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Stochastic Optimization of a Vendor-Managed Inventory Problem in a Two-Echelon Supply Chain

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Abstract**—**The purpose of this paper is to develop a multi-product economic production quantity model under vendor management inventory policy and restrictions including limited warehouse space, budget, and number of orders, average shortage time and maximum permissible shortage. Since the "costs" cannot be predicted with certainty, it is assumed that data behave under uncertain environment. The problem is first formulated into the framework of a bi-objective of multi-product economic production quantity model. Then, the problem is solved with three multi-objective decision-making (MODM) methods. Then following this, three methods had been compared on information on the optimal value of the two objective functions and the central processing unit (CPU) time with the statistical analysis method and the multi-attribute decision-making (MADM). The results are compared with statistical analysis method and the MADM. The results of the study demonstrate that augmented-constraint in terms of optimal value of the two objective functions and the CPU time perform better than global criteria, and goal programming. Sensitivity analysis is done to illustrate the effect of parameter variations on the optimal solution. The contribution of this research is the use of random costs data in developing a multi-product economic production quantity model under vendor management inventory policy with several constraints.

Keywords**—**Economic production quantity, random cost, supply chain management, vendor-managed inventory.

I. Introduction

NOWADAYS, organizations need to use a suitable model, such as Chain Management (CM) to acquire and maintain an appropriate place in the national and global market and meet the expectations of customers [1]. Supply Chain Management (SCM) refers to a set of methods used to integrate suppliers, manufacturers, warehouses, stores and to produce and distribute products efficiently and to provide services to the customers at the right time and place to minimize extensive costs in the system and increase customer satisfaction [2]. An appropriate approach to achieve these goals is the inventory management and control. In classic models, the inventory Economic Order Quantity (EOQ) model was the first model proposed by [3], and the Economic Production Quantity (EPQ), was the model announced by [4] five years later.

Programming issues for inventory production and control are one of the issues that have widely been considered in recent decades and different organizations have to deal with it. In some cases, neglecting inventory production and control increases the cost of inventory control. Considering the importance of inventory control in the production of products in the industry, the models of EPQ have been addressed in some studies [5]-[7]. Most of the companies apply EPQ inventory model to control their current situations and strategies. For example, the paper manufacturing industry to help companies determine the optimal quantity of production under conditions that minimize the cost of the whole supply chain [5]. However, in reality, most of the factors are unpredictable. Companies may face some restrictions, such as the circumscription of space in warehousing, limitation of budget, limitation on the number of orders and other restrictions. Since the classical model of EPQ has certain conditions and assumptions that are less applicable in real situations; this model needs to be developed from different aspects to achieve better results. See, for instance, researchers' effort in recent years has led to the development of a model of EPQ from a variety of perspectives [6]-[10].

Determining the ordering quantity is a classic inventory control problem because, in the traditional supply chain, every member of the chain only controls its circumstances and orders the supplier according to its inventory and order levels. Given the insufficient visibility of the last customer behavior for the supplier and the independent effort of each level for an optimal cost of inventory, a non-optimal result for the entire supply chain is achieved. Hence, in an attempt to solve this problem, the concept of Vendor Management Inventory (VMI) was established in the late 1980s. Vendor inventory management is one of the most successful approaches, which enhances supply chain integration. Inventory management by the vendor is an industrial approach to supply chain collaboration whereby the inventory manager is responsible for the retailer and decides on time and amount of re-inventory. Under the inventory management program by the vendor, the vendor can determine the timing and amount of re-inventory and could have access to data on the demand and inventory of the retailer. Subsequently, the seller can coordinate their long-term programs and control the flow of goods and materials daily. On the other hand, retailers will not be charged for the ordering cost and will be protected by contractual arrangements against additional charges [11]. Some researchers have tried to develop the VMI systems, which include the possibility of deficiency, the possibility of re-ordering, rebroadcasting based on different rates of goods, opening the sequence, additional constraints, such as green restrictions (emissions of greenhouse gases), and limitations on the number of orders and budget available [12]-[15].

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Probable strategies in the inventory management system by the vendor, which reduce the cost of ordering and shipping and ultimately lead to a reduction in the total cost of the supply chain, have also been adopted.

In short, although the growing popularity of the EPQ models has attracted researchers to study various aspects of inventory control under VMI policy, based on the literature summarized in Table I, it can be concluded that this paper tries to optimize a Bi-objective EPQ model.

TABLE I
A REVIEW OF VENDOR MANAGED INVENTORY (VMI)

Year	Author(s)	Inventory Model		Product		Objective function		Constraints						Uncertainty
		EOQ	EPQ	Single	Multi	Single	Multi	Shortage	Store space	Budget	No. of Orders	Capacity production	Vehicles capacity	
2010	[16]	✓	-	✓	-	✓	-	-	-	-	✓	✓	-	-
2011	[17]	✓	-	-	✓	✓	-	-	✓	-	✓	-	-	-
2012	[18]	✓	-	-	✓	✓	-	-	✓	-	-	✓	-	-
2012	[19]	✓	-	✓	-	✓	-	-	-	-	-	✓	-	✓
2012	[20]	✓	-	✓	-	✓	-	-	-	-	✓	✓	-	-
2013	[21]	✓	-	✓	-	✓	-	-	✓	-	✓	-		-
2013	[22]	✓	-	✓	-	✓	-	-	-	-	✓	✓	✓	-
2014	[23]	-	✓	-	✓	✓	-	-	✓	✓	-	✓	-	-
2014	[24]	✓	-	✓	-	✓	-	✓	✓	-	✓	✓	-	✓
2014	[25]	✓	-	✓	-	-	✓	-	✓	-	-	-	-	✓
2015	[15]	✓	-	✓	-	✓	-	-	✓	✓	✓	-	-	✓
2015	[26]	✓	-	-	✓	-	-	✓	✓	-	✓	-	-	-
2016	[27]	✓	-	✓	-	✓	-	✓	-	-	✓	-	-	✓
2017	[12]	✓	-	-	✓	-	✓	-	✓	✓	-	✓	✓	-
2017	[13]	-	✓	-	✓	-	-	-	-	-	-	✓	-	-
2018	[28]	✓	-	-	✓	✓	-	-	-	-	✓	-	✓	-
2018	[29]	✓	-	-	✓	✓	-	-	-	-	-	✓	-	-
2018	[30]	✓	-	✓	-	✓	-	-	✓	-	-	✓	-	-
This paper		-	✓	-	✓	-	✓	✓	✓	✓	✓	✓	✓	✓

Phase 1. Propose a bi-objective of multi-product economic production quantity model under VMI

Phase 2. Solve the model with three multi-objective decision-making methods

Phase 3. Compare the results from three methods

Phase 4. Do a sensitive analysis

Fig. 1 Flowchart of the paper procedure

The objective of this research is to expand the previous EPQ inventory models under a VIM policy to consider warehouse space, budget, and number of orders, average shortage time and maximum permissible shortage limitations at the same time. The main contribution of this research is that all costs considered in the developed EPQ model under VMI policy are uncertain, while the most previous papers have only focused on uncertain demand such as [31], [32]. The previous papers have only focused on demand and a few research studies have used costs. The research is also to solve a bi-objective model including the expected value and variance of the total cost. Three types of methods, namely the augmented ε-constraint, global criteria and goal programming, are used to determine the optimum order quantity and maximum level of shortage for the EPQ inventory models. Furthermore, this study compares three methods and selects the most suitable method based on the best optimal solution and CPU time.

This paper is structured in different sections. Methods of solving the model are described in Section II and the developed model is presented in Section III. Moreover, a numerical example for solving a two-objective model is encoded using the augmented ε-constraint, global criteria and goal programming methods in GAMS software in the following section whereby the three methods are compared using the Tukey and WASPAS techniques. In Section V, the sensitivity analysis of random parameters of the model is discussed, and conclusions and future suggestions are presented. The flowchart of the paper procedure is shown in Fig. 1.

II. PRELIMINARIES

A. MODM

1) Augmented ε -Constraint Method

The augmented ε-constraint method provides Pareto's efficient optimal responses. In this method, one of the objective functions is considered as the main objective function and the other objective functions are limited to the permissible limit of the epsilon [25].

$$Min/Max\ (f_1(x) + \upsilon \times (\frac{s_2}{r_2} + \frac{s_3}{r_3} + ... + \frac{s_i}{r_i} + ... + \frac{s_n}{r_n})) \tag{1}$$

s.t.

$$f_2(x) - s_2 = \varepsilon_2 \tag{2}$$

$$f_3(x) - s_3 = \varepsilon_3 \tag{3}$$

$$i \in [2, n] \tag{4}$$

$$s_i \in \Re^+ \tag{5}$$

The augmented ε-constraint method can be shown as: According to (1)-(5), Pareto optimal solutions are obtained. In that domain, r_i is the i^{th} objective function, υ is a small number between 0.001 and 0.000001, and s_i is an additional non-negative variable. The first stage is obtained by a pay-off matrix as:

$$\begin{pmatrix} f_1^*(x_1^*) & \dots & f_i^*(x_1^*) & \dots & f_n^*(x_1^*) \\ \vdots & \ddots & & & \vdots \\ f_1^*(x_i^*) & \dots & f_i^*(x_i^*) & \dots & f_n^*(x_i^*) \\ \vdots & & & \ddots & \vdots \\ f_1^*(x_n^*) & \dots & f_i^*(x_n^*) & \dots & f_n^*(x_n^*) \end{pmatrix} \tag{6}$$

In the pay-off matrix in each column, the best value for the objective function and the worst value of the objective function will be named PIS_{f_i}, NIS_{f_i}. Then, the value of the domain of the i^{th} objective function is calculated by:

$$r_i = PIS_{f_i} - NIS_{f_i} \tag{7}$$

where r_i is divided into equal intervals l_i. Then, $l_i + 1$ points are obtained, according to (8), the amount of epsilons is obtained based on these points (grid point). In this method, for all the resulting epsilons must be solved, in which case, η, the number of points (i.e., grid points) is obtained.

$$\varepsilon_i^\eta = NIS_{f_i} + \frac{r_i}{l_i} * \eta \qquad \eta = 0, 1, ..., l_i \tag{8}$$

The final stage of selection is obtained by the decision-maker through optimal Pareto points.

2) Global Criteria Method

The purpose of this method is to minimize the deviation of objective functions in a multi-objective model to solve the idea of them. Since there may not be any unique solution that is optimal for all functions, each objective function is optimized individually, and then in the resulting model, this objective function is minimized with ideal solutions [31]. The multi-objective problem is represented by:

$$Max/Min\ z_1 \tag{9}$$

$$Max/Min\ z_2 \tag{10}$$

$$Max/Min\ z_3 \tag{11}$$

.
.
.

$$Max/Min\ z_k \tag{12}$$

s.t.

$$g(x) \geq b \tag{13}$$

$$g(x) = b \tag{14}$$

$$g(x) \leq b \tag{15}$$

$$X \geq 0 \tag{16}$$

where $g(x)$ shows the problem constraints. Each of the above functions is considered alone with the problem constraints and we obtain the optimal solution. We call it z_i^*. Then, the final solution to the problem is obtained using the comprehensive criteria method by solving the following model:

$$Min\ z = \sum \left(\frac{z_k^* - z_k}{z_k^*} \right)^p \qquad k = 1, 2, ... \tag{17}$$

s.t.

$$g(x) \geq b \tag{18}$$

$$g(x) = b \tag{19}$$

$$g(x) \leq b \tag{20}$$

$$X \geq 0 \tag{21}$$

3) Goal Programming Method

The purpose of goal programming (GP) method is to find a solution that minimizes undesirable deviations between the objective functions and their corresponding goals. The mathematical formulation of this method is as follows:

$$Min \sum_{g=1}^{P} a_g . h_g(d_g^+, d_g^-) \tag{22}$$

s.t.

$$f_g - d_g^+ + d_g^- = f_g^*; \quad \forall g \tag{23}$$

$$d_g^+ \geq 0, d_g^- \geq 0; \quad \forall g \tag{24}$$

where d_g^+ and d_g^- show the positive and the negative deviations of the objective functions from their goals, respectively, a_g are the preferred positive weights directly assigned to deviations, and f_g^* is the ideal value of the objective function g .

$$h_g(d_g^+, d_g^-) \tag{25}$$
$$= \begin{cases} d_g^+; & \text{if } f_g \text{ is an objective function of a minimization problem} \\ d_g^-; & \text{if } f_g \text{ is an objective function of a maximization problem} \\ d_g^+ + d_g^-; & \text{Otherwise} \end{cases}$$

B. Multi-Criteria Decision Making (MCDM)

The weighted aggregated sum product assessment (WASPAS) method for solving MCDM problems was proposed by [30]. The main procedure of the WASPAS method solving MCDM problems includes several steps.

Step1. Set the initial decision matrix.

Step2. Normalize the decision matrix by using the equations:

$$\bar{x}_{ij} = x_{ij} / max_i\, x_{ij} \tag{26a}$$

$$\bar{x}_{ij} = min_i\, x_{ij} / x_{ij} \tag{26b}$$

where x_{ij} is the assessment value of the i^{th} alternative with respect to the j^{th} criterion, and (26a) and (26b) are used for maximization and minimization criteria, respectively.

Step3. The total relative importance of the i^{th} alternative, based on weighted sum method (WSM), is calculated as [32]:

$$\sigma^2(Q_i^{(1)}) = \sum_{j=1}^{n} w_j^2 \sigma^2(\bar{x}_{ij}) \tag{27}$$

Step4. The total relative importance of the i^{th} alternative based on the weighted product method (WPM) is calculated by:

$$Q_i^{(2)} = \prod_{j=1}^{n} \bar{x}_{ij}^{w_j} \tag{28}$$

Step5. To increase the ranking accuracy and effectiveness of the decision-making process, in the WASPAS method, a more generalized equation for determining the total relative importance of alternatives is developed below [30]:

$$Q_i = \lambda\, Q_i^{(1)} + (1-\lambda)\, Q_i^{(2)} \tag{29}$$

where $\lambda = 0, 0.1, \ldots, 1$.

For a given decision-making problem, the optimal values of λ can be determined while searching the following extreme function:

$$\lambda = \frac{\sigma^2(Q_i^{(2)})}{\sigma^2(Q_i^{(1)}) + \sigma^2(Q_i^{(2)})} \tag{30}$$

Variances $\sigma^2(Q_i^{(1)})$ and $\sigma^2(Q_i^{(2)})$ can be computed by applying:

$$\sigma^2(Q_i^{(1)}) = \sum_{j=1}^{n} w_j^2 \sigma^2(\bar{x}_{ij}) \tag{31}$$

$$\sigma^2(Q_i^{(2)}) = \sum_{j=1}^{n} \left(\frac{\prod_{j=1}^{n} \bar{x}_{ij}^{w_j} w_j}{(\bar{x}_{ij}^{w_j})(\bar{x}_{ij}^{1-w_j})} \right)^2 \sigma^2(\bar{x}_{ij}) \tag{32}$$

The estimates of variances of the normalized initial criteria values are calculated by:

$$\sigma^2(\bar{x}_{ij}) = (0.05\bar{x}_{ij})^2 \tag{33}$$

Now, the candidate alternatives are ranked based on the Q values (i.e., the best alternative will be that one having the highest Q value). When the value of λ is 0, the WASPAS method is transformed to the WPM. When λ is 1, it becomes the WSM [32].

C. Statistical Analysis

The Tukey method is a single-step multiple comparison procedure and statistical test. It can be used on raw data or in conjunction with an ANOVA method to find means that are significantly different from each other.

III. Mathematical Model

In this section, the assumptions and symbols used to formulate the problem mathematically are described below. Then, the EPQ model of multi-product under the VMI policy that consists of a single manufacturer and a single retailer in the supply chain is expressed. Finally, the development of the model is presented.

A. Assumptions, Problem Parameters and Decision Variables

1) Basic Assumptions for the Proposed Model

- Order is continuous.
- Setup time is ignored.
- Order is always made during the ordering period.
- The model is designed for a period of time.
- The amount of shortage is less than that of orders in each period.

2) Problem Parameters, Decision Variables

j	Index for products (j = 1, …, n)
B	Index for a buyer
S	Index for a supplier
A_{j_s}	Order cost of a manufacturer for product j
$\overline{A_{j_s}}$	Random order cost of a manufacturer for product j

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A_{j_B}	Order cost of a retailer for product *j*
$\overline{A_{j_B}}$	Random order cost of a retailer for product *j*
B	Maximum permissible shortage
C_j	Production cost for product *j*
D_j	Demand for product *j*
f_j	Area of the warehouse that occupies each unit of product *j*
F	Maximum warehouse space
h_{j_B}	Holding cost of a retailer for product *j*
$\overline{h_{j_B}}$	Random holding cost of a retailer for product *j*
L_j	Minimum orders for product *j*
M	Upper limit for the average shortage time
N	Number of products
P_j	Production rate for product *j*
TC_B	Total cost of buyer's inventory in the VMI system
TC_S	Total cost of supplier's inventory in the VMI system
$TC(O_j)$	Total ordering cost
$TC(H_j)$	Total holding cost
$TC(b_j)$	Total shortage cost
TC_{VMI}	Total cost in the VMI system
U_j	Maximum orders for product *j*
X	Maximum available budget
π_{1j}	Shortage cost per unit for product *j*
$\overline{\pi_{1j}}$	Random shortage cost per unit for product *j*
π_{2j}	Shortage cost related to time for product *j*
$\overline{\pi_{2j}}$	Random shortage cost related to time for product *j*
Q_j	Order quantity from product *j* (decision variables)
b_j	Maximum level of shortage for product *j* (decision variables)

B. EPQ Model under VMI Policy

1) Buyer's Total Cost

In the SC under the VMI policy, based on their own inventory cost (which equals to the total cost of the SC), any supplier determines the timing and the quantity of production in a cycle. When VMI is applied, the supplier determines the buyer's order quantity in a VMI policy, in which it is assumed that the supplier pays the ordering and the holding cost on behalf of the buyer [18], [33]. Thus, the buyer pays no cost as indicated by:

$$TC_B = 0 \tag{34}$$

2) Supplier's Total Cost

In EPQ model with shortage under the VMI policy, the supplier's total cost per unit time of the j^{th} item is determined by adding the cost of ordering, holding, and shortage as:

$$TC_S = TC(b_j) + TC(O_j) + TC(H_j) \tag{35}$$

where

$$TC(O_j) = A_{j_S}\frac{D_j}{Q_j} + A_{j_B}\frac{D_j}{Q_j} \tag{36}$$

The order cost is defined as the sum of customer order cost and producer order cost, in which that costs (customer and producer) are computed as the multiplication of each order cost by some ordering times.

$$TC(H_j) = h_{j_B}\left[\frac{(Q_j(1-\frac{D_j}{P_j}) - b_j)^2}{2Q_j(1-\frac{D_j}{P_j})}\right] \tag{37}$$

The holding cost is defined as multiplying the inventory in its cost.

$$TC(b_j) = \pi_{2j}\frac{b_j^2}{2Q_j(1-\frac{D_j}{P_j})} + \pi_{1j} b_j \frac{D_j}{Q_j} \tag{38}$$

The shortage cost is defined as the total of time-dependent and independent of shortage costs [24]. As a result, the supplier's total cost is [13]:

$$TC_s = \sum_{j=1}^{n}\left(\begin{array}{l} A_{j_S}\frac{D_j}{Q_j} + A_{j_B}\frac{D_j}{Q_j} + h_{j_B}\left[\frac{(Q_j(1-\frac{D_j}{P_j}) - b_j)^2}{2Q_j(1-\frac{D_j}{P_j})}\right] + \pi_{2j}\frac{b_j^2}{2Q_j(1-\frac{D_j}{P_j})} \\ +\pi_{1j} b_j \frac{D_j}{Q_j} \end{array}\right) \tag{39}$$

3) Chain Total Cost

Based on (34) and (39), the total cost of the SC under the VMI policy is determined by:

$$TC_{VMI} = \sum_{j=1}^{n}\left(\begin{array}{l} A_{j_S}\frac{D_j}{Q_j} + A_{j_B}\frac{D_j}{Q_j} + h_{j_B}\left[\frac{(Q_j(1-\frac{D_j}{P_j}) - b_j)^2}{2Q_j(1-\frac{D_j}{P_j})}\right] + \pi_{2j}\frac{b_j^2}{2Q_j(1-\frac{D_j}{P_j})} \\ +\pi_{1j} b_j \frac{D_j}{Q_j} \end{array}\right) \tag{40}$$

$$Q_j > 0 \qquad j = 1, 2, ..., n \tag{41}$$

$$b_j \geq 0 \qquad j = 1, 2, ..., n \tag{42}$$

C. Development of the EPQ Model under the VMI Policy

1) Objective Function

In reality, costs vary and there is no definite amount, thus, all the costs of the above statements are regarded as random, and consequently, the objective function or the sum of these costs is also random. As a result, the target function can be updated by:

$$
\begin{aligned}
Z_1 = Min \quad E(TC_{VMI}) &= \sum_{j=1}^{n}\frac{D_j}{Q_j}E(\overline{A_{j_S}}) + \sum_{j=1}^{n}\frac{D_j}{Q_j}E(\overline{A_{j_B}}) \\
&+ \sum_{j=1}^{n}\left(\frac{(Q_j(1-\frac{D_j}{P_j})-b_j)^2}{2Q_j(1-\frac{D_j}{P_j})}\right)E(\overline{h_{j_B}}) \\
&+ \sum_{j=1}^{n}\frac{b_j^2}{2Q_j(1-\frac{D_j}{P_j})}E(\overline{\pi_{2j}}) + \sum_{j=1}^{n}b_j\frac{D_j}{Q_j}E(\overline{\pi_{1j}})
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
Z_2 = Min \; Var(TC_{VMI}) &= \sum_{j=1}^{n}(\frac{D_j}{Q_j})^2 Var(\overline{A_{j_S}}) + \sum_{j=1}^{n}(\frac{D_j}{Q_j})^2 Var(\overline{A_{j_B}}) \\
&+ \sum_{j=1}^{n}\left(\frac{(Q_j(1-\frac{D_j}{P_j})-b_j)^2}{2Q_j(1-\frac{D_j}{P_j})}\right)^2 Var(\overline{h_{j_B}}) \\
&+ \sum_{j=1}^{n}\left(\frac{b_j^2}{2Q_j(1-\frac{D_j}{P_j})}\right)^2 Var(\overline{\pi_{2j}}) \\
&+ \sum_{j=1}^{n}\left(\frac{b_jD_j}{Q_j}\right)^2 Var(\overline{\pi_{1j}})
\end{aligned} \tag{44}
$$

2) Constraints

As mentioned before, there is a contractual agreement between the supplier and the buyer that accounts for the constraints of the model.

$$\sum_{j=1}^{n} f_j(Q_j(1-\frac{D_j}{P_j})-b_j) \le F \tag{45}$$

Constraint (45) enforces that the vendor storage capacity is limited and since the average inventory of the j^{th} item is $(Q_j(1-\frac{D_j}{P_j})-b_j)$, the space constraint will be [18], [17].

$$L_j \le Q_j \le U_j \tag{46}$$

Constraint (46) shows the bounds on the buyer's order quantity of the j^{th} item [16].

$$\sum_{j=1}^{n} C_j Q_j \le X \tag{47}$$

Also, constraint (47) enforces the manufacturer is financially limited, here called, a budget constraint [14].

$$\sum_{j=1}^{n} b_j \frac{D_j}{Q_j} \le B \tag{48}$$

In the following, two constraints are also considered to be lacking. The first limitation is the maximum allowed shortage. The second constraint states that the maximum level of shortage for product *j* in a cycle must be less than or equal to its order quantity [24] as follows:

$$b_j \le Q_j \qquad j = 1,\dots,n \tag{49}$$

Before submitting the last constraint, the time of a course should be divided. A course can be divided into four parts. The time starts from the production period when the inventory is zero (to produce a shortage).

$$t_{1_j} = \frac{b_j}{p_j - D_j} \tag{50}$$

The time has come from a period of production and inventory is increasing.

$$t_{2_j} = \frac{Q_j(1-\frac{D_j}{p_j})-b_j}{p_j - D_j} \tag{51}$$

When the production is still progressing and the product is being consumed but the inventory is positive:

$$t_{3_j} = \frac{Q_j(1-\frac{D_j}{p_j})-b_j}{D_j} \tag{52}$$

When production is still progressing and there is no inventory:

$$t_{4_j} = \frac{b_j}{D_j} \tag{53}$$

Therefore, a period is equal to:

$$T_j = t_{1_j} + t_{2_j} + t_{3_j} + t_{4_j} = \frac{Q_j}{D_j} \tag{54}$$

When there is a shortage, the time of a period is equal to:

$$td_j = t_{1_j} + t_{4_j} = \frac{b_j p_j}{D_j(p_j - D_j)} \tag{55}$$

This constraint concludes that the average deficiency time for all products is high (*M*).

$$\frac{\sum_{j=1}^{n}\frac{td_j}{T_j}}{n} \le M \rightarrow \frac{\sum_{j=1}^{n}\frac{b_j p_j}{Q_j(p_j - D_j)}}{n} \le M \tag{56}$$

The model consists of two nonlinear objective functions (the expected value of total system cost and the variance of total system cost) in which costs are considered random. This model also includes storage capacity, budget constraint, and the frequency of order's constraint, the average shortage time, and the maximum permissible shortage.

The goal is to determine the order quantities, the maximum level of shortage for product j in a cycle so that the total cost of the SC under the VMI policy given in (43) and (44) is minimized and all the constraints are fulfilled.

IV. Numerical Experiments

To demonstrate the application of the proposed methodology and to compare the performances of the above-mentioned solution methods in terms of the objective function values and required CPU time, various test problems of a different number of products are provided in this section. Moreover, for each problem size, 10 problems are solved. All the parameters of the model are randomly generated based on uniform distributions in pre-specified intervals shown in Table II.

This study uses two classic methods (i.e., global criteria and GP), and i.e., augmented ε-constraint method to solve this bi-objective model. The model is solved using the GAMS software with the AP Pavilion dv, CPU 2.3, RAM 8GB and the optimal values and CPU time are shown in the Appendix.

Figs. 2-4 show the values of Z_1, Z_2, and CPU time for 30 problems obtained employing all the methods, respectively. These figures show that the MODM methods have different objective function values and CPU times. Furthermore, the averages of the local optimum feasible solutions (Z_1 & Z_2) and the averages of their CPU times obtained by GAMS are summarized in Table III.

Table IV indicates that each of the solution methods has a different performance towards the others so that the deviation among them decreases while the number of products increases. Hence, three methods of the augmented ε-constraint, the global criteria, and the GP are compared based on information on the optimal value of Z_1, Z_2 and also the CPU time with the statistical analysis methods and the MADM.

TABLE II
DOMAIN PARAMETERS

Parameters	Variation range
L_j	(1, 10)
U_j	(100, 2150)
$E(\overline{A_{j_S}})$	(10, 209)
$Var(\overline{A_{j_S}})$	(1.1, 2.5)
$E(\overline{A_{j_B}})$	(11, 110)
$Var(\overline{A_{j_B}})$	(0.3, 1.9)
$E(\overline{h_{j_B}})$	(2, 24)
$Var(\overline{h_{j_B}})$	(0.1, 0.9)
f_j	(1, 26)
$E(\overline{\pi_{1j}})$	(0.2, 4.6)
$Var(\overline{\pi_{1j}})$	(0.1, 0.6)
$E(\overline{\pi_{2j}})$	(13, 39)
$Var(\overline{\pi_{2j}})$	(0.1, 1)
D_j	(30, 85)
P_j	(80, 100)
C_j	(40, 590)

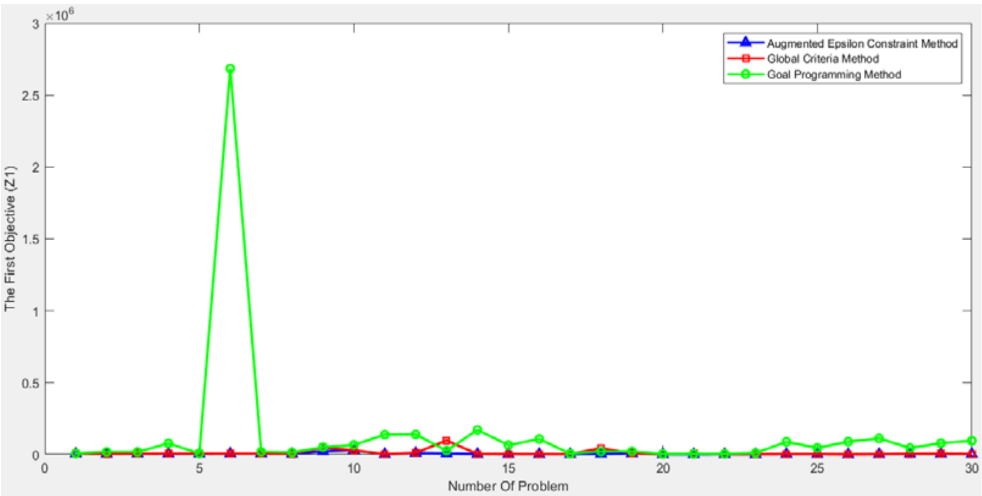

Fig. 2 Values of Z_1 for 30 problems applying MODM methods

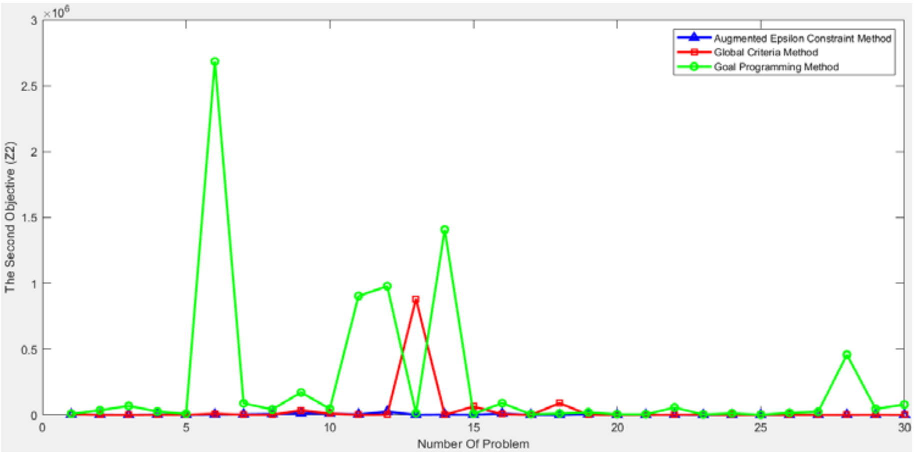

Fig. 3 Values of Z_2 for 30 problems applying MODM methods

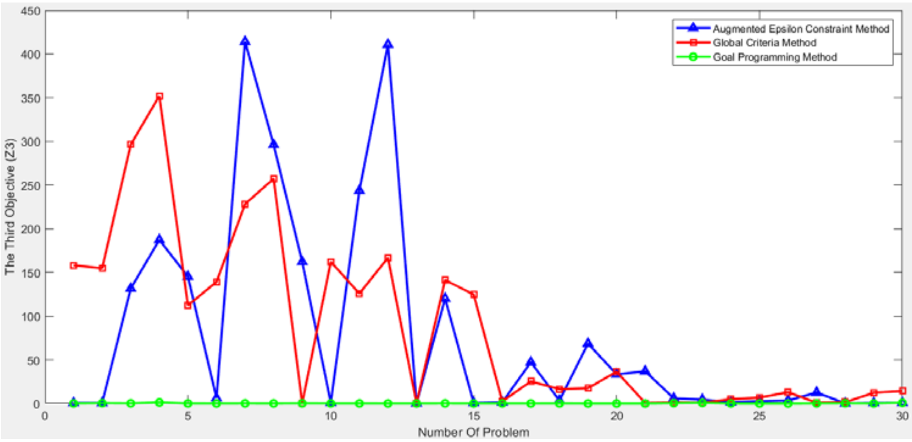

Fig. 4 Values of CPU time for 30 problems applying MODM methods

TABLE III
AVERAGES OF THE RESULTS OBTAINED BY THE SOLUTION METHODS

No. of products	No. of examples	Solving methods	Average Z_1	Average Z_2	Average CPU time
5	10	Augmented ε-Constraint Method	9776.0066	5080.0343	134.5374
		Global Criteria Method	12852.730	3462.6333	186.1432
		GP Method	297027.90	25.100	0.30760
7	10	Augmented ε-Constraint Method	4549.3922	5607.2258	93.0155
		Global Criteria Method	17779.380	104679.3	65.7392
		GP Method	70467.960	366000.4	0.12510
10	10	Augmented ε-Constraint Method	2806.6495	843.8111	6.9662
		Global Criteria Method	3583.5560	1274.373	5.6924
		GP Method	57897.850	70375.21	0.2891

A. Statistical Analysis (Tukey)

For the statistical analysis, we use the Tukey test at the 5% confidence level ($\alpha = 0.05$). In this study, three hypotheses are done. In the first hypothesis testing, the basic assumption of the equality of the mean value of the first objective function (Z_1) is obtained by the three methods. In the second hypothesis testing, the main assumption is the equality of the mean value of the second objective function (Z_2) obtained by the three methods, and in the third hypothesis testing, the main assumption is the equality of the average value of the CPU time in the three methods. The Tukey method is then performed using the SPSS software to test the hypotheses. The results in Tables IV-VI indicate the assumption of the equality of the mean value of the second objective function obtained from the three methods is rejected ($\alpha > 0.05$). Therefore, it can be concluded from the results obtained by the Tukey method that the above three methods are in the same competitive state. It should be noted that codes 1, 2 and 3 in the software are allocated to the augmented -constraint, global criteria and GP methods, respectively.

According to Tables IV-VI, the result is that based on the variance of the total cost of the system, the augmented ε-constraint acts better than the GP at the 5% confidence level. Therefore, to determine the most appropriate method, an alternative method (i.e., WASPAS) is used.

TABLE IV
OUTPUT RESULTS OF COMPARING THE FIRST OBJECTIVE FUNCTION (Z_1)

Z1		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-5783.44075	72024.55725	.996	-177524.5273	165957.6458
	3	-136087.23020	72024.55725	.148	-307828.3167	35653.8563
2	1	5783.44075	72024.55725	.996	-165957.6458	177524.5273
	3	-130303.78945	72024.55725	.173	-302044.8760	41437.2971
3	1	136087.23020	72024.55725	.148	-35653.8563	307828.3167
	2	130303.78945	72024.55725	.173	-41437.2971	302044.8760

TABLE V
OUTPUT RESULTS OF COMPARING THE SECOND OBJECTIVE FUNCTION (Z_2)

Z2		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-33818.03969	88408.02644	.923	-244625.1740	176989.0947
	3	-240430.47940*	88408.02644	.021*	-451237.6137	-29623.3451
2	1	33818.03969	88408.02644	.923	-176989.0947	244625.1740
	3	-206612.43971	88408.02644	.056	-417419.5741	4194.6946
3	1	240430.47940*	88408.02644	.021*	29623.3451	451237.6137
	2	206612.43971	88408.02644	.056	-4194.6946	417419.5741

TABLE VI
THE OUTPUT RESULTS TO COMPARE THE CPU TIME

CPU time		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-.055127700212016	.069931471161393	.711	-22187786566	.11162246524
	3	-.004509715373557	.069931471161393	.998	-17125988082	.16224045008
2	1	.055127700212016	.069931471161393	.711	-.1116224652	.22187786566
	3	.050617984838459	.069931471161393	.750	-.1161321806	.21736815029
3	1	.004509715373557	.069931471161393	.998	-.16224045008	.17125988082
	2	-.050617984838459	.069931471161393	.750	-.21736815029	.11613218061

B. Multi-Criteria Decision Making (WASPAS)

The application of the WASPAS method starts with normalization of the decision matrix using (26a) and (26b). Subsequently, the total relative importance of alternatives as per WSM and WPM are calculated by using (27) and (28), respectively. Finally, joint criterion of optimality of the WASPAS method is calculated by using (29). Then, the optimal λ values are determined by using (30). Table X provides the values of total relative importance (performance scores) for all the considered alternatives. Results of the WASPAS analysis indicate that the augmented ε-constraint method has the highest retention limit. Global Criteria and the GP method were the next methods with the highest retention limit, respectively. The output of the WASPAS analysis is summarized in Tables VII to X.

TABLE VII
DECISION MATRIX

Method	Z_1	Z_2	CPU time
Augmented ε-Constraint	5710.682767	3843.6904	0.18871
Global Criteria	11405.22	36472.09	0.243837
GP	141797.91	244274.1698	0.19322

V. SENSITIVITY ANALYSIS

Sensitivity analysis is a post-optimization method that attempts to detect the effect of random and non-random parameters on the optimal response and to examine the changes in the optimum value of the objective function and the optimum answer of the problem under the influence of changing the important and random parameters. In this method, every time the effect of only one changing one parameter is examined while the other parameters are constant. Figs. 6 and 7 show the effect of the random parameters of cost, including the order cost of the manufacturer, the order cost of the retailer, the holding cost of the retailer, the shortage cost per unit product, the time-dependent shortage cost. The results of the studies are shown in Figs. 5 and 6. According to these figures, it is concluded that with an increase in every cost, the expected value of system total costs and the variance of system total costs increase.

TABLE VIII
NORMALIZED DECISION MATRIX

Method	Z_1	Z_2	CPU time
Augmented ε-Constraint	1	1	1
Global Criteria	0.500707813	0.105387171	0.773918642
GP	0.040273392	0.015735149	0.976658731

TABLE IX
VARIANCE OF NORMALIZED DECISION MATRIX ((1 / 3, 1 / 3, 1 / 3) WEIGHTED VECTOR)

Method	Z_1	Z_2	CPU time
Augmented ε -Constraint	0.0025	0.0025	0.0025
Global Criteria	0.000626771	2.77661E-05	0.001497375
GP	4.05487E-06	6.18987E-07	0.002384656

In Fig. 5, the order cost of the manufacturer (AS) is exponential. According to the form of the function, Z_1 is more sensitive to the order cost of the manufacturer (AS). The effect of the order cost of the retailer (AB) and the holding cost of the retailer (H) on the objective function of the expected value of system total cost is linear. The effect of the cost of time-dependent shortage (W_2) and the cost of shortage per unit of product (W_1) are negligible.

TABLE X
VALUES OF TOTAL RELATIVE IMPORTANCE (PERFORMANCE SCORES)

Method	WSM	WPM	Q (1)	Q (2)	λ	Score
Augmented ε -Constraint	1	1	0.000833	0.000833	0.5	1
Global Criteria	0.4600045	0.3443677	0.000239	9.88243	0.29244	0.37818
GP	0.344222	0.08522	0.00026	6.05211	0.0222	0.0909

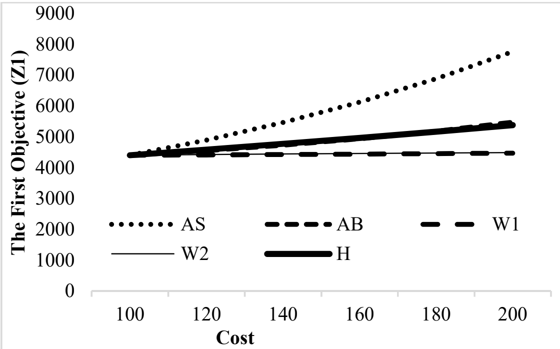

Fig. 5 Sensitive analysis results based on Z_1

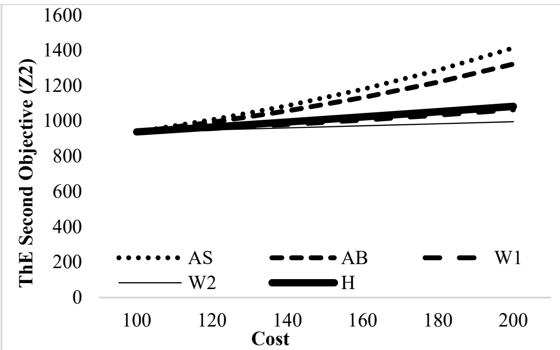

Fig. 6 Sensitive analysis results based on Z_2

In Fig. 6, the order cost of the manufacturer (AS) has an exponential growth in Fig. 5 and has the most impact on the total variance of the system costs. An increase in the order cost of the retailer (AB) than the other costs makes the math variance of system total costs have a steeper slope while it corresponds to the cost of time-dependent shortages (W_2), which have little impact on the total cost variance (Z_2). Also, the holding cost of the retailor (H) and the shortage cost per unit of product (W_1) increases Z_2 almost with the same slope. Since the goal of this study is to minimize total costs, more attention should be paid to the order cost of the manufacturer and the order cost of the buyer than the other costs. This can help managers to decide about minimizing the total system cost.

VI. CONCLUSIONS

Planning for production and inventory management is one of the major concerns of most organizations. In this study, the inventory model of integrated EPQ model was developed based on the vendor managed inventory approach in a two-level chain, and for efficiency in real-world some assumptions were made. The proposed model was assessed through three methods described in the study using GAMS software. The three methods were then compared based on some criteria using Tukey, WASPAS methods. In the Tukey method, the augmented ε -constraint approach to the GP yielded better results based on the variance of system total cost. The results showed that in the WASPAS method the augmented ε -constraint approach provided better concessions in less time compared to other methods.

In the end, the sensitivity analysis of the random parameters of the cost was considered on two objective functions indicating that with an increase in every cost with a different stroke, the cost of the two objective functions increased. Since the goal in this paper is to minimize total costs, it should pay more attention to the order cost of the manufacturer and the order cost of buyer than the other costs. Although some work has been undertaken by researchers in recent years around the vendor managed inventory concept (see Table I), this area of study is still under-researched and could be extended as a future research path from different perspectives, for instance by (1) considering other constraints such as producing capacity for manufacturing; (2) developing the proposed model by taking into account the assumption of 'set up time' in the vendor managed inventory system; (3) using other methods, such as goal attainment or Max-Min methods for solving two-objective models and comparing their results obtained from different methods; (4) developing the proposed model by combining discount models.

APPENDIX

Output of solving the proposed model based on three methods for 30 problems are presented in the Table XI.

TABLE XI
OUTPUT OF SOLVING THE PROPOSED MODEL BASED ON THREE METHODS

Number	Augmented Epsilon Constraint Method			Global Criteria Method			GP Method		
	Z_1	Z_2	CPU Time	Z_1	Z_2	CPU Time	Z_1	Z_2	CPU Time
1	5698.123	3654.147	0.698	5864.78	5632.85	158.32	7465.02	8713.65	0.125
2	4398.112	2752.63	0.594	4835.66	975.896	154.844	19099.199	35283.825	0.469
3	5679.002	1321.195	131.641	5780.887	1163.005	296.437	18562.04	68920.12	0.236
4	5864.246	3417.255	187.36	6978.047	1245.36	352.112	78450.23	26593.02	1.23
5	5239.365	1278.365	145.3	6894.35	1456.07	112.36	7406.01	8913.243	0.125
6	6425.455	3252.109	6.281	6545.19	9998.23	139.203	2685312.053	2685312.056	0.109
7	6860.616	4506.663	414.25	7470.99	1642.285	228.344	19871.681	88598.456	0.203
8	4576.453	7798.553	296.219	5015.336	1722.778	257.39	14756.994	42134.678	0.141
9	22676.072	12029.634	162.219	48799.36	35689.25	0.157	52216.805	171361.95	0.266
10	30342.622	10789.792	0.812	30342.741	10789.854	162.265	67139.292	46420.02	0.172
11	2301.897	4284.788	243.75	2528.575	877.221	125.829	138094.202	903360.38	0.172
12	10183.497	25688.105	410.968	11807.865	2476.169	166.812	139931.205	977788.138	0.078
13	7582.226	1554.853	0.312	98562.023	879865.3	0.328	27549.104	13269.53	0.125
14	3384.196	3321.996	120.156	3737.295	843.903	141.359	171371.063	1407670.33	0.203
15	3080.667	2174.83	0.656	3350.477	64020.02	124.69	65815.32	8291.067	0.11
16	2719.367	10861.61	1.36	2518.18	6420.988	2.562	108171.007	89752.103	0.172
17	2510.55	942.391	47.36	2667.043	541.767	25.312	6507.816	5612.505	0.094
18	4528.944	380.633	3.468	45127.025	89562.31	16.469	21127.062	10589.378	0.157
19	7494.139	6117.679	68.656	8338.823	1626.335	17.547	22218.676	20972.8	0.062
20	1708.439	745.373	33.469	1823.497	558.582	36.484	3894.15	4915.705	0.078
21	1765.135	845.036	36.98	5986.32	945.047	0.689	4598.36	4897.32	0.012
22	1948.061	1941.246	6.016	1606.401	2522.284	1.21	4578.09	56943.02	0.45
23	1559.973	305.293	4.578	2608.608	2516.979	0.843	11222.98	3546.12	0.78
24	3006.911	229.413	1.532	3019.243	225.391	5.156	88453.05	12458.36	0.098
25	3215.345	245.87	2.658	3398.14	468.25	6.78	46598.23	1548.03	0.045
26	1054.98	569.023	3.56	1124.65	789.12	12.98	89789.25	16549.23	0.013
27	2458.23	948.32	12.65	2858.36	1002.3	0.986	112236.98	24589.32	0.15
28	4589.54	1021.32	0.25	5012.68	1320.9	1.58	45698.32	457830.02	0.325
29	4789.34	1457.14	0.654	5986.15	1945.23	12.35	79461.05	46258.14	0.124
30	3678.98	875.45	0.784	4235.01	1008.23	14.35	96342.15	79132.58	0.894

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