International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:14, No:7, 2020

Kalman Filter Gain Elimination in Linear Estimation

Nicholas D. Assimakis

***Abstract*—**In linear estimation, the traditional Kalman filter uses the Kalman filter gain in order to produce estimation and prediction of the n-dimensional state vector using the m-dimensional measurement vector. The computation of the Kalman filter gain requires the inversion of an m x m matrix in every iteration. In this paper, a variation of the Kalman filter eliminating the Kalman filter gain is proposed. In the time varying case, the elimination of the Kalman filter gain requires the inversion of an n x n matrix and the inversion of an m x m matrix in every iteration. In the time invariant case, the elimination of the Kalman filter gain requires the inversion of an n x n matrix in every iteration. The proposed Kalman filter gain elimination algorithm may be faster than the conventional Kalman filter, depending on the model dimensions.

***Keywords*—**Discrete time, linear estimation, Kalman filter, Kalman filter gain.

I. Introduction

ESTIMATION plays a very important role in many fields of science and engineering, such as in applications to communication systems, control systems, power systems, aerospace industry [1].

The estimation problem arises in linear estimation and is associated with discrete time systems described by the following state space equations:

$$x(k+1) = F(k+1,k)x(k) + w(k) \tag{1}$$

$$z(k) = H(k)x(k) + v(k) \tag{2}$$

where $x(k)$ is the $n \times 1$ state vector, $z(k)$ is the $m \times 1$ measurement vector, $F(k+1,k)$ is the $n \times n$ transition matrix, $H(k)$ is the $m \times n$ output matrix, $w(k)$ is the $n \times 1$ state noise and $v(k)$ is the $m \times 1$ measurement noise at time $k \geq 0$.

The statistical model expresses the nature of the state and the measurements. The basic assumption is that the state noise $\{w(k)\}$ and the measurement noise $\{v(k)\}$ are white noises, i.e. a stochastic process with uncorrelated successive values: $\{w(k)\}$ is a zero mean, Gaussian process with known covariance $Q(k)$ of dimension $n \times n$ $R(k)$ of dimension $m \times m$, respectively. The following assumptions also hold: (a) the initial value of the state $x(0)$ is a Gaussian random variable with mean x_0 and covariance P_0; (b) the stochastic processes $\{w(k)\}$, $\{v(k)\}$ and the random variable $x(0)$ are independent.

The discrete time Kalman filter is the most well-known algorithm that solves the filtering problem. The linear discrete time Kalman filter for solving the linear estimation problem was introduced by Kalman in 1960 [2].

The Conventional Kalman Filter or Traditional Kalman Filter [2] computes the state estimation $x(k/k)$ and the corresponding estimation covariance matrix $P(k/k)$ as well as the state prediction $x(k+1/k)$ and the corresponding prediction covariance matrix $P(k+1/k)$, using measurements until time k.

The Kalman filter has been evolved as follows: The Square Root Kalman Filter [3] uses the square root $S(k/k)$ of the estimation covariance matrix $P(k/k)$, i.e. the covariance matrix $P(k/k)$ is replaced by $P(k/k) = S(k/k)S^T(k/k)$, where $S(k/k)$ is a triangular matrix. S^T denotes the transpose of matrix S. In UDU Kalman Filter [4] the covariance matrix $P(k/k)$ is replaced by a diagonal matrix $D(k/k)$ and an upper triangular matrix $U(k/k)$ with ones on the main diagonal, such that $P(k/k) = U(k/k)D(k/k)U^T(k/k)$. Information Kalman Filter [5] uses of the inverse $P^{-1}(k/k)$ of the covariance matrix $P(k/k)$ and the information state vector $y(k/k) = P^{-1}(k/k)x(k/k)$.

In UDU Information Kalman Filter [6] the inverse $P^{-1}(k/k)$ of the covariance matrix $P(k/k)$ is replaced by two factors: a diagonal matrix $D(k/k)$ and a lower triangular matrix $U(k/k)$ with ones on the main diagonal, such that $P^{-1}(k/k) = U(k/k)D(k/k)U^T(k/k)$. The Kalman Filter using General Chandrasekhar Algorithm [7] uses the difference between two successive prediction covariance matrices. The Information Kalman Filter using General Chandrasekhar Algorithm [8] has been developed.

Sigma-Rho Kalman Filter [9] propagates standard deviation σ and correlation coefficients ρ rather than covariance matrix. In Schmidt-Kalman Filter or Consider Kalman Filter [10] the state and covariance are augmented with both parameters to be estimated and parameters to be considered. Steady State Kalman Filter [11] propagates the estimation using measurement and previous estimation; the steady state Kalman Filter prediction covariance is computed off-line by solving the Riccati equation and then the steady state Kalman Filter gain is computed and the steady state Kalman Filter coefficients are derived.

In the Finite Impulse Response (FIR) Steady State Kalman Filter [12] the FIR filter coefficients are calculated a-priori and the estimation depends only on a well-defined set of measurements. The Periodic Steady State Kalman Filter [13] deals with periodic systems with periodic parameters and the periodic steady state Kalman Filter prediction covariance is computed off-line. The Implicit Kalman Filter [14] has been developed for implicit systems and can be readily applied to ill-conditioned systems.

The Square Root Implicit Kalman Filter [15] concerns implicitly defined discrete systems and propagates the factors

Nicholas Assimakis is with the National and Kapodistrian University of Athens, Greece (e-mail: nasimakis@uoa.gr).

of the covariance matrix, rather than the covariance matrix itself. The Complex Kalman Filter [16] uses augmented complex statistics and has been proposed for sequential state space estimation of the generality of complex signals. The Interval Kalman Filter [17] concerns linear systems which contain uncertainties; all the matrices and vectors involved are interval quantities except the noise covariance matrices and, as a result, this algorithm produces interval estimation vectors.

The Fuzzy Kalman Filter [18] for interval systems uses fuzzification of the interval system matrices. The Monte Carlo Kalman Filter [19] concerns binary probit models. The Maximum Correntropy Kalman Filter [20] adopts the robust maximum correntropy criterion (MCC) as the optimality criterion, instead of using the minimum mean square error (MMSE).

All the estimation algorithms developed during the Kalman filter evolution in linear estimation are based on the conventional or traditional Kalman filter, which uses the Kalman filter gain.

The paper is organized as follows: Section II summarizes the conventional Kalman filter. The estimation algorithm based on Kalman filter gain elimination is derived in Section III. In Section IV the conventional Kalman filter and the proposed estimation algorithm based on Kalman filter gain elimination are compared with respect to their calculation burdens. Finally, Section V summarizes the conclusions.

II. Conventional Kalman Filter

In this section the linear discrete time Kalman filter is summarized for time varying and invariant systems. The discrete time Kalman filter is the most well-known algorithm that solves the filtering problem, producing the state estimation $x(k/k)$ and the corresponding estimation covariance matrix $P(k/k)$ as well as the state prediction $x(k+1/k)$ and the corresponding prediction covariance matrix $P(k+1/k)$.

For time varying systems, the Time Varying Kalman Filter (TVKF) is derived [1]:

$$K(k) = P(k/k-1)H^T(k) [H(k)P(k/k-1)H^T(k) + R(k)]^{-1}$$
$$x(k/k) = [I - K(k)H(k)]x(k/k-1) + K(k)z(k)$$
$$P(k/k) = [I - K(k)H(k)]P(k/k-1)$$
$$x(k+1/k) = F(k+1,k)x(k/k)$$
$$P(k+1/k) = Q(k) + F(k+1,k)P(k/k)F^T(k+1,k)$$
$$for\ k = 0,1,\ldots,$$
$$with\ initial\ conditions\ x(0/-1) = x_0, P(0/-1) = P_0.$$

$K(k)$ is the Kalman filter gain. I denotes the identity matrix.

Note that the existence of the inverse of the matrices in the Kalman filter gain equation is ensured assuming that every covariance matrix $R(k)$ is positive definite; this has the significance that no measurement is exact.

For time invariant systems, the system transition matrix, the output matrix, and the noise covariance matrices are constant. For time invariant systems the resulting Time Invariant Kalman filter (TIKF) takes the following form [1]:

$$K(k) = P(k/k-1)H^T[HP(k/k-1)H^T + R]^{-1}$$
$$x(k/k) = [I - K(k)H]x(k/k-1) + K(k)z(k)$$
$$P(k/k) = [I - K(k)H]P(k/k-1)$$
$$x(k+1/k) = Fx(k/k)$$
$$P(k+1/k) = Q + FP(k/k)F^T$$
$$for\ k = 0,1,\ldots,$$
$$with\ initial\ conditions\ x(0/-1) = x_0, P(0/-1) = P_0.$$

For time invariant systems, in [1] it is well known that if the signal process model is asymptotically stable, then there exists a unique steady state value of the prediction covariance matrix. In the steady state case, the resulting discrete time Steady State Kalman Filter (SSKF) takes the following form:

$$x(k/k) = [I - KH]Fx(k-1/k-1) + Kz(k)$$
$$for\ k = 1,2,\ldots,$$
$$with\ initial\ condition$$
$$x(0/0) = [I - K(0)H]x(0/-1) + K(0)z(0)$$
$$where\ K(0) = P(0/-1)H^T\left[HP(0/-1)H^T + R\right]^{-1}$$
$$and\ x(0/-1) = x_0, P(0/-1) = P_0.$$

The steady state value P of the prediction covariance matrix is calculated *off-line* by solving the corresponding discrete time Riccati equation [1]:

$$P = Q + FPF^T - FPH^T[HPH^T + R]^{-1}HPF^T \tag{3}$$

and computing the SSKF gain:

$$K = PH^T[HPH^T + R]^{-1} \tag{4}$$

III. Estimation Algorithm Based on Kalman Filter Gain Elimination

The basic idea is to eliminate the Kalman filter gain computation in order to derive an estimation algorithm which requires the inversion of an $n \times n$ matrix in every iteration. This can be achieved by defining the ratio $\Lambda(k)$ (the term Ratio corresponds to the Greek term Λόγος):

$$\Lambda(k) = [I - K(k)H(k)]^{-1}K(k) \tag{5}$$

This ratio describes the relation between the coefficients of prediction and measurement in the estimation equation:

$$x(k/k) = [I - K(k)H(k)]x(k/k-1) + K(k)z(k).$$

From the Kalman filter gain equation we get:

$$K(k) = P(k/k-1)H^T(k) [H(k)P(k/k-1)H^T(k) + R(k)]^{-1}$$
$$\Rightarrow K(k)H(k)P(k/k-1)H^T(k) + K(k)R(k) = P(k/k-1)H^T(k)$$
$$\Rightarrow K(k)H(k)P(k/k-1)H^T(k)R^{-1}(k) + K(k) = P(k/k-1)H^T(k)R^{-1}(k)$$
$$\Rightarrow K(k) = [I - K(k)H(k)]P(k/k-1)H^T(k)R^{-1}(k)$$

Then using (5) we derive

$$\Lambda(k) = P(k/k-1)H^T(k)R^{-1}(k) \tag{6}$$

From the Kalman filter gain equation we get:

$$K(k) = P(k/k-1)H^T(k)$$
$$[H(k)P(k/k-1)H^T(k) + R(k)]^{-1}$$
$$\Rightarrow P(k/k-1) - K(k)H(k)P(k/k-1)H^T(k) = P(k/k-1)$$
$$-P(k/k-1)H^T(k)$$
$$[H(k)P(k/k-1)H^T(k) + R(k)]^{-1}H(k)P(k/k-1)$$
$$= [P^{-1}(k/k-1) + H^T(k)R^{-1}(k)H(k)\ ]^{-1}$$
$$\Rightarrow [I - K(k)H(k)]P(k/k-1)$$
$$= [P^{-1}(k/k-1) + H^T(k)R^{-1}(k)H(k)]^{-1}$$
$$\Rightarrow [I - K(k)H(k)]$$
$$= [P^{-1}(k/k-1) + H^T(k)R^{-1}(k)H(k)]^{-1}P^{-1}(k/k-1)$$
$$\Rightarrow [I - K(k)H(k)] = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1}$$

Then we are able to eliminate the Kalman filter gain
a) from the estimation equation

$$x(k/k) = [I - K(k)H(k)]x(k/k-1) + K(k)z(k)$$
$$= [I - K(k)H(k)]$$
$$\{x(k/k-1) + [I - K(k)H(k)]^{-1}K(k)z(k)\}$$

by writing:

$$x(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} [x(k/k-1) + \Lambda(k)z(k)] \quad (7)$$

b) from the estimation covariance equation

$$P(k/k) = [I - K(k)H(k)]P(k/k-1)$$

by writing:

$$P(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} P(k/k-1) \quad (8)$$

For time varying systems, the TVKF gain elimination algorithm (TVKFge) has been derived:

$$\Lambda(k) = P(k/k-1)H^T(k)R^{-1}(k)$$
$$x(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} [x(k/k-1) + \Lambda(k)z(k)]$$
$$P(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} P(k/k-1)$$
$$x(k+1/k) = F(k+1,k)x(k/k)$$
$$P(k+1/k) = Q(k) + F(k+1,k)P(k/k)F^T(k+1,k)$$
$$for\ k = 0,1,\ldots,$$
$$with\ initial\ conditions\ x(0/-1) = x_0, P(0/-1) = P_0.$$

Note that the existence of the inverse of the matrices in the Kalman filter gain elimination algorithm equations is ensured assuming that every covariance matrix $R(k)$ is positive definite and that the initial condition $P(0/-1) = P_0$ is positive definite.

Note that the Kalman filter gain can be easily retrieved from the proposed algorithm using

$$K(k) = [I - K(k)H(k)]P(k/k-1)H^T(k)R^{-1}(k)$$
$$= P(k/k)H^T(k)R^{-1}(k) \quad (9)$$

For time invariant systems the resulting TIKF algorithm (TIKFge) takes the following form:

$$\Lambda(k) = P(k/k-1)H^T R^{-1}$$
$$x(k/k) = [I + P(k/k-1)H^T R^{-1}H]^{-1} [x(k/k-1) + \Lambda(k)z(k)]$$
$$P(k/k) = [I + P(k/k-1)H^T R^{-1}H]^{-1}P(k/k-1)$$
$$x(k+1/k) = Fx(k/k)$$
$$P(k+1/k) = Q + FP(k/k)F^T$$
$$for\ k = 0,1,\ldots,$$
$$with\ initial\ conditions\ x(0/-1) = x_0, P(0/-1) = P_0.$$

Note that the matrices R^{-1}, $H^T R^{-1}$ and $H^T R^{-1} H$ are calculated once and *off-line*.

In the steady state case, the resulting discrete time SSKF gain elimination algorithm (SSKFge) takes the following form:

$$x(k/k) = \{[I + PH^T R^{-1}H]^{-1}F\}\, x(k-1/k-1) + \{[I + PH^T R^{-1}H]^{-1}\Lambda\}z(k)$$
$$for\ k = 1,2,\ldots,$$
$$with\ initial\ condition$$
$$x(0/0) = [I - K(0)H]x(0/-1) + K(0)z(0)$$
$$where\ K(0) = P(0/-1)H^T[HP(0/-1)H^T + R]^{-1}$$
$$and\ x(0/-1) = x_0, P(0/-1) = P_0.$$

The steady state value P of the prediction covariance matrix is calculated *off-line* by solving the corresponding discrete time Riccati equation. The steady state ratio is

$$\Lambda = PH^T R^{-1} \quad (10)$$

IV. Comparison of the Algorithms

It is established that the Kalman filter gain elimination algorithm equations have been derived by the conventional Kalman filter equations. Thus the conventional Kalman filter and the proposed Kalman filter gain elimination algorithm are equivalent filters with respect to their behavior, since they calculate theoretically the same estimates. Both filters are iterative algorithms; then, it is reasonable to assume that all the filters compute the estimation $x(k/k)$ executing the same number of iterations. Thus, in order to compare the algorithms with respect to their computational time, we have to compare their per step (iteration) calculation burden (CB) required for the on-line calculations; the CB of the off-line calculations (initialization process for time invariant filters and for the steady state filters) is not taken into account.

Scalar operations are involved in matrix manipulation operations, which are needed for the implementation of the filtering algorithms. Table I summarizes the CB of needed matrix operations. Note that the identity matrix is denoted by I and a symmetric matrix by S. The details are given in [21].

The per iteration CB of the conventional Kalman Filter are given in [21]. The per iteration CB of the estimation algorithms based on Kalman filter gain elimination for the general multidimensional case, where $n \geq 2, m \geq 2$, are analytically calculated in the Appendix. The per iteration CB of the conventional Kalman filter and of the Kalman filter gain elimination algorithm are summarized in Tables II and III, respectively.

International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:14, No:7, 2020

TABLE I
CB OF MATRIX OPERATIONS

Matrix Operation	Matrix Dimensions	CB
$C = A + B$	$(n \times m) + (n \times m)$	nm
$S = A + B$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$B = I + A$	$(n \times n) + (n \times n)$	n
$C = A \cdot B$	$(n \times m) \cdot (m \times \ell)$	$2nm\ell - n\ell$
$S = A \cdot B$	$(n \times m) \cdot (m \times n)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$B = A^{-1}$	$n \times n$	$\begin{cases} \frac{1}{6}(16n^3 - 3n^2 - n), n \geq 2 \\ 1, \quad n = 1 \end{cases}$

TABLE II
PER ITERATION CB OF CONVENTIONAL KALMAN FILTER

System	Matrix Dimensions	CB
Time Varying	$n \geq 1$ $m \geq 2$	$CB_{TVKF} = \frac{1}{2}(8n^3 + 7n^2 - 3n)$ $+4n^2m + nm + 3nm^2$ $+\frac{1}{6}(16m^3 - 3m^2 - m)$
	$n \geq 1$ $m = 1$	$CB_{TVKFm1} = \frac{1}{2}(8n^3 + 15n^2 + 5n) + 1$
Time Invariant	$n \geq 1$ $m \geq 2$	$CB_{TIKF} = \frac{1}{2}(8n^3 + 7n^2 - 3n)$ $+4n^2m + nm + 3nm^2$ $+\frac{1}{6}(16m^3 - 3m^2 - m)$
	$n \geq 1$ $m = 1$	$CB_{TIKFm1} = \frac{1}{2}(8n^3 + 15n^2 + 5n) + 1$
Steady State	$n \geq 1$ $m \geq 1$	$CB_{SSKF} = 2n^2 + 2nm - n$

TABLE III
PER ITERATION CB OF KALMAN FILTER GAIN ELIMINATION ALGORITHM

System	Matrix Dimensions	CB
Time Varying	$n \geq 2$ $m \geq 2$	$CB_{TVKFge} = \frac{1}{6}(52n^3 + 15n^2 - 13n)$ $+3n^2m + nm2nm^2$ $+\frac{1}{6}(16m^3 - 3m^2 - m)$
	$n = 1$ $m = 2$	$CB_{TVKFgen1} = \frac{1}{6}(16m^3 + 9m^2 + 23m) + 8$
	$n \geq 2$ $m = 1$	$CB_{TVKFgem1} = \frac{1}{6}(52n^3 + 33n^2 + 5n) + 1$
	$n = 1$ $m = 1$	$CB_{TVKFgenm1} = 15$
Time Invariant	$n \geq 2$ $m \geq 2$	$CB_{TIKFge} = \frac{1}{3}(26n^3 + 9n^2 - 5n)$ $+2n^2m + nm$
	$n = 1$ $m \geq 1$	$CB_{TIKFgen1} = 3m + 9$
Steady State	$n \geq 1$ $m \geq 1$	$CB_{SSKFge} = 2n^2 + 2nm - n$

From Tables II and III we get:

$$CB_{TVKF} - CB_{TVKFge} = \frac{1}{3}(-14n^3 + 3n^2 + 7n) + n^2m + nm^2$$
$$CB_{TIKF} - CB_{TIKFge} = \frac{1}{6}(-28n^3 + 3n^2 + n) + 2n^2m + 3nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$$
$$CB_{SSKF} = CB_{SSKFge}$$

for the general multidimensional case, where $n \geq 2, m \geq 2$. Then we lead to the fact that the knowledge of the system dimensions can determine which filter is faster. In fact:

a) For time varying multidimensional systems, the areas depending on the model dimensions, where the proposed Kalman filter gain elimination algorithm or the conventional Kalman filter is faster, are shown in Fig. 1. The following Rule of Thumb for time varying systems is derived: The TVKFge algorithm is faster than the time varying conventional Kalman filter, when $m/n > 1.7$

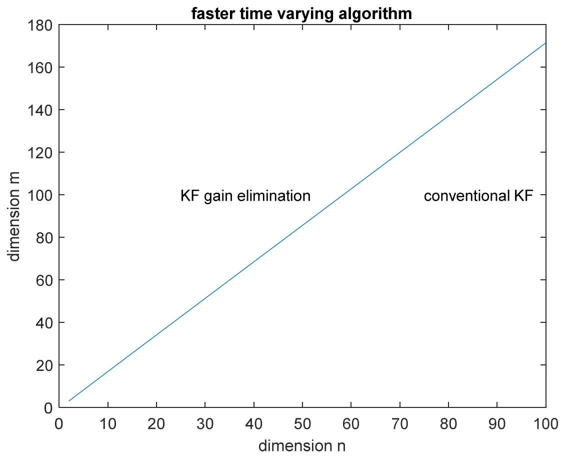

Fig. 1 Time varying systems: The faster filter depends on the model dimensions

b) For time invariant multidimensional systems, the areas depending on the model dimensions, where the proposed Kalman filter gain elimination algorithm or the conventional Kalman filter is faster, are shown in Fig. 2. The following Rule of Thumb for time invariant systems is derived: The TIKFge algorithm is faster than the time invariant conventional Kalman filter, when $m/n > 0.8$

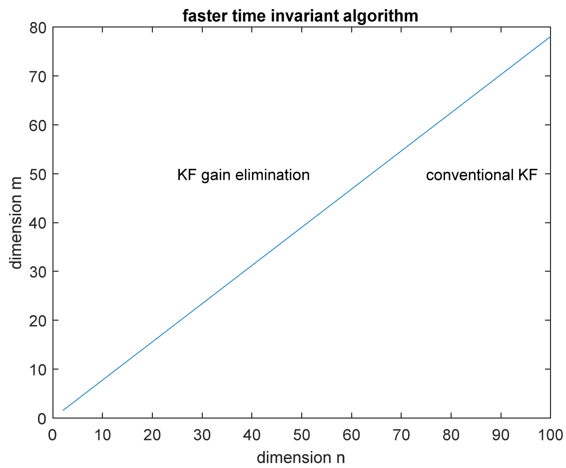

Fig. 2 Time invariant systems: The faster filter depends on the model dimensions

c) For the steady state case, the proposed Kalman filter gain elimination algorithm is as fast the conventional Kalman filter is.

V. CONCLUSIONS

The discrete Kalman filter solves the linear estimation problem using the Kalman filter gain; the conventional Kalman filter algorithm computes the state estimation through measurements till estimation time.

Elimination of the Kalman filter gain has been proposed. It was established that the Kalman filter gain elimination algorithm equations have been directly derived by the Kalman filter equations. Thus the conventional Kalman filter algorithm and the proposed Kalman filter gain elimination algorithm compute the same estimates and hence they are equivalent algorithms.

The basic idea was to eliminate the Kalman filter gain computation from the Kalman filter equations. This has been achieved by replacing the Kalman filter gain by a ratio which describes the relation between the coefficients of prediction and measurement in the estimation equation. The proposed filtering algorithm computes the state estimation and the corresponding estimation covariance matrix as well as the state prediction and the corresponding prediction covariance matrix using this ratio.

It was shown that the proposed Kalman filter gain elimination algorithm is competitive to the conventional Kalman filter concerning the computational burden. The basic conclusion is that the knowledge of the system dimensions leads to the ability to a priori (before the filter implementation) determine which filter is faster.

APPENDIX

The per iteration CB of the estimation algorithms based on Kalman filter gain elimination for the general multidimensional case, where $n \geq 2, m \geq 2$, are analytically calculated in the Appendix.

TABLE IV
TVKFGE ALGORITHM

Matrix Operation	Matrix Dimensions	Calculation Burden
$R^{-1}(k)$	$m \times m$	$\frac{1}{6}(16m^3 - 3m^2 - m)$
$H^T(k)R^{-1}(k)$	$(n \times m) \cdot (m \times m)$	$2nm^2 - nm$
$H^T(k)R^{-1}(k)H(k)$	$(n \times m) \cdot (m \times n)$	$n^2m + nm - \frac{1}{2}n^2 - \frac{1}{2}n$
$\Lambda(k) = P(k/k-1)H^T(k)R^{-1}(k)$	$(n \times n) \cdot (n \times m)$	$2n^2m - nm$
$P(k/k-1)H^T(k)R^{-1}(k)H(k)$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$I + P(k/k-1)H^T(k)R^{-1}(k)H(k)$	$(n \times n) + (n \times n)$	n
$[I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1}$	$n \times n$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$r(k)z(k)$	$(n \times m) \cdot (m \times 1)$	$2nm - n$
$x(k/k-1) + r(k)z(k)$	$(n \times 1) + (n \times 1)$	n
$x(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} [x(k/k-1) + \Lambda(k)z(k)]$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$P(k/k) = [I + P(k/k-1)H^T(k)R^{-1}(k)H(k)]^{-1} P(k/k-1)$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$x(k+1/k) = F(k+1,k)x(k/k)$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$F(k+1,k)P(k/k)$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$F(k+1,k)P(k/k)F^T(k+1,k)$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k+1/k) = Q(k) + F(k+1,k)P(k/k)F^T(k+1,k)$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$CB_{TVKFge} = \frac{1}{6}(52n^3 + 15n^2 - 13n) + 3n^2m + nm + 2nm^2 + \frac{1}{6}(16m^3 - 3m^2 - m)$		

TABLE V
TIKFGE ALGORITHM

Matrix Operation	Matrix Dimensions	Calculation Burden
$\Lambda(k) = P(k/k-1)H^T R^{-1}$	$(n \times n) \cdot (n \times m)$	$2n^2m - nm$
$P(k/k-1)H^T R^{-1} H$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$I + P(k/k-1)H^T R^{-1} H$	$(n \times n) + (n \times n)$	n
$[I + P(k/k-1)H^T R^{-1} H]^{-1}$	$n \times n$	$\frac{1}{6}(16n^3 - 3n^2 - n)$
$r(k)z(k)$	$(n \times m) \cdot (m \times 1)$	$2nm - n$
$x(k/k-1) + r(k)z(k)$	$(n \times 1) + (n \times 1)$	n
$x(k/k) = [I + P(k/k-1)H^T R^{-1} H]^{-1} [x(k/k-1) + \Lambda(k)z(k)]$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$P(k/k) = [I + P(k/k-1)H^T R^{-1} H]^{-1} P(k/k-1)$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$x(k+1/k) = Fx(k/k)$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$FP(k/k)$	$(n \times n) \cdot (n \times n)$	$2n^3 - n^2$
$FP(k/k)F^T$	$(n \times n) \cdot (n \times n)$	$n^3 + \frac{1}{2}n^2 - \frac{1}{2}n$
$P(k+1/k) = Q + FP(k/k)F^T$	$(n \times n) + (n \times n)$	$\frac{1}{2}n^2 + \frac{1}{2}n$
$CB_{TIKFge} = \frac{1}{3}(26n^3 + 9n^2 - 5n) + 2n^2m + nm$		

TABLE VI
SSKFGE ALGORITHM

Matrix Operation	Matrix Dimensions	Calculation Burden
$\{[I + PH^T R^{-1} H]^{-1} F\} x(k-1/k-1)$	$(n \times n) \cdot (n \times 1)$	$2n^2 - n$
$\{[I + PH^T R^{-1} H]^{-1} r\} z(k)$	$(n \times m) \cdot (m \times n)$	$2nm - n$
$x(k/k) = \{[I + PH^T R^{-1} H]^{-1} F\} x(k-1/k-1) + \{[I + PH^T R^{-1} H]^{-1} \Lambda\} z(k)$	$(n \times 1) \cdot (n \times 1)$	n
$CB_{SSKFge} = 2n^2 + 2nm - n$		

REFERENCES

[1] B. D .O Anderson, J. B. Moore, *Optimal Filtering*, Dover Publications, New York, 2005.

[2] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Bas. Eng., Trans. ASME*, Ser. D, vol. 8, no. 1, pp. 34-45, 1960.

[3] J. E. Potter, "New statistical formulas. Instrumentation Laboratory," MIT, Cambridge, Massachusetts , Space Guidance Memo 40, 1963.

[4] C. Thornton, "Triangular Covariance Factorizations for Kalman Filtering," Ph.D. dissertation, University of California at Los Angeles, 1976.

[5] N. Assimakis, M. Adam, A. Douladiris, "Information Filter and Kalman Filter Comparison: Selection of the Faster Filter," *International Journal of Information Engineering*, vol. 2, no. 1, pp. 1-5, 2012.

[6] C. D'Souza, R. Zanetti, "Information Formulation of the UDU Kalman Filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 1, pp. 493–498, 2019.

[7] N. Assimakis, A. Kechriniotis, S. Voliotis, F. Tassis, M. Kousteri, "Analysis of the time invariant Kalman filter implementation via general Chandrasekhar algorithm," *International Journal of Signal and Imaging Systems Engineering (IJSISE)*, vol. 1, no. 1, pp. 51-57, 2008.

[8] M. Morf, G. S. Sidhu, T. Kailath, "Some New Algorithms for Recursive Estimation in Constant, Linear, Discrete-time Systems," *IEEE Trans. Automatic Control*, vol. 19, issue. 4, pp. 315–323, 1974.

[9] M. Grewal, J. Kain, "Kalman Filter Implementation With Improved Numerical Properties," *IEEE Transactions on Automatic Control*, vol. 55, issue. 9, pp. 2058-2068, 2010.

[10] D. Woodbury, J. Junkins, "On the Consider Kalman Filter. In: Proceedings of the AIAA Guidance," *Navigation and Control Conference*, 2010.

[11] N. Assimakis, "A new algorithm for the steady state Kalman filter," *Neural, Parallel and Scientific Computations*, vol. 14, no. 1, pp. 69-74, 2006.

[12] N. Assimakis, M. Adam, "FIR implementation of the steady state Kalman filter," *International Journal of Signal and Imaging Systems Engineering (IJSISE)*, vol.1, no 3/4, pp. 279-286, 2008.

[13] M. Adam, N. Assimakis, "Periodic Kalman filter: Steady state from the

beginning," *Journal of Mathematical Sciences: Advances and Applications*, vol. 1, no. 3, pp. 505-520, 2008.
[14] M. Skliar, W. F. Ramirez, "Implicit Kalman filtering," *Int. J. Control*, nol. 66, no. 3, pp. 393-412, 1995.
[15] M. Skliar, W. F. Ramirez, "Square Root Implicit Kalman Filtering," *13th Triennial World Congress*, USA, 1996.
[16] D. H. Dini, D. P. Mandic, "Class of Widely Linear Complex Kalman Filters," *IEEE Trans. On Neural Networks and Learning Systems*, vol. 23, no. 5, pp. 775-786, 2012.
[17] G. Chen, J. Wang, L. S. Shieh, "Interval Kalman filtering," *IEEE Trans. Aerospace Electron. Systems*, vol. 33, pp. 250-259, 1997.
[18] G. Chen, Q. Xie, L. S. Shieh, "Fuzzy Kalman filtering," *Journal of Information Sciences*, vol. 109, pp. 197-209, 1998.
[19] P. Song, "Monte Carlo Kalman filter and smoothing for multivariate discrete state space models," *The Canadian Journal of Statistics*, vol. 28, no. 4, pp. 641-652, 2000.
[20] B. Chen, X. Liu, H. Zhao, J. C. Principe, "Maximum Correntropy Kalman filter," *Automatica*, vol. 76, pp. 70–77, 2017.
[21] N. Assimakis, M. Adam, "Discrete time Kalman and Lainiotis filters comparison," *Int. Journal of Mathematical Analysis (IJMA)*, vol. 1, no. 13, pp. 635-659, 2007.

Nicholas D. Assimakis was born in Athens in 1962. He received the Diploma in Computer Engineering and Informatics from the Department of Computer Engineering and Informatics of the Polytechnic School of the University of Patras, Greece in 1985. He obtained a Ph.D. in Signal Processing from the Department of Computer Engineering and Informatics of the Polytechnic School of the University of Patras, Greece in 1993.

He is currently a Professor with the General Department of the National and Kapodistrian University of Athens, Greece. He has 50 scientific publications in international journals and 25 scientific publications in international conferences. He authored 4 books and co-authored 7 books. His research interests are in digital signal processing, estimation theory, filtering, algorithm design and development.

Prof. N. D. Assimakis is member of the Technical Chamber of Greece.