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Solitons and Universes with Acceleration Driven by Bulk Particles

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Abstract—Considering a scenario where our universe is taken as a 3d domain wall embedded in a 5d dimensional Minkowski space-time, we explore the existence of a richer class of solitonic solutions and their consequences for accelerating universes driven by collisions of bulk particle excitations with the walls. In particular it is shown that some of these solutions should play a fundamental role at the beginning of the expansion process. We present some of these solutions in cosmological scenarios that can be applied to models that describe the inflationary period of the Universe.

Keywords—Solitons, topological defects, Branes, kinks, accelerating universes in Brane scenarios.

I. Introduction

THE great majority of problems in physics is described by nonlinear differential equations. Fortunately it has been possible to simplify and obtain reliable and measurable results for some physical systems by means of approximations, effective theories and phenomenological models. Particularly, very important systems described by quantum field theories are intrinsically nonlinear; the Standard Model and the Quantum Chromodynamics are classical examples. By exploring deeply such systems or effective non-linear models, even at the classical level, one has shown the increasing importance of the soliton solutions (classical solutions with finite and localized energy) and their broad applications [2]-[5]. Soliton solutions in quantum field theory describe, for instance, monopoles, magnetic vortices, instantons in quantum chromodynamics, cosmic strings and magnetic domain walls. Finding exact classical solutions, particularly solitons, is one of the problems on nonlinear models with interacting fields. When one has in hands a systematic method, as the one offered by Rajaraman [6], things become much easier, even though one has to explore the consequences of the classical solutions, as well as their possible realizations in the nature. As pointed out by Rajaraman and Weinberg [7], in such nonlinear models more than one time-independent classical solution can exist and each one of them corresponds to a different family of quantum states, which come into play when one performs a perturbation around those classical solutions. One can raise several questions about those classical solutions: How to find different soliton solutions? Are the quantum states stable? What further consequences do different soliton solutions bring into play?

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The method in [6], usually called *trial orbits method*, is a very powerful one presented for finding exact soliton solutions for non-linear second-order differential equations of models with two interacting relativistic scalar fields in 1+1 dimensions, and it is model independent. Bazeia et al. [8] have applied the trial orbits method to the special cases whose soliton solutions of the non-linear second-order differential equations are equivalent to the soliton solutions of first-order non-linear coupled differential equations, the so called Bolgomol'nyi-Prasad-Sommerfeld (BPS) topological soliton solutions [9]. A couple of years ago one us presented a method for finding additional soliton solutions for those special cases whose soliton solutions are the BPS ones [10] and last year that approach was extended, allowing more general models [11]. Furthermore, that method shows the general equation of the orbits, explains how the different solutions connect the different vacua of the model under analysis and, as a novelty, presents a class of soliton solution with a kink-like profile for both fields with its minimum energy (BPS energy) smaller than that of the usual solution which exhibits a kink-like configuration for one of the fields and a lump-like configuration for the other one. Moreover, the stability of the quantum states corresponding to these new soliton solutions can be shown on the same basis presented in [12].

The BPS soliton solutions have found applications in a great variety of natural systems whose dynamics can be approximately described by non-linear quantum field models for interacting scalar fields in 1+1 dimensions [13]. Those kind of models have, for example, been generalized by including into the Lagrangian density some minimal terms that break Lorentz and CPT symmetries [14]. Following the important route in analyzing the consequences of additional soliton solutions in a given non-linear model, some of us have shown [15], by using the method developed in [10], that those nonlinear Lorentz breaking models in 1+1 dimensions exhibits additional soliton solutions whose BPS energies are smaller than those found in [14] and that even more general Lorentz breaking models in 1+1 dimensions, which admits soliton solutions, can be built.

In this paper we explore more deeply the classical solutions found in [10] in the nonlinear model of two interacting scalar fields in 1+1 dimensions [8]. In special, we analyze the consequences that those additional soliton solutions bring for the scenario of accelerating universes. This scenario was recently conceived by Brito et al. [1] and it is within the context of the extra-dimensions [16]-[22]. Our analysis is done by following a similar approach to that of [1]. The impact over the expansion scenario of a class of degenerate soliton solutions [23] is here explored. It is shown that, for a critical value of the degeneracy parameter, the reflection of the bulk particles over the wall becomes total, and this happens for the lightest particles which, due to this, should be the first to be created.

In the second section we present the model we are going to work with and review the approach introduced in [10] to find classical soliton solutions. In the third section we discuss the variety of soliton solutions that have been found up to now, by using the method described in the second section; we also present the BPS energy of each set of solutions. In the fourth section, we discuss the influence of each set of soliton solutions in the context of accelerating universes driven by the quantum states (bulk particles) found in the third section. Finally we address final comments about the impact of some soliton solutions over the original expanding process in the last section.

II. A Nonlinear Model for Two Interacting Scalar Fields: Finding Soliton Solutions

The model we consider here was introduced before to study BPS soliton solutions [8]. It consists of two interacting real scalar fields in 1+1 dimensions and it is reminiscent of other models studied previously [7], [24]. It is described by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - V(\phi,\chi), \tag{1}$$

where the potential is given by

$$V(\phi,\chi) = \frac{1}{2}\lambda^2(\phi^2-a^2)^2+(2\mu^2+\lambda\,\mu)\phi^2\chi^2-\lambda\,\mu\,a^2\,\chi^2+\frac{1}{2}\mu^2\chi^4. \tag{2}$$

The distinctive property of this model is that its potential can be written in terms of a so called superpotential as

$$V(\phi,\chi) = \frac{1}{2}\left(\frac{\partial W(\phi,\chi)}{\partial\phi}\right)^2 + \frac{1}{2}\left(\frac{\partial W(\phi,\chi)}{\partial\chi}\right)^2, \tag{3}$$

where the superpotential is:

$$W(\phi,\chi) = \phi\left[\lambda\left(\frac{\phi^2}{3}-a^2\right)+\mu\chi^2\right]. \tag{4}$$

Hence, finding the classical solutions with minimum energy for the time-independent equations of motion

$$\frac{d^2\phi}{dx^2} = \frac{\partial V}{\partial \phi} \qquad \text{and} \qquad \frac{d^2\chi}{dx^2} = \frac{\partial V}{\partial \chi}, \tag{5}$$

is equivalent to find the classical solutions with minimum energy for the time-independent first-order differential equations

$$\phi' = W_\phi(\phi,\chi) \tag{6}$$

and

$$\chi' = W_\chi(\phi,\chi). \tag{7}$$

In the above equations the prime means the derivative with respect to the space coordinate, and W_ϕ (W_χ) stands for the partial derivative of $W(\phi,\chi)$ with respect to the ϕ (χ) field. The minimum energy (BPS energy) [9] for non-linear systems described by the Lagrangian density (1) with potentials written as (3) are found to be given by

$$E_{BPS} = |W(\phi_j,\chi_j) - W(\phi_i,\chi_i)|\,, \tag{8}$$

where ϕ_i and χ_i mean the $i-th$ vacuum states of the model. Here, it is important to remark that the BPS solutions settle into vacuum states asymptotically. In other words, the vacuum states act as implicit boundary conditions of the BPS equations. This particular model we are working with has two degenerate absolute minima at $\phi = \pm a,\ \chi = 0$.

From now on, in order to solve the equations, we follow the method of [10] instead of applying the usual trial orbits method. We note that it is possible to write the relation $d\phi/W_\phi = dx = d\chi/W_\chi$, where the differential element dx is a kind of invariant. Thus, one is led to

$$\frac{d\phi}{d\chi} = \frac{W_\phi}{W_\chi}. \tag{9}$$

This is in general a nonlinear differential equation relating the scalar fields of the model. If one is able to solve it completely for a given model, the function $\phi(\chi)$ can be used to eliminate one of the fields, rendering (6) and (7) uncoupled and equivalent to a single one. Finally, the resulting uncoupled first-order nonlinear equation can be solved in general, even if numerically. By substituting the derivatives of the superpotential (4) with respect to the fields in (9) we have

$$\frac{d\phi}{d\chi} = \frac{\lambda(\phi^2-a^2)+\mu\,\chi^2}{2\,\mu\,\phi\,\chi}, \tag{10}$$

which can be rewritten as a linear differential equation,

$$\frac{d\rho}{d\chi} - \frac{\lambda}{\mu\,\chi} = \chi, \tag{11}$$

by the redefinition of the fields, $\rho = \phi^2 - a^2$. Now, the general solutions are easily obtained as

$$\rho(\chi) = \phi^2 - a^2 = c_0\,\chi^{\lambda/\mu} - \frac{\mu}{\lambda-2\mu}\,\chi^2, \qquad \text{for} \quad \lambda \neq 2\mu, \tag{12}$$

and

$$\rho(\chi) = \phi^2 - a^2 = \chi^2[\ln(\chi)+c_1], \qquad \text{for} \quad \lambda = 2\mu, \tag{13}$$

where c_0 and c_1 are arbitrary integration constants. We substitute the above solutions in the differential equation (7) and obtain the following first-order differential equations for the field $\chi(x)$

$$\frac{d\chi}{dx} = \pm\, 2\,\mu\chi\sqrt{a^2 + c_0\,\chi^{\lambda/\mu} - \frac{\mu}{\lambda-2\mu}\,\chi^2}\,, \qquad \lambda \neq 2\mu, \tag{14}$$

and

$$\frac{d\chi}{dx} = \pm\, 2\,\mu\chi\sqrt{a^2 + \chi^2[\ln(\chi)+c_1]}\,, \qquad \lambda = 2\mu. \tag{15}$$

Despite the fact that in general an explicit solution for each one of the above equations can not be obtained, one can verify numerically that the solutions belong to the same classes, and some of those classes of solutions can be written in closed explicit forms. In those last cases we are able to obtain the several types of soliton solutions we discuss in

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the next section. We have found this method simpler than the trial orbits method, broadly and successfully applied to study the mapping of the soliton solutions and defect structures in problems involving the interaction of two scalar fields [13]. Despite of being simpler, the method applied here furnishes at once all the orbits which, otherwise, should be guessed in the usual procedure of the trial orbits method.

III. Soliton Solutions

In this section we obtain particular solutions for (14), we present several resulting soliton solutions for the model under consideration and also obtain their BPS energies. Before proceeding with the program, we would like to stress that the model admits a particular set of solutions which can not be obtained from the method described in the previous section.

A. Isolated Solutions: Type-I Kink

We consider here a set of classical solutions, which we call isolated solutions because it is characterized by $\bar{\chi}_I(x) = 0$, such that there is no sense in writing the differential equation (10) for this case. Even though, (6) admits a soliton solution given by $\bar{\phi}_I(x) = \pm a\ \tanh(\lambda a x)$, where the (lower) upper sign refers to a (anti-)kink solution. The BPS energy can easily be obtained from (8) and (4) and is given by

$$E_B = \frac{4}{3}\lambda a^3. \tag{16}$$

B. Type-II Kinks

The usual set of solutions, called type-II kink, can be obtained by means of the method described in the previous section. It is obtained when we take $c_0 = 0$ in (14). In this case that equation can be solved analytically for any value of λ and μ, in the range $\lambda > 2\,\mu$, and we get the following solutions for $\chi(x)$

$$\chi_{IIA}(x) = a\sqrt{\frac{\lambda - 2\mu}{\mu}}\mathrm{sech}(2\mu a x), \tag{17}$$

which is called a lump-like solution. One can observe that this solution vanishes when $x \to \pm\infty$. The corresponding kink solution, which is also called type-II kink, is given by

$$\phi_{IIA}(x) = \pm a\ \tanh(2\mu a x), \tag{18}$$

which connect the vacua of the model. In this case the BPS energy is again given by (16). We call this type of kink as type-IIA kink to distinguish it from other types of kink solutions we present for this model.

1) Double Kinks: Others soliton solutions can be found when one considers the integration constant $c_0 \neq 0$ [23]. It was found in [10] that in three particular cases (14) can be solved analytically. For $c_0 < -2$ and $\lambda = \mu$ it was found that the solutions for the $\chi(x)$ field are lump-like solutions, which vanish when $x \to \pm\infty$. On its turn, the field $\phi(x)$ exhibits a kink-like profile. They also connect the vacua of the model and also have BPS energy given by (16). These classical solutions can be written as

$$\tilde{\chi}^{(1)}_{IIA}(x) = \frac{2a}{\sqrt{c_0^2 - 4}\cosh(2\mu a x) - c_0}, \mathrm{for}\lambda = \mu,\ c_0 < -2, \tag{19}$$

and

$$\tilde{\phi}^{(1)}_{IIA}(x) = a\frac{\sqrt{c_0^2 - 4}\sinh(2\mu a x)}{\sqrt{c_0^2 - 4}\cosh(2\mu a x) - c_0}, \quad \mathrm{for}\lambda = \mu,\ c_0 < -2. \tag{20}$$

An interesting aspect of these solutions is that, for some values of $c_0 < -2$, $\tilde{\phi}^{(1)}_{IIA}(x)$ exhibits a double kink profile. We can speak of a formation of a double wall structure, extended along a third dimension perpendicular to the page. In Fig. 1 we plot some typical profiles of the soliton solutions in the case where $\lambda = \mu$, both when c_0 is close to its critical value ($c_0 = -2$ in this case) and far from it. Both fields are there represented. One can verify that the distance from one wall to the other one increases as c_0 approaches its critical value. For the critical value of c_0 the double wall structure merges into a single one.

Similar behavior is also noted in the classical solutions for $\lambda = 4\mu$ and $c_0 < 1/16$. In this case the field $\chi(x)$ has a lump-like profile given by

$$\tilde{\chi}^{(2)}_{IIA}(x) = -\frac{2a}{\sqrt{\sqrt{1 - 16c_0}\ \cosh(4\mu a x) + 1}}, \quad \mathrm{for}\ \ \lambda = 4\mu,\ c_0 < 1/16, \tag{21}$$

and the solution for the field $\phi(x)$ is

$$\tilde{\phi}^{(2)}_{IIA}(x) = \sqrt{1 - 16c_0}\,a\frac{\sinh(4\mu a x)}{\sqrt{1 - 16c_0}\ \cosh(4\mu a x) + 1}, \quad \mathrm{for}\lambda = 4\mu,\ c_0 < 1/16. \tag{22}$$

In the case of $\tilde{\phi}^{(2)}_{IIA}(x)$ also, the typical behavior is such that one can see the double kink profile for some values of c_0, and the increasing of the distance from one wall to the another as c_0 approaches its critical value (c_0 = 1/16 in this case). Once again, at the critical value of c_0 the double wall structure coalesces into a single wall. In the Fig. 2 it is shown the behavior of the energy density, where it becomes quite evident the appearance of the double walls when one approaches $c_0 = 1/16$.

2) Two Kinks: Finally, very interesting analytical soliton solutions were shown to exist when one takes $\lambda = \mu$ and the critical parameter $c_0 = -2$ and for $\lambda = 4\mu$ and the critical parameter $c_0 = 1/16$, in (14). The novelty in these cases is the fact that both, the $\chi(x)$ field and the $\phi(x)$ field present a kink-like profile and the BPS energy is half of that of type-IIA kinks. We call this set of solutions as type-IIB kinks. For $\lambda = \mu$ and $c_0 = -2$ the classical solution for the $\chi(x)$ field can be shown to be given by

$$\chi^{(1)}_{IIB}(x) = \frac{a}{2}(1 \pm \tanh(\mu a\, x)), \tag{23}$$

and the solution for the $\phi(x)$ field is given by

$$\phi^{(1)}_{IIB}(x) = \frac{a}{2}(\tanh(\mu a x) \mp 1). \tag{24}$$

For $c_0 = 1/16$ and $\lambda = 4\mu$, the following set of type-IIB kinks is obtained

$$\chi^{(2)}_{IIB}(x) = -\sqrt{2}a\frac{\cosh(\mu a x) \pm \sinh(\mu a x)}{\sqrt{\cosh(2\mu a x)}}, \tag{25}$$

and

$$\phi^{(2)}_{IIB}(x) = \frac{a}{2}(1 \mp \tanh(2\mu a x)). \tag{26}$$

For type-IIB kink solutions it was found that the BPS energy is

$$E_{(II-B)} = \frac{2}{3}\lambda a^3, \tag{27}$$

that is, it is half of that for the type-I and type-IIA kink solutions. The type-IIB solutions also connect the vacua of the model, but this time one could interpret these solutions as representing two kinds of torsion in a chain, represented through an orthogonal set of coordinates ϕ and χ. So that, in the plane (ϕ,χ), the types I and IIA kinks correspond to a complete torsion going from $(-1, 0)$ to $(0, 0)$ while the type IIB corresponds to a half torsion, where the system goes from $(-1, 0)$ to $(0, 1)$, in the case where ($\lambda = \mu$) for instance. In the next section, we analyze the influence of collisions of quantum particles with 3d double domain walls in the context of accelerating universes.

IV. Domain Walls and Accelerating Universes

A very important and intriguing modern physical problem is that of finding a way to explain the observed accelerated expansion of the universe. On the other hand, the recent cosmological data indicates that a relevant part of the energy of the universe would be a kind of *dark energy* [25]. In fact, that *dark energy* is supposed to be one the responsible for that acceleration. As a consequence, many authors look for an deep understanding of these subjects and, one very interesting possibility is the one associated to the so called brane worlds [22]. In a recent work, Brito et al. [1] conceived a scenario where our universe is taken as a 3d domain wall embedded in a 5d dimensional Minkowsky space-time, where the bulk particles elastic collisions with the 3d domain walls would be the ultimate reason for the universe acceleration.

The model under analysis in this work is precisely the same one considered in [1], which is the scalar sector of a five-dimensional supergravity theory obtained by means of dimensional compactification of a higher dimensional supergravity. As a matter of fact we are working with a nonlinear model with interacting scalar fields in 1+1 dimensions whose time-independent equations of motion, (5) are those one-dimensional static equations for the scalar fields taken into account in [1]. We follow the same route as in that reference, but here we consider the more general set of static solutions presented in the previous section and, then, compare our results with those obtained in [1] where the bulk particles, which are the quantum excitations around the classical solutions, collide with a 3d domain wall given by the type-IIA kink of (18). The essential idea is to show that the situation is richer than analyzed in [1], and that from a complete solution as the one we present here, important consequences for the expanding scenario shows up.

By proceeding as in [1], we perform a linear perturbation of the $\chi(r, t)$ field around the classical solutions, that is

$$\chi(r, t) = \bar{\chi}(r) + \zeta(r, t) \qquad \text{and} \qquad \phi(r, t) = \bar{\phi}(r), \tag{28}$$

where $\bar{\chi}(r)$ and $\bar{\phi}(r)$ are the classical solutions (background fields) and $\zeta(r, t)$ is the quantum field. By expanding the action up to quadratic terms in the quantum fields we obtain second-order differential equations for the quantum fields

$$\partial_\mu \partial^\mu \zeta + \bar{V}_{\chi\chi}(r)\zeta = 0, \tag{29}$$

that is, the quantum field obeys a Klein-Gordon equation with an effective potential $\bar{V}_{\chi\chi}(r)$ which is obtained by taking the second derivative of the potential given in (2) with respect to $\chi(r)$ and evaluated at the classical solutions $\bar{\chi}(r)$ and $\bar{\phi}(r)$

$$\bar{V}_{\chi\chi}(x) = 2(2\mu^2 + \lambda\ \mu)\bar{\phi}^2 + 6\mu^2\bar{\chi}^2 - 2\lambda\ \mu\ a^2. \tag{30}$$

If we consider our model as five-dimensional one and that the quantum field can be expanded in terms of plane-waves in the space-time coordinates, that is

$$\zeta(r, t) = \zeta(r) \exp[-i(\omega t - k_x x - k_y y - k_z z), \tag{31}$$

with $\zeta(r)$ a function of the fifth coordinate, we obtain the following Schrödinger-like equation

$$\left(-\frac{d^2}{dr^2} + \bar{V}_{\chi\chi}(r)\right)\zeta(r) = \varepsilon\ \zeta(r), \tag{32}$$

where $\varepsilon = \omega^2 + k_x^2 + k_y^2 + k_z^2$ and we are considering that the static solutions described in the previous section are now functions of the coordinate r. Everything happens as you had started with the model in five-dimensions and considered the static solutions in only one-dimension, namely the ADS dimension r.

Now it is time to compare the effect of all of above solitonic solutions over the acceleration scenario proposed in [1]. For this we begin by considering the static solutions given by (19) and (20) and substituting them in the effective potential expressed in (30). The Schrödinger-like equation for the quantum excitations is

$$\left(-\frac{d^2}{dr^2} + V^{(IIA-1)}_{eff}(r)\right)\zeta(r) = \varepsilon\ \zeta(r), \tag{33}$$

where

$$V^{(IIA-1)}_{eff}(r) = 6\mu^2 a^2 \frac{(c_0^2 - 4)\sinh^2(2\,\mu\,a\,r) + 4}{\left(\sqrt{c_0^2 - 4}\cosh(2\,\mu\,a\,r) - c_0\right)^2} - 2\mu^2 a^2, \tag{34}$$

and, when $c_0 = -2$, the effective potential becomes ($a = \mu = \lambda = 1$)

$$V^{(IIB-1)}_{eff}(r) = \mu^2\,a^2(1 + 3\ \tanh^2(\mu\,a\,r)). \tag{35}$$

Note that, in the case analyzed in [1], the effective potential looks like

$$V^{(IIA)}_{eff}(r) = 4\,\mu\,a^2 - 4\,\mu\,a^2\left(4 - \frac{\lambda}{\mu}\right)\mathrm{sech}^2\,(2\,\mu\,a\,r)\,. \tag{36}$$

At this point it is important to remark that, if $\lambda = 4\mu$, this effective potential becomes constant, so that there will exist no reflection, once the bulk particles becomes free. Then a natural question arises, is it truly a behavior of the system, or an artifact of a particular solution? In fact, as we will see below, the other solutions do not have this behavior and, in fact the reflection coefficient for this particular choice of the potential parameters is even bigger. In the Fig. 3 it is shown the behavior of the effective potential coming from the domain wall solutions in four situations: the case studied by Brito et al. and for the case of degenerate solitons when c_0 is far from its critical value, near it and at the critical point ($c_0 = -2$). From that figure one can perceive that, apart from the appearance of the two wells potential, for c_0 close to the critical value, the case where the reflection coefficient is bigger is probably the case studied in [1]. Really, the more remarkable result comes to life for the next example, where $\lambda = 4\mu$.

Let us now discuss the case where the potential parameters are such that $\lambda = 4\,\mu$. By substituting (21) and (22) in (30), one can verify that the quantum excitations of the $\chi(r,t)$ field satisfies the effective Schrödinger equation (32) with an effective potential given by

$$V_{eff}^{(IIA-2)}(r) = 12\mu^2 a^2 \frac{(1-16c_0)\sinh^2(4\,\mu\,a\,r)+2}{\left(\sqrt{1-16c_0}\cosh(4\,\mu\,a\,r)+1\right)} - 8\mu^2 a^2, \tag{37}$$

and, when $c_0 = 1/16$, the effective potential becomes

$$V_{eff}^{(IIB-2)}(r) = \mu^2\, a^2\, \mathrm{sech}^2(2\,\mu\,a\,r)[2 + 5\,\cosh(4\,\mu\,a\,r) \pm 3\,\sinh(2\,\mu\,a\,r)]. \tag{38}$$

As observed above, in this case the solution analyzed in [1] has a constant effective potential $V_{eff}^{(IIA)}(r) = 4\,\mu\,a^2$, so rendering no reflection of the bulk particles. In contrast with this, the solutions presented here are such that the reflection coefficient becomes larger and larger, when c_0 approaches its critical value. In fact, for the limit case, the effective potential acquires a step potential shape, so granting that the bulk particles with kinetic energy below a certain limit will certainly be reflected on that infinitely large barrier.

V. Final Remarks

In this work we have analyzed the impact of a general set of soliton solutions over the reflection coefficient of the bulk particle collisions with a 3d domain wall, as originally proposed in [1]. We have shown that when the potential parameters are such that $\lambda = 4\mu$, the effective potential interacting with the bulk particles have its reflection coefficient arbitrarily larger, depending on the value of the degeneracy parameter c_0. The most remarkable situation is that when c_0 reaches its critical value. In that case, the effective potential becomes a kind of step potential, so that the bulk particles having energy lesser than that of the step potential will always be reflected, so producing a maximum effect on the driven acceleration. Moreover, by remembering that in this precise situation, the domain wall comes from a type B kink which have an energy smaller than the type A one, we can conclude that they can be produced more easily and, as a consequence, they would be responsible for the most part of the initial inflationary expansion. Furthermore, from the existence of the double wall solution, which is degenerate through the parameter c_0, and the fact that the distance between the wall increases more and more as c_0 approaches its critical value, and thinking that we live between those walls, one could try to construct a procedure where the value of c_0 depends on some dynamical parameters, such that the universe expansion could be understood through this hypothetical mechanism.

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