditions. Pradhan and Kumar [9] proposed the small scale effect on the vibration analysis of orthotropic single-layered graphene sheets embedded in an elastic medium that was obtained using nonlocal elasticity and classical shear deformation plate theory. They considered the principle of virtual work; the governing differential equations were derived and solved by DQM for various boundary conditions. Arani et al. [10] described the free vibration of rectangular nanoplate made of MsMs on orthotropic patterns of the Pasternak foundation. Reddy's third-order shear deformation theory along with Eringen's nonlocal continuum model was utilized to derive motion equations at the nanoscale using Hamilton's principle. A size-dependent model for bending and free vibration of functionally graded Reddy plate based on a MCS theory was analyzed by Thai and Kim [11]. They resulted that the inclusion of small scale effects increases plate stiffness and frequency. Akgöz and Civalek [12] developed modeling and analysis of micro-sized plates for bending, buckling, and

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vibration resting on elastic medium using the MCS theory and Hamilton's principles. Arani et al. [13] presented free vibration of the magnetostrictive sandwich composite microplate with magnetostrictive core and composite face sheets. The MCS theory was taken into account to consider the small scale effects.

In this research, the free vibration response of the rectangular nanoplate made of MsM by FSDT is studied and two different theories are compared to consider the small scale effects.

II. GOVERNING EQUATIONS

An embedded MsNP system by two parameters foundation under the in-plane force N_x, N_y is considered in Fig. 1 in which geometrical parameters of length a, width b and thickness h are indicated and the Cartesian coordinate system (x,y,z) is introduced.

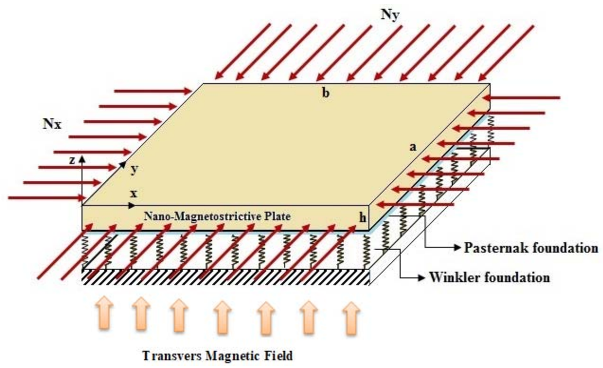

Fig. 1 Geometry and coordinate of MsNP

A. FSDT

According to the FSDT of the plate, the transverse shear strain is assumed to be constant and shear correction factors are introduced to correct the discrepancy between the actual transverse shear force distributions, the displacement components of the middle surface along the x, y, and z axes, shown by $\tilde{U}, \tilde{V}, \tilde{W}$ can be expressed as [14]:

$$\begin{aligned} \tilde{U}(x,y,z,t) &= u_0(x,y,t) + z\,\varphi_1(x,y,t) \\ \tilde{V}(x,y,z,t) &= v_0(x,y,t) + z\,\varphi_2(x,y,t) \\ \tilde{W}(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where $u_0(\mathrm{x,y,t})$, $v_0(\mathrm{x,y,t})$, $w_0(\mathrm{x,y,t})$ are displacements along with (x, y, z) directions and $\varphi_1(x,y,t), \varphi_2(x,y,t)$ are rotations about x and y axes.

The linear strain field for FSDT is obtained by using Hooke's law that can be represented as:

$$\varepsilon_{ij} = \frac{1}{2}(\mathrm{u}_{ij} + \mathrm{u}_{ji}) \tag{2}$$

B. Constitutive Equations

Stress-strain and magnetic field relations for MsMs are shown in (3) [15]:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & 0 & 0 & 0 \\ q_{21} & q_{22} & q_{23} & 0 & 0 & 0 \\ q_{31} & q_{32} & q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{34} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ H_z \end{bmatrix} \tag{3}$$

where σ_{ij} and q_{ij} are stress and the terms of engineering constants, respectively.

$$q_{ij} = \begin{bmatrix} \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & & & \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E\upsilon}{(1-2\upsilon)(1+\upsilon)} & \frac{E(1-\upsilon)}{(1-2\upsilon)(1+\upsilon)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} \end{bmatrix} \tag{4}$$

E and υ are Young modulus and Poisson's ratio, also e_{ij} are magnetostrictive coupling modules determined as [15]:

$$\begin{aligned} e_{31} &= \tilde{e}_{31}\cos^2\theta + \tilde{e}_{32}\sin^2\theta \\ e_{32} &= \tilde{e}_{31}\sin^2\theta + \tilde{e}_{32}\cos^2\theta \\ e_{34} &= (\tilde{e}_{31} - e_{32})\sin\theta\sin\theta \end{aligned} \tag{5}$$

where θ represents the direction along which a given magnetic anisotropy may have been induced. H_z is the magnetic field intensity and can be expressed as follows [15]-[17]:

$$H_z = K_c I(x,y,t) = K_c C(t)\frac{\partial w(x,y,z,t)}{\partial t} \tag{6}$$

where K_c, $I(t)$ and $C(t)$ are the coil constant, coil current and the control gain in which $K_c C(t)$ is introduced as velocity feedback gain.

C. Strain Energy Based on Nonlocal Elasticity Theory (Eringen's Theory)

The nonlocal elasticity theory is assumed that the stress at a point is a function of strains at all points in the continuum. The nonlocal constitutive equation given by Eringen is [18]:

$$(1-\mu\nabla^2)\sigma_{ij}^{nl} = \sigma_{ij}^{l} \qquad for: i,j = x,y,z \tag{7}$$

where σ_{ij}^{nl} and σ_{ij}^{l} are the nonlocal stress tensor and local stress tensor, e_0 denotes a constant appropriate to each material, and a is an internal characteristic length of the material. Consequently, $e_0 a$ is a constant parameter and ∇^2 is the Laplacian operator in the above equations.

According to (7) and the magneto-mechanical coupling for isotropic MsM, stress-strain relation can be observed in a matrix (8) [15], [18]:

$$\left[\sigma_{ij} - \mu\nabla^2\sigma_{ij}\right] = \left[q_{ij}\right]\left\{\varepsilon_{ij}\right\} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{34} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ H_z \end{bmatrix} \tag{8}$$

Therefore, the strain energy of an elastic body for rectangular nano-plate based on Nonlocal elasticity theory is expressed as [19]:

$$U = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^b\int_0^a \left(\sigma^{nl}{}_{xx}\varepsilon_{xx} + \sigma^{nl}{}_{yy}\varepsilon_{yy} + \sigma^{nl}{}_{zz}\varepsilon_{zz} + \tau^{nl}{}_{xy}\gamma_{xy} + \tau^{nl}{}_{xz}\gamma_{xz} + \tau^{nl}{}_{yz}\gamma_{yz}\right) dx\, dy\, dz \tag{9}$$

D. Strain Energy Based on MCS Theory

Based on the MCS theory, the density of strain energy is a function of curvature (conjugated with couple stress) and strain (conjugated with stress); the strain energy in an isotropic linear elastic material is given by [20], [21]:

$$U = \frac{1}{2}\int_{-h/2}^{h/2}\int_0^b\int_0^a (\sigma_{ij}\varepsilon_{jk} + 2l_0^2 G\,\chi_{ij}\,\chi_{ij})dx\; dy\; dz \tag{10}$$

where l_0 and G are material length scale parameter and shear module; also, χ_{ij} is symmetric curvature tensor which is defined as [21]:

$$\chi_{ij} = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}), \quad \theta_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{11}$$

where e_{ijk} is the permutation symbol.

E. Kinetic Energy

The kinetic energy of the rectangular plate is calculated as [19]:

$$K = \frac{\rho A}{2}\int_0^b\int_0^a\left[\left(\frac{\partial\tilde{U}}{\partial t}\right)^2 + \left(\frac{\partial\tilde{V}}{\partial t}\right)^2 + \left(\frac{\partial\tilde{W}}{\partial t}\right)^2\right]dx\; dy \tag{12}$$

where ρ and A are the mass density and area of the MsNP.

F. In-Plane Forces

Rectangular plates are usually subjected to in-plane forces; therefore, the in-plane stresses effects must be considered in their analysis and vibrations. Uniform in-plane forces *Nx* and *Ny* are applied in *x* and *y* directions as shown in Fig. 1 and calculated as [22]:

$$F_I = N_x\left(\frac{\partial^2 W}{\partial x^2}\right) + N_y\left(\frac{\partial^2 W}{\partial y^2}\right) \tag{13}$$

G. Elastic Medium

Pasternak foundation is capable to consider transverse shear loads and normal loads. The effect of surrounding elastic medium on the nano-plate which is simulated with Pasternak model is considered as follows [23]:

$$F_{II} = k_w W - k_G\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) \tag{14}$$

where k_w and k_G are the Winkler modulus for a normal load and the shear modulus for transverse shear loads.

H. External Work

The external work due to in-plane forces and the elastic medium is calculated as:

$$\Sigma = \frac{1}{2}\int_0^b\int_0^a F_I\, W\; dxdy + \frac{1}{2}\int_0^b\int_0^a F_{II}\, W\; dxdy \tag{15}$$

I. Hamilton's Principle

In this step, Hamilton's principle is employed to obtain the motion equations and corresponding boundary conditions. This principle can be expressed as follows:

$$\delta\int_{t_1}^{t_2}\left[U - (K + \Sigma)\right]dt = 0 \tag{16}$$

where δU, δK and $\delta\Sigma$ are a variation of strain energy, the variation of Kinetic energy and variation of external work. Substituting (9)-815) into (16) for FSDT and afterward using dimensionless parameters introduced in (17)-(19):

Both theories

$$(\zeta,\eta) = \left(\frac{x}{a},\frac{y}{b}\right), \quad (U, V, W) = \left(\frac{u_0}{a},\frac{v_0}{b},\frac{w_0}{h}\right),$$
$$(\alpha,\beta,\gamma) = \left(\frac{h}{a},\frac{h}{b},\frac{a}{b}\right), \phi_1 = \frac{\varphi_1}{h}, \quad \phi_2 = \frac{\varphi_2}{h}, Q_{ij} = \frac{q_{ij}}{E},$$
$$G_{ij} = \frac{e_{ij}\,C(t)K_c}{\sqrt{E\rho}}, \quad \tau = \frac{t}{a}\sqrt{\frac{E}{\rho}}, \quad K^*_w = \frac{k_w h}{E},$$
$$K^*_g = \frac{K_G}{a.E}, \; N^*_x = \frac{N_x}{a.E}, \; N^*_y = \frac{N_y}{b.E}, \; \bar{\omega} = \frac{\omega}{a}\sqrt{\frac{E}{\rho_m}} \tag{17}$$

Only Nonlocal theory

$$e_n = \frac{\mu}{a^2}, \quad \psi = \frac{\mu}{b^2} = e_n.\gamma^2 \tag{18}$$

Only couple stress theory

$$L_0 = \frac{l_0}{a}, \; G^* = \frac{G}{E} \tag{19}$$

J. Motion Equations

The motion equations for nonlocal elasticity theory and FSDT are obtained by setting the coefficient δU, δV, δW, $\delta\varphi_1$, $\delta\varphi_2$ equal to zero as:

$$\delta U: \quad -\frac{1}{2}Q_{66}\gamma\beta\frac{d^2U}{d\eta^2} - Q_{11}\alpha\frac{d^2U}{d\zeta^2} - \frac{1}{2}Q_{66}\alpha\frac{d^2V}{d\eta d\zeta} - Q_{21}\alpha\frac{d^2V}{d\eta d\zeta}$$
$$+\alpha\frac{d^2U}{d\tau^2} - \alpha e_n\frac{d^4U}{d\tau^2 d\zeta^2} - \alpha\psi\frac{d^4U}{d\tau^2 d\eta^2} + \frac{1}{2}G_{31}\alpha^2\frac{d^2W}{d\tau d\zeta} = 0 \tag{20}$$

$$\delta V: \quad -\frac{1}{2}Q_{66}\beta\frac{d^2U}{d\eta d\zeta} - Q_{21}\beta\frac{d^2U}{d\eta d\zeta} - Q_{22}\beta\frac{d^2V}{d\eta^2} - \frac{1}{2}\frac{Q_{66}\alpha}{\gamma}\frac{d^2V}{d\zeta^2}$$
$$+\frac{\alpha}{\gamma}\frac{d^2V}{d\tau^2} - \frac{\alpha e_n}{\gamma}\frac{d^4V}{d\tau^2 d\zeta^2} - \beta\phi\frac{d^4V}{d\tau^2 d\eta^2} + \frac{1}{2}G_{32}\alpha\beta\frac{d^2W}{d\tau d\eta} = 0 \tag{21}$$

$$\begin{aligned}\delta W:\quad & -\frac{1}{2}\alpha^2 Q_{55}\frac{d^2W}{d\zeta^2}-\frac{1}{2}\beta^2 Q_{44}\frac{d^2W}{d\eta^2}-\frac{1}{2}\alpha Q_{55}\frac{d\phi_1}{d\zeta}-\frac{1}{2}\beta Q_{44}\frac{d\phi_2}{d\eta}\\ & +\alpha^2\frac{d^2W}{d\tau^2}-e_n\alpha^2\frac{d^4W}{d\tau^2 d\zeta^2}-\psi\alpha^2\frac{d^4W}{d\tau^2 d\eta^2}+\frac{1}{2}G_{31}\alpha\frac{d^2U}{d\tau d\zeta}\\ & +\frac{1}{2}G_{32}\alpha\frac{d^2V}{d\tau d\eta}-K_w^*W+e_nK_w^*\frac{d^2W}{d\zeta^2}+\psi K_w^*\frac{d^2W}{d\eta^2}+K_g^*\alpha\frac{d^2W}{d\zeta^2}\\ & -e_nK_g^*\alpha\frac{d^4W}{d\zeta^4}-\psi K_g^*\alpha\frac{d^4W}{d\zeta^2 d\eta^2}-K_g^*\gamma\beta\frac{d^2W}{d\eta^2}-e_nK_g^*\gamma\beta\frac{d^4W}{d\zeta^2 d\eta^2}\\ & -\psi K_g^*\gamma\beta\frac{d^4W}{d\eta^4}+N_x^*\alpha\frac{d^2W}{d\zeta^2}-e_nN_x^*\alpha\frac{d^4W}{d\zeta^4}-\psi N_x^*\alpha\frac{d^4W}{d\eta^2 d\zeta^2}\\ & +N_y^*\beta\frac{d^2W}{d\eta^2}-e_nN_y^*\beta\frac{d^4W}{d\eta^2 d\zeta^2}-\psi N_y^*\beta\frac{d^4W}{d\eta^4}=0\end{aligned}\tag{22}$$

$$\begin{aligned}\delta\phi_1:\quad & \frac{1}{2}\alpha Q_{55}\frac{dW}{d\zeta}-\frac{1}{24}Q_{66}\beta^2\frac{d^2\phi_1}{d\eta^2}-\frac{1}{12}Q_{11}\alpha^2\frac{d^2\phi_1}{d\zeta^2}+\frac{1}{2}Q_{55}\phi_1-\frac{1}{24}Q_{66}\beta\alpha\frac{d^2\phi_2}{d\eta d\zeta}\\ & -\frac{1}{12}Q_{21}\beta\alpha\frac{d^2\phi_2}{d\eta d\zeta}+\frac{1}{12}\alpha^2\frac{d^2\phi_1}{d\tau^2}-\frac{1}{12}e_n\alpha^2\frac{d^4\phi_1}{d\tau^2 d\zeta^2}-\frac{1}{12}\psi\alpha^2\frac{d^4\phi_1}{d\tau^2 d\eta^2}=0\end{aligned}\tag{23}$$

$$\begin{aligned}\delta\phi_2:\quad & \frac{1}{2}\beta Q_{44}\frac{dW}{d\eta}-\frac{1}{24}Q_{66}\beta\alpha\frac{d^2\phi_1}{d\eta d\zeta}-\frac{1}{12}Q_{21}\beta\alpha\frac{d^2\phi_1}{d\eta d\zeta}-\frac{1}{24}Q_{66}\alpha^2\frac{d^2\phi_2}{d\zeta^2}\\ & +\frac{1}{2}Q_{44}\phi_2-\frac{1}{12}Q_{22}\beta^2\frac{d^2\phi_2}{d\eta^2}+\frac{1}{12}\alpha^2\frac{d^2\phi_2}{d\tau^2}-\frac{1}{12}e_n\alpha^2\frac{d^4\phi_2}{d\tau^2 d\zeta^2}-\frac{1}{12}\psi\alpha^2\frac{d^4\phi_2}{d\tau^2 d\eta^2}=0\end{aligned}\tag{24}$$

Also; the motion equations for MCS theory and FSDT are obtained by setting the coefficient δU ,δV ,δW ,$\delta\varphi_1$,$\delta\varphi_2$ equal to zero as:

$$\begin{aligned}\delta U:\quad & -Q_{11}\alpha\frac{\partial^2U}{\partial\zeta^2}-Q_{66}\alpha\frac{\partial^2V}{\partial\eta\partial\zeta}-\frac{1}{2}Q_{21}\alpha\frac{\partial^2V}{\partial\eta\partial\zeta}-\frac{1}{2}Q_{12}\alpha\frac{\partial^2V}{\partial\eta\partial\zeta}\\ & -\frac{1}{4}L_0^2G^*\beta\gamma\frac{\partial^4V}{\partial\eta^3\partial\zeta}-\frac{1}{4}L_0^2G^*\alpha\frac{\partial^4V}{\partial\eta\partial\zeta^3}-Q_{66}\beta\gamma\frac{\partial^2U}{\partial\eta^2}+\alpha\frac{\partial^2U}{\partial\tau^2}\\ & +\frac{1}{4}L_0^2G^*\beta\gamma\frac{\partial^4U}{\partial\eta^2\partial\zeta^2}+\frac{1}{4}L_0^2G^*\beta\gamma^3\frac{\partial^4U}{\partial\eta^4}+\frac{1}{2}G_{31}\alpha^2\frac{\partial^2W}{\partial\tau\partial\zeta}=0\end{aligned}\tag{25}$$

$$\begin{aligned}\delta V:\quad & -Q_{22}\beta\frac{\partial^2V}{\partial\eta^2}-Q_{66}\beta\frac{\partial^2U}{\partial\eta\partial\zeta}-\frac{1}{2}Q_{21}\beta\frac{\partial^2U}{\partial\eta\partial\zeta}-\frac{1}{2}Q_{12}\beta\frac{\partial^2U}{\partial\eta\partial\zeta}\\ & -\frac{1}{4}L_0^2\gamma^2\frac{\partial^4U}{\partial\eta^3\partial\zeta}-\frac{1}{4}L_0^2G^*\beta\frac{\partial^4U}{\partial\eta\partial\zeta^3}-\frac{Q_{66}\alpha}{\gamma}\frac{\partial^2V}{\partial\zeta^2}+\frac{\alpha}{\gamma}\frac{\partial^2V}{\partial\tau^2}\\ & +\frac{1}{4}\frac{L_0^2G^*\alpha}{\gamma}\frac{\partial^4V}{\partial\zeta^4}+\frac{1}{4}L_0^2G^*\beta\frac{\partial^4V}{\partial\eta^2\partial\zeta^2}+\frac{1}{2}G_{32}\alpha\beta\frac{\partial^2W}{\partial\tau\partial\eta}=0\end{aligned}\tag{26}$$

$$\begin{aligned}\delta W:\quad & -Q_{44}\beta\frac{\partial\phi_2}{\partial\eta}-Q_{55}\alpha\frac{\partial\phi_1}{\partial\zeta}-\frac{1}{4}L_0^2G^*\beta\gamma\frac{\partial^3\phi_1}{\partial\eta^2\partial\zeta}-\frac{1}{4}L_0^2G^*\alpha\frac{\partial^3\phi_1}{\partial\zeta^3}\\ & -\frac{1}{4}L_0^2G^*\beta\gamma^2\frac{\partial^3\phi_2}{\partial\eta^3}-\frac{1}{4}L_0^2G^*\beta\frac{\partial^3\phi_2}{\partial\eta\partial\zeta^2}-Q_{44}\beta^2\frac{\partial^2W}{\partial\eta^2}\\ & -Q_{55}\alpha^2\frac{\partial^2W}{\partial\zeta^2}+\frac{1}{2}L_0G^*\beta^2\frac{\partial^4W}{\partial\eta^2\partial\zeta^2}+\frac{1}{4}L_0^2G^*\alpha^2\frac{\partial^4W}{\partial\zeta^4}\\ & +\frac{1}{4}L_0G^*\beta^2\frac{\partial^4W}{\partial\eta^4}-K_g^*\alpha\frac{d^2W}{d\zeta^2}-K_g^*\beta\frac{d^2W}{d\eta^2}\\ & +K_w^*W+\alpha\frac{\partial^2W}{\partial\tau^2}+N_x^*\alpha\frac{d^2W}{d\zeta^2}+N_y^*\beta\frac{d^2W}{d\eta^2}=0\end{aligned}\tag{27}$$

$$\begin{aligned}\delta\phi_1:\quad & -\frac{1}{12}Q_{11}\alpha^2\frac{\partial^2\phi_1}{\partial\zeta^2}+Q_{55}\alpha\frac{\partial W}{\partial\zeta}+\frac{1}{48}L_0^2G^*\beta^2\frac{\partial^4\phi_1}{\partial\eta^2\partial\zeta^2}-L_0^2G^*\gamma^2\frac{\partial^2\phi_1}{\partial\eta^2}\\ & +\frac{1}{48}L_0^2G^*\beta^2\gamma^2\frac{\partial^4\phi_1}{\partial\eta^4}-\frac{1}{24}Q_{21}\alpha\beta\frac{\partial^2\phi_2}{\partial\eta\partial\zeta}-\frac{1}{12}Q_{66}\alpha\beta\frac{\partial^2\phi_2}{\partial\eta\partial\zeta}\\ & +\frac{1}{4}L_0^2G^*\beta\gamma\frac{\partial^3W}{\partial\eta^2\partial\zeta}+\frac{1}{4}L_0^2G^*\alpha\frac{\partial^3W}{\partial\zeta^3}-\frac{1}{24}Q_{12}\alpha\beta\frac{\partial^2\phi_2}{\partial\eta\partial\zeta}\\ & +\frac{3}{4}L_0^2G^*\beta\gamma\frac{\partial^2\phi_2}{\partial\eta\partial\zeta}+Q_{55}\phi_1-\frac{1}{4}L_0^2G^*\frac{\partial^2\phi_1}{\partial\zeta^2}-\frac{1}{12}Q_{66}\beta^2\frac{\partial^2\phi_1}{\partial\eta^2}\\ & -\frac{1}{48}L_0^2G^*\beta^2\gamma\frac{\partial^4\phi_2}{\partial\eta^3\partial\zeta}-48L_0^2G^*\alpha\beta\frac{\partial^4\phi_2}{\partial\eta\partial\zeta^3}+\frac{1}{12}\alpha^2\frac{\partial^2\phi_1}{\partial\tau^2}=0\end{aligned}\tag{28}$$

$$\begin{aligned}\delta\phi_2:\quad & Q_{44}\beta_m\frac{\partial W}{\partial\eta}+\frac{1}{48}l_{0b}{}^2G^*\alpha_m{}^2\frac{\partial^4\phi_2}{\partial\zeta^4}+\frac{1}{48}l_{0b}{}^2G^*\beta_m{}^2\frac{\partial^4\phi_2}{\partial\eta^2\partial\zeta^2}\\ & -\frac{1}{24}Q_{21}\alpha_m\beta_m\frac{\partial^2\phi_1}{\partial\eta\partial\zeta}-\frac{1}{12}Q_{66}\alpha_m\beta_m\frac{\partial^2\phi_1}{\partial\eta\partial\zeta}-\frac{1}{24}Q_{12}\alpha_m\beta_m\frac{\partial^2\phi_1}{\partial\eta\partial\zeta}\\ & +\frac{1}{4}L_0^2G^*\beta\gamma^2\frac{\partial^3W}{\partial\eta^3}+\frac{3}{4}L_0^2G^*\beta\gamma\frac{\partial^2\phi_1}{\partial\eta\partial\zeta}+\frac{1}{4}L_0^2G^*\beta\frac{\partial^3W}{\partial\eta\partial\zeta^2}\\ & +Q_{44}\phi_2-L_0^2G^*\frac{\partial^2\phi_2}{\partial\zeta^2}-\frac{1}{4}l_{0b}{}^2G^*\gamma^2\frac{\partial^2\phi_2}{\partial\eta^2}-\frac{1}{12}Q_{66}\alpha^2\frac{\partial^2\phi_2}{\partial\zeta^2}\\ & -\frac{1}{48}L_0^2G^*\beta^2\gamma\frac{\partial^4\phi_1}{\partial\eta^3\partial\zeta}-\frac{1}{48}L_0^2G^*\alpha_m\beta\frac{\partial^4\phi_1}{\partial\eta\partial\zeta^3}+\frac{1}{12}\alpha^2\frac{\partial^2\phi_2}{\partial\tau^2}=0\end{aligned}\tag{29}$$

III. Solution Method

In this study, DQM has been utilized to solve motion equations and obtain frequency. In the DQM, the derivatives of a function are approximated with weighted sums of the function values at a group of grid points. Thus the partial derivatives of a function F (representing w) at a given point are expressed as [24]:

$$\frac{\partial^k F}{\partial R^k}=\sum_{k=1}^{N}A_{pq}^{(k)}F\left(R_i\right),\tag{30}$$

where N is the number of grid points in the radial direction and $A_{pq}^{(k)}$ is the respective weighting coefficients matrix.

Applying DQM and considering boundary conditions yields:

$$\begin{bmatrix}[\mathbf{A_{bb}}] & [\mathbf{A_{bd}}]\\ [\mathbf{A_{db}}] & [\mathbf{A_{dd}}]\end{bmatrix}\begin{Bmatrix}\{\mathbf{d_b}\}\\ \{\mathbf{d_d}\}\end{Bmatrix}=\begin{Bmatrix}\{\mathbf{B}\}\\ \{0\}\end{Bmatrix},\tag{31}$$

in which $\{B\}$ is boundary condition ($b: boundary$, $d: domain$). The eigenvalues of (31) are the frequency of the system.

IV. Numerical Results and Discussion

In this study, the vibration analysis of MsNP by two different theories for simply support boundary conditions is investigated. Table I shows the material properties of MsM.

TABLE I
ELASTIC PROPERTIES OF TERFENOL-D [8]

Properties	E (Pa)	υ	$\rho(kg/m^3)$	$e_{31}=e_{32}$
Terfenol-D	$30e9$	0.25	9.25×10^3	$442.55N/(m.A)$

Fig. 2 shows the variation of dimensionless natural frequency versus thickness ratio (α) by using MSC theory and nonlocal theory. α changes from 0.01 to 0.2 for thin and thick plates and this figure demonstrates that the increasing thickness ratio leads to an increase of frequency ($\sqrt{k/m}$) due to the effect of mass ($m=\rho a.\mathrm{b.h}$) and moment of inertia ($k\sim I\rightarrow I=\frac{1}{12}bh^3$) and consequently stability of MsNP increase. Also, it is obvious that the frequency of nano-plate by using MSC theory is more than nonlocal theory and the frequency of Macro-plate ($L_0=0$, $e_n=0$) is between them.

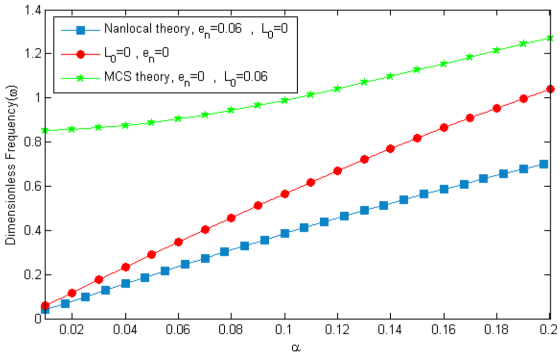

Fig. 2 Variation of dimensionless natural frequency versus thickness ratios in MCS and nonlocal theories ($\gamma = 1$, $K_c C(t) = 10^4$)

Fig. 3 illustrates the effect of the magnetic field on MsNP by changing the velocity feedback gain. It is worth to mention that when the MsMs are subjected to the magnetic field, they deform due to its reciprocal nature. As can be seen from the figure, the frequency of MsNP decreases with increasing velocity feedback gain from $K_c C(t) = 10^4$ *to* 10^5, especially at MCS theory.

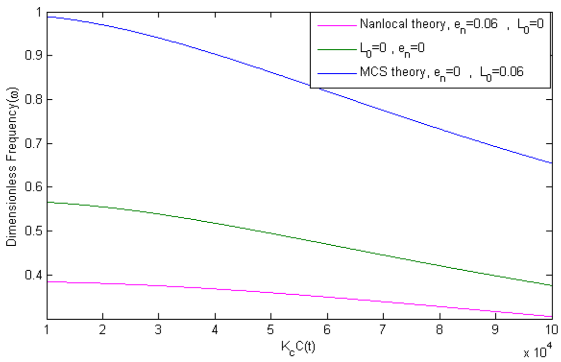

Fig. 3 Effect of magnetic field on the dimensionless frequency in MCS and nonlocal theories ($\gamma = 1$, $\alpha = 0.1$)

Figs. 4 and 5 depict the effect of the length scale parameter in the MCS where it changes from 0 to 0.1 in different aspect ratios. It clearly shows that the frequency increases with the increase of length scale parameter (L_0) at MCS theory and it decreases with the increase of length scale parameter (e_n) at Eringen's theory. The explanation might be that couple stress is needed to generate the gradient of rotation; the stiffness owing to the couple stress effect is added to the classical stiffness; thus, the total stiffness of the nano-plate is larger than that of its classical counterpart ($L_0 = 0$). Undoubtedly, with the increase of length scale parameter, the couple stress effect becomes more significant, therefore the stiffness of the nano-plate increases, which leads to the increase of the frequency and more stability while Fig. 5 shows that the effect of size in nonlocal theory which this figure has opposite results of MCS theory. Physical intuition reveals that increasing nanoscale stress leads to increasing the stiffness of nanostructure which is firstly established by Eringen's theory [10] while many papers [21], [23] have concluded otherwise. On the basis, MCS theory is more accurate than nonlocal theory in such problems. It is obvious that like thickness ratio, aspect ratio also increases the dimensionless frequency of MsNP.

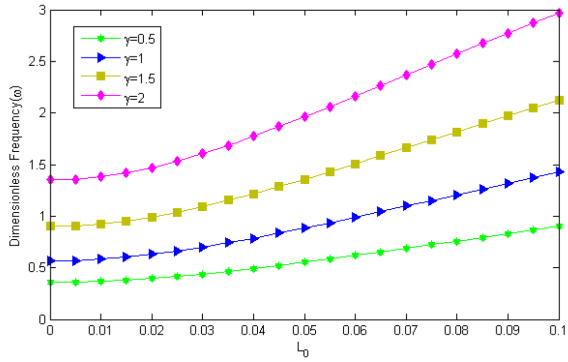

Fig. 4 Effect of size on the dimensionless frequency in the MCS theory ($\alpha = 0.1$, $K_c C(t) = 10^4$)

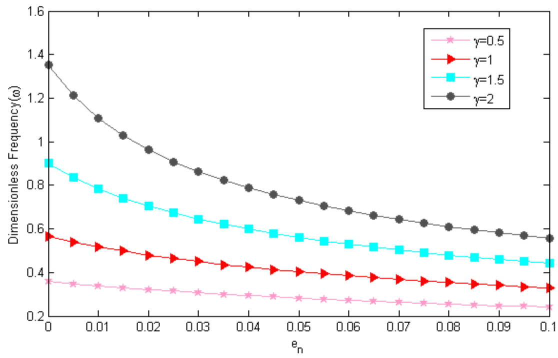

Fig. 5 Effect of size on the dimensionless frequency in the Nonlocal theory ($\alpha = 0.1$, $K_c C(t) = 10^4$)

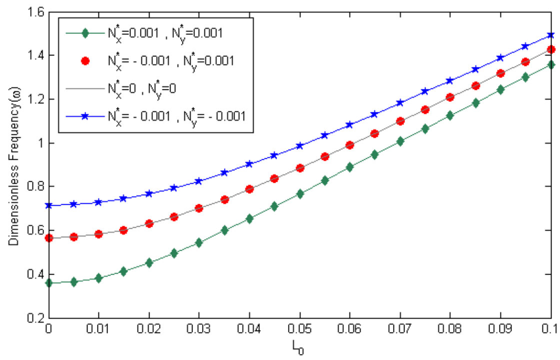

Fig. 6 Variation of dimensionless frequency versus length scale parameter in different in-plane forces ($\alpha = 0.1$, $\gamma = 1$, $K_c C(t) = 10^4$)

The effect of mechanical in-plane loadings is especially studied in Fig. 6; the result shows that in-plane forces change effectively the vibration response of embedded MsNP. Since the in-plane forces are vector quantity, the positive value

$(N_x^*, N_y^* \succ 0)$ indicates the compression force and negative value $(N_x^*, N_y^* \prec 0)$ shows the extensional or tension force where in-plane compression forces decrease the dimensionless frequency and cause the instability of the system, but tension force increases the frequency and more stability.

TABLE II
DIMENSIONLESS FREQUENCY FOR DIFFERENT WAVE NUMBERS
($\gamma = 1, \alpha = 0.1, K_c C(t) = 10^4$)

Dimensionless frequency	n=1	n=2	n=3	n=4
Eringen theory ($e_n = 0.06$)	0.3841	0.6827	0.9690	1.2081
MCS theory ($L_0 = 0.06$)	0.9893	2.2637	4.1219	6.4305

All of the figures in the present work were plotted for the first wavenumber, but at Table II, the natural frequency has been plotted for wavenumbers from 1 to 4. The natural frequency increases with an increase in the wavenumber.

V. CONCLUSION

At the present work, the free vibration of MsNP in the magnetic field was studied. Considering nonlocal elasticity and MCS theories, FSDT were utilized and motion equation was derived using Hamilton's principle. The vibration equation was solved by DQM in simply supported boundary conditions. The effects of various parameters such as aspect ratio, thickness ratio, small scale parameter, magnetic field and compression and tension loads were investigated. The brief result of this study is listed as follow:

- MSC theory is more accurate than nonlocal theory in such problems.
- Increasing the aspect and thickness ratio leads to an increase of natural frequency due to the effect of mass and moment of inertia.
- Increasing tension force leads to an increase of natural frequency while the compression force has a contrary effect.
- Velocity feedback gain as a control parameter can be used to reduce the frequency of MsP and control its vibration behavior.

According to the above results, MsNP can be used for active noise and vibration cancellation systems in nano/micro smart structures.

ACKNOWLEDGMENT

The authors would like to thank the reviewers for their comments and suggestions to improve the clarity of this article. This work was supported by KNTU and Iran's National Elites Foundation.

REFERENCES

[1] Y.S. Li, Z.Y. Cai, and S.Y. Shi, *Buckling and free vibration of magnetoelectro elastic nanoplate based on nonlocal theory*. Compos. Struct., 2014, vol. 111, pp.522-529.

[2] N. Radic, D. Jeremic, and S. Trifkovic, *buckling analysis of double-orthotropic nano-plates embedded in Pasternak elastic medium using nonlocal elasticity theory*. Compos Part B., 2014; vol. 61, pp. 162–171.

[3] J.P. Liu, E. Fullerton, O. Gutfleisch, D.J. Sellmyer, Nanoscale Magnetic Materials and Applications, Springer Publisher, New York, 2009.

[4] M. Di Sciuva, and M. Sorrenti, *Bending, free vibration and buckling of functionally graded carbon nanotube-reinforced sandwich plates, using the extended Refined Zigzag Theory*. Compos. Struct., 2019, vol. 227.

[5] C. Albrechts, Magnetostrictive materials M205. Technische Fakultat der Christian- Albrechts, University Zu Kiel, 2015.

[6] L. Jing Hua, J. Cheng Bao, and X. Hui Bin, *Giant magnetostrictive materials*, Science China Technological Sciences, 2012, vol. 5, pp. 1319–1326.

[7] C.C. Hong, *Application of a magnetostrictive actuator*, Mater. Des. 2013, vol. 46, pp. 617-621.

[8] C.C. Hong, *Thermal vibration and transient response of magnetostrictive functionally graded material plates*. Eur J Mech A Solids, 2014, vol. 43, pp. 78–88

[9] S.C. Pradhan, and A. Kumar, *Vibration analysis of orthotropic graphene sheets embedded in Pasternak elastic medium using nonlocal elasticity theory and differential quadrature method*, Comput. Mater. Sci, 2010, vol. 50, pp. 239- 245.

[10] A. Ghorbanpour Arani, Z. Khoddami Maraghiand, and H. Khani Arani, *Orthotropic patterns of Pasternak foundation in smart vibration analysis of magnetostrictive nanoplate*. Proc IMechE Part C, 2015, vol. 230, pp. 1–14.

[11] H.T. Thai, and S.E Kim, *A size-dependent functionally graded reddy plate model based on a modified couple stress theory*, Compos. Part B, 2013, vol. 45, pp. 1636–1645.

[12] B. Akgöz, and Ö. Civalek, *Modeling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory, Meccanica*, 2013, vol. 48, pp. 863–873.

[13] A. Ghorbanpour Arani, H. Khani Arani, and Z. Khoddami Maraghi, *Size-dependent in vibration analysis of magnetostrictive sandwich composite micro-plate in magnetic field using modified couple stress theory*, J. Sandwich Struct. Mater, 2017, vol. 21, pp.580–603.

[14] R. D. Mindlin, *Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates*, ASME Journal of Applied Mechanics., 1951, vol. 18, pp. 31–38.

[15] C.C. Hong, *Transient responses of magnetostrictive plates by using the GDQ method*. Eur J Mech A Solids, 2010, vol. 29, pp. 1015–1021.

[16] S.P. Timoshenko, *On the transverse vibrations of bars of uniform cross-section*, Philos, Mag. A., 1922, vol. 43, pp. 125–131.

[17] M. Krishna, M. Anjanappa, and Y.F. Wu, *The use of magnetostrictive particle actuators for vibration attenuation of flexible beams*, J. Sound Vib., 1997, vol. 206, pp. 133-149.

[18] A.C. Eringen, Nonlocal continuum field theories, Springer Publisher, New York, 2002.

[19] J.N. Reddy, Energy principles and variational methods in applied mechanics, Texas John Wiley & Sons Publishers, 2004.

[20] Y.S. Li, *buckling analysis of magneto electro elastic plate resting on Pasternak elastic foundation.* Mechanics Research Communications. 2015, vol. 56, pp. 104–114.

[21] E. Jomehzadeh, H.R. Noori, and A.R. Saidi, *the size-dependent vibration analysis of micro-plates based on a modified couple stress theory.* Physica E., 2011, vol. 43, pp. 877–883.

[22] K. Malekzadeh, S.M.R. Khalili, and p. Abbaspour, *Vibration of non-ideal simply supported laminated plate on an elastic foundation subjected to in-plane stresses*, Compos. Struct., 2010, vol. 92, pp. 1478-1484.

[23] A. Ghorbanpour Arani, H. Khani Arani, and Z. Khoddami Maraghi, *Vibration analysis of sandwich composite micro-plate under electro-magneto-mechanical loadings*. Appl. Math. Modell, 2016, vol. 40, pp. 10596–10615.

[24] C. Shu, "Differential quadrature and its application in engineering. Singapore," Springer, 2000.