

# Modeling Non-Darcy Natural Convection Flow of a Micropolar Dusty Fluid with Convective Boundary Condition

F. M. Hady, A. Mahdy, R. A. Mohamed, Omima A. Abo Zaid

**Abstract**—A numerical approach of the effectiveness of numerous parameters on magnetohydrodynamic (MHD) natural convection heat and mass transfer problem of a dusty micropolar fluid in a non-Darcy porous regime is prepared in the current paper. In addition, a convective boundary condition is scrutinized into the micropolar dusty fluid model. The governing boundary layer equations are converted utilizing similarity transformations to a system of dimensionless equations to be convenient for numerical treatment. The resulting equations for fluid phase and dust phases of momentum, angular momentum, energy, and concentration with the appropriate boundary conditions are solved numerically applying the Runge-Kutta method of fourth-order. In accordance with the numerical study, it is obtained that the magnitude of the velocity of both fluid phase and particle phase reduces with an increasing magnetic parameter, the mass concentration of the dust particles, and Forchheimer number. While rises due to an increment in convective parameter and Darcy number. Also, the results refer that high values of the magnetic parameter, convective parameter, and Forchheimer number support the temperature distributions. However, deterioration occurs as the mass concentration of the dust particles and Darcy number increases. The angular velocity behavior is described by progress when studying the effect of the magnetic parameter and microrotation parameter.

**Keywords**—micropolar dusty fluid, convective heating, natural convection, MHD, porous media.

## I. INTRODUCTION

**M**ICROPOLAR fluids are treated as a model of non-Newtonian fluids models which do not adhere to the Newtonian law of the viscosity. As well micropolar fluids are seen as consisting of suspended particles in a viscous medium and these particles are in the form of solid particles, spherical particles or random-oriented with their private spins and microrotations. In addition, micropolar fluids are characterized by microstructures. Eringen [1], [2] introduced the theory of thermomicropolar and micropolar fluids which can be utilized to portray fluid conduct in a lot of practical applications. Out of many of these applications animal blood, liquid crystal and the mathematical model for fluids with the suspensions as real fluids, colloidal fluids, and polymeric fluids. It is worth mentioning that a comprehensive and excellent review of microbial fluid mechanics and its

applications was displayed by Ariman et al. [3], [4]. When studying micropolar fluids, it was found that there are many mathematical models for the natural convection flow of a micropolar fluid. The problem of natural convection boundary layer flow of micropolar fluids over a vertical cylinder has been analyzed by Rani and Kim [5]. Cheng [6] investigated boundary layer flow around natural convection heat transfer of a micropolar fluid with the power-law variance in surface temperature near a vertical truncated cone. A numerical discussion was prepared by Damseh et al. [7] about unsteady natural convection in the boundary layer flow of a micropolar fluid with fixed heat flux over a vertical surface. The natural convection flow problem of a micropolar fluid over a vertical plate and permeability in a porous medium with uniform heat flux has been studied by Hassanien et al. [8]. Ferdows and Liu [9] presented the natural convective flow of micropolar fluid past a vertical plate in the presence of a magnetic field effect. Recently, Unsteady MHD natural convection boundary layer flow over a radiated stretching sheet in a micropolar fluid with viscous dissipation and thermal radiation has been grasped by Rao et al. [10].

Based on the many MHD applications, it is studied to understand fluid motion behavior where it is studying the fluid attitude electrically conducting and the magnetic properties. Among of MHD applications are mention as plasma, power generation, liquid metals, etc. Numerous analyses have been executed with various aspects of unsteady MHD flows of Newtonian and non-Newtonian fluid [11]-[15]. Sheri and Shamshuddin [16] discussed the influences of a chemical reaction and viscous dissipation on mass and heat transfer flow of magnetohydrodynamic (MHD) of micropolar fluid. An investigation has been done by Shehzad et al. [17] into the magnetohydrodynamic flow problem of non-Newtonian nanofluid with convective boundary conditions.

Darcy's law has been discovered to study the flow of fluids through porous media in 1856 and the Darcy equation is defined as strong to describe this flow. According to Lee and Yang [18], the flow of fluids through a bank of circular cylinders was studied and they modeled it as Darcy-Forchheimer drag. As for the equations of Darcy-Brinkman-Forchheimer have been applied by Prasad and Kladias [19] to solve the heat transfer problem in horizontal porous layers which are heated from below under the impacts of boundary viscous diffusion and Inertia. Dye et al. [20] presented a description of fluid flow over the systems of porous medium for a set of Reynolds numbers congruous

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to both Darcy and non-Darcy regimes. Mixed convection problem from a vertical plate in a non-Darcy porous medium saturated with a non-Newtonian fluid under the effects of both magnetic field and melting has been analyzed by Prasad and Hemalatha [21]. It can be noted that a public pattern of the porous media is utilized to investigate the problem of the flow of Darcy and non-Darcy regimes in an axisymmetric porous cavity by Nithiarasu et al. [22]. Bakier [23] explained the problem of mass and heat transfer by natural convection with thermophoresis and radiation effects in a micropolar fluid through a non-Darcy porous medium. He found out that the dimensionless velocity and the function of mass transfer increase with enhancement thermophoresis, while reduces angular velocity and temperature. There are researches that have been displayed [24]-[26] about the flow of non-Darcy natural and mixed convection.

On the other hand, dusty fluid can be defined as due to the addition of dust particles in the size of the micrometer to the base fluids in nature. In addition, the dust fluid improves the thermal conductivity and heat transfer process of these fluids. The phenomenon of natural convection was associated with dusty fluid flow in problems addressed by Siddiqua et al. [27]-[29]. Also, Unsteady natural convection flow problem in a rectangular channel of a dusty fluid has been studied by Dalal et al. [30]. The problem of flow MHD boundary layer over a semi-infinite surface of dusty fluid under induced magnetic field effects was examined by Silu et al. [31]. There is an issue that deals with the influences of Biot number and magnetic field on the flow of the boundary layer and heat transfer over a stretching surface of dusty fluid containing silver (Ag) nanoparticles have been investigated by Gireesha et al. [32]. Recently, Krishnamurthy [33] presented a study on MHD flow of nano micropolar fluid in the presence of dust particles over a permeable stretching sheet in a porous medium. Finally, the objective of the current investigation is to present a discussion about magnetohydrodynamic (MHD) natural convection boundary layer flow and mass and heat transfer of a dusty micropolar fluid in a non-Darcy porous regime in the presence of a convective boundary condition. In other words, it is to present a discussion about the impacts of a magnetic field and a convective boundary condition on incompressible boundary layer flow and heat transfer of a dusty micropolar fluid through a non-Darcy porous regime. Non-linear equations of momentum, heat and mass transfer and angular momentum are solved numerically using the fourth-order Runge-Kutta method by software algebraic Matlab.

## II. MODEL FORMULATION

The mathematical description of the current problem of natural convection laminar boundary layer flow of a micropolar fluid in a non-Darcy porous medium in the presence of dust particles and magnetic field with a convective boundary condition is explained in this section. Characterized by this fluid as viscous, incompressible and electrically conducted because of an applied magnetic field  $B_0$  in the  $\hat{y}$  direction, as well as the flow is in two dimensional and stable. Fig.

1 displays the physical diagram and coordinate system of this problem. Cartesian coordinates  $\hat{x}$  and  $\hat{y}$  are used such that  $\hat{x}$  is along the vertical surface and  $\hat{y}$  perpendicular it. It is assumed that  $T_\infty$  and  $C_\infty$  are the ambient temperature and species concentration, respectively,  $n$  is a constant lay between 0 and 1 ( $0 \leq n \leq 1$ ),  $T_w$  is the surface temperature which will determine later result from a convective heating process which is described by heat transfer coefficient  $h_f$  and temperature  $T_f$ , and  $C_w$  represents stationary value of  $C$  at  $\hat{y} = 0$ . Based on using approximations of the boundary-layer and Oberbeck-Boussinesq and assumption that number density of the dust particle remains constant during the flow and the dust particles have the same size, the governing equations of the fluid phase and particle phase follow the next system of equations:

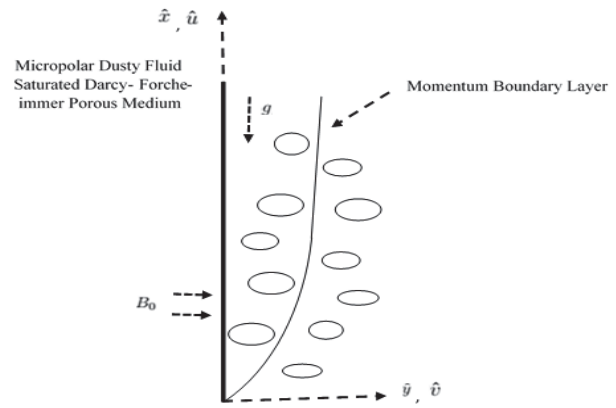


Fig. 1 Physical diagram representation for the problem

For the fluid phase:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \quad (1)$$

$$\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} = \nu \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{k^*}{\rho} \frac{\partial E}{\partial \hat{y}} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu}{K} \hat{u} - \frac{\sigma B_0^2}{\rho} \hat{u} - \frac{b}{K} \hat{u}^2 + \frac{\rho_p}{\rho \tau_m} (\hat{u}_p - \hat{u}) \quad (2)$$

$$\hat{u} \frac{\partial E}{\partial \hat{x}} + \hat{v} \frac{\partial E}{\partial \hat{y}} = \frac{\varepsilon}{\rho j} \frac{\partial^2 E}{\partial \hat{y}^2} - \frac{k^*}{\rho j} \left( 2E + \frac{\partial \hat{u}}{\partial \hat{y}} \right) \quad (3)$$

$$(\rho c_p)_f \left( \hat{u} \frac{\partial T}{\partial \hat{x}} + \hat{v} \frac{\partial T}{\partial \hat{y}} \right) = k \frac{\partial^2 T}{\partial \hat{y}^2} + \frac{\rho_p c_s}{\tau_T} (T_p - T) + \frac{\rho_p}{\tau_m} (\hat{u}_p - \hat{u})^2 \quad (4)$$

$$\hat{u} \frac{\partial C}{\partial \hat{x}} + \hat{v} \frac{\partial C}{\partial \hat{y}} = D \frac{\partial^2 C}{\partial \hat{y}^2} \quad (5)$$

For the dust phase:

$$\frac{\partial \hat{u}_p}{\partial \hat{x}} + \frac{\partial \hat{v}_p}{\partial \hat{y}} = 0 \quad (6)$$

$$\hat{u}_p \frac{\partial \hat{u}_p}{\partial \hat{x}} + \hat{v}_p \frac{\partial \hat{u}_p}{\partial \hat{y}} = -\frac{1}{\tau_m} (\hat{u}_p - \hat{u}) \quad (7)$$

$$\rho_p c_s \left( \hat{u}_p \frac{\partial T_p}{\partial \hat{x}} + \hat{v}_p \frac{\partial T_p}{\partial \hat{y}} \right) = -\frac{\rho_p c_s}{\tau_T} (T_p - T) \quad (8)$$

Here,  $\hat{u}$ ,  $\hat{v}$ ,  $\hat{u}_p$  and  $\hat{v}_p$  are the components of velocity in two phases fluid and dust along  $\hat{x}$  and  $\hat{y}$  axes, respectively,  $\rho$  and  $\rho_p$  reference the density of fluid and dust particles, respectively,  $k^*$  and  $\nu$  are the coefficient of rotational and kinematic viscosity, respectively,  $g$  refers to the acceleration of gravity vector,  $\sigma$  signify to the fluid electrical conductivity,  $B_0$  gives the strength of the magnetic field,  $\beta$  and  $\beta^*$  present the coefficient of thermal and concentration expansion, respectively,  $b$  points out the Forchheimer constant,  $K$  represents the permeability of the porous medium,  $k$  gives the thermal conductivity of the fluid,  $E$  refers to the angular velocity,  $C$  refers to the species concentration,  $D$  points out the coefficient of mass diffusion,  $j$  is the density of micro-inertia,  $\varepsilon$  represents the viscosity of spin gradient,  $T$  and  $T_p$  determine the temperature of fluid and dust particles, respectively,  $c_p$  and  $c_s$  give the specific heat of fluid and dust particles, respectively,  $(\rho c_p)_f$  indicates the heat capacity of the fluid,  $\tau_m$  and  $\tau_T$  aim the velocity relaxation time and thermal relaxation time of the dust particle, respectively and  $\tau_T = \frac{3}{2} \frac{c_s}{c_p} Pr \tau_m$ . Based on the following boundary conditions:

$$\hat{u} = U_0, \quad \hat{v} = 0, \quad -k \frac{\partial T}{\partial \hat{y}} = h_f (T_f - T) \quad (9)$$

$$C = C_w, \quad E = -n \frac{\partial \hat{u}}{\partial \hat{y}}, \quad \hat{y} = 0$$

$$\hat{u}, \hat{u}_p \rightarrow 0, \quad \hat{v}_p \rightarrow \hat{v}, \quad T, T_p \rightarrow T_\infty \quad (10)$$

$$C \rightarrow C_\infty, \quad E \rightarrow 0, \quad \hat{y} \rightarrow \infty$$

The dimensionless form for a micropolar dusty fluid flow equations of momentum, angular momentum, energy, and concentration with previous conditions can be reached by introducing the following transformations:

$$\psi = \left( \sqrt{2\nu\hat{x}U_0} \right) f(\eta) = \left( \sqrt{2\nu\hat{x}U_0} \right) F(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_f - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad E(\eta) = \left( \sqrt{\frac{U_0}{2\nu\hat{x}}} \right) U_0 S,$$

$$\eta = \left( \sqrt{\frac{U_0}{2\nu\hat{x}}} \right) \hat{y} \quad (11)$$

where  $\eta$  represents the transverse coordinate in non-dimension form,  $\psi$  is the stream function such that  $\hat{u} = \frac{\partial \psi}{\partial \hat{y}}$  and  $\hat{v} = -\frac{\partial \psi}{\partial \hat{x}}$ ,  $f$  and  $F$  are the stream function in non-dimension form for fluid phase and particle phase, respectively,  $\phi$  points out concentration function in non-dimension form,  $\theta$  and  $\theta_p$  indicate the temperature function in non-dimension form for fluid phase and particle phase, respectively,  $S$  gives the angular velocity in non-dimension form.

Now the dimensionless form of the system of above equations with the corresponding boundary conditions are:

For the fluid phase:

$$f''' + ff'' + BS' + Gr\theta + Gm\phi - \frac{1}{DaRe} f' - Mf' - \frac{Fs}{Da} f'^2 + D\rho\alpha_d(F' - f') = 0 \quad (12)$$

$$\lambda S'' - 2\frac{\lambda}{G^*} (2S + f'') + f'S + fS' = 0 \quad (13)$$

$$\theta'' + Prf\theta' + \frac{2}{3} D\rho\alpha_d(\theta_p - \theta) + PrD\rho\alpha_d Ec(F' - f')^2 = 0 \quad (14)$$

$$\phi'' + Scf\phi' = 0 \quad (15)$$

For the dust phase:

$$FF'' + \alpha_d(f' - F') = 0 \quad (16)$$

$$F\theta'_p + \frac{2}{3} \frac{1}{\Gamma Pr} \alpha_d(\theta - \theta_p) = 0 \quad (17)$$

Transformed boundary conditions are:

$$f' = 1, \quad f = 0, \quad \theta' = -Bi(1 - \theta) \quad (18)$$

$$\phi = 1, \quad S = -nf'', \quad \eta = 0$$

$$f', F' \rightarrow 0, \quad F \rightarrow f, \quad \theta, \theta_p \rightarrow 0 \quad (19)$$

$$\phi \rightarrow 0, \quad S \rightarrow 0, \quad \eta \rightarrow \infty$$

where  $\lambda = \frac{\varepsilon}{\rho j \nu}$  and  $G^* = \frac{\varepsilon U_0}{k^* \nu \hat{x}}$  represented the micropolar material parameters,  $B = \frac{k^*}{\rho \nu}$  expresses the coupling constant parameter,  $Da = \frac{K}{L^2}$  points out the Darcy number,  $Re = \frac{U_0 L^2}{2\nu \hat{x}}$  gives the local Reynolds number,  $M = \frac{\nu \sigma B_0^2}{\rho U_0^3}$  refers to the magnetic parameter,  $Fs = \frac{2b\hat{x}}{L^2}$  denotes the Forchheimer number,  $Sc = \frac{\nu}{D}$  presents the Schmidt number,  $Pr = \frac{\mu c_p}{k}$  defines the Prandtl number,  $Gr = \frac{g\beta\nu(T_f - T_\infty)}{U_0^3}$  and  $Gm = \frac{g\beta^*\nu(C_w - C_\infty)}{U_0^3}$  indicate the local Grashof number and modified Grashof number, respectively,  $Ec = \frac{U_0^2}{c_p(T_f - T_\infty)}$  Known as the Eckert number,  $\alpha_d = \frac{2\hat{x}}{U_0} \frac{1}{\tau_m}$  determines fluid particle interaction parameter,  $D\rho = \frac{\rho_p}{\rho}$  signify to the mass concentration of the dust particles,  $\Gamma^p = \frac{c_s}{c_p}$  represents the specific heat ratio of the mixture and  $Bi = \frac{h_f}{k} \sqrt{\frac{2\nu\hat{x}}{U_0}} = \frac{h_f}{k} \frac{L}{\sqrt{Re}}$  gives the Biot number.

### III. NUMERICAL METHOD

The set of ordinary and nonlinear differential equations (12)-(17) with the boundary conditions (18) and (19) have been solved numerically utilized the Runge-Kutta method of fourth-order. Our equations are dealt with in a Matlab based on the fact that each nth-order equation is mutated into n of the first-order equations. Then using the bvp4c function, the system of first-order equations is solved. The edge of the boundary layer at infinity ( $\eta_\infty$ ) was selected in this method equal to  $\eta_{max} = 8$  that correctly assures that each numerical solutions approach the asymptotic values. Additionally, numerical solutions have not a salient significant

change when rising values of  $\eta_{max}$ . In order to clarify this method more, we rewrite the momentum equation (12) as an example as follow:

TABLE I  
VALUES OF  $-\theta'(0)$  AND  $f''(0)$  FOR DIFFERENT VALUES OF THE  
PARAMETERS

$Pr$	$M$	$Gr$	$Bi$	$n$	$D_\rho$	$B$	$Da$	$-\theta'(0)$	$f''(0)$
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
0.8								0.44981	-0.96026
0.9								0.47688	-0.97672
1								0.50249	-0.99222
0.7	0	6	10	0.5	1	0.5	0.3	0.43955	-0.44823
2								0.42101	-0.94268
4								0.40443	-1.38461
6								0.38960	-1.78554
0.7	2	5	10	0.5	1	0.5	0.3	0.40739	-1.19047
6								0.42101	-0.94268
7								0.43294	-0.69666
8								0.44346	-0.45223
0.7	2	6	1	0.5	1	0.5	0.3	0.28648	-1.32278
5								0.39950	-1.00117
10								0.42101	-0.94268
$\infty$								0.44527	-0.87766
0.7	2	6	10	0	1	0.5	0.3	0.42225	-0.81574
0.3								0.42101	-0.94268
0.5								0.42101	-0.94268
1								0.41931	-1.11652
0.7	2	6	10	0.5	0	0.5	0.3	0.43908	-0.92639
1								0.42101	-0.94268
4								0.36992	-0.99253
10								0.28059	-1.09575
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
5								0.40258	-0.57653
7								0.38523	-0.88054
9								0.37554	-0.81323
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
0.4								0.44243	-0.32777
1								0.48892	1.16705
1.1								0.49196	1.28521

$$f''' = -ff'' - BS' - Gr\theta - Gm\phi + \frac{1}{DaRe}f' + Mf' + \frac{Fs}{Da}f'^2 - D_\rho\alpha_d(F' - f') \quad (20)$$

The following system of equations represents how to place the momentum equation in MATLAB.

$$\begin{aligned} F(1) &= Y(2), \\ F(2) &= Y(3), \\ F(3) &= -Y(1)Y(3) - BY(9) - GrY(4) - GmY(6) + \\ & (1/DaRe)Y(2) + MY(2) + (Fs/Da)Y^2(2) - \\ & D_\rho\alpha_d(Y(11) - Y(2)) \end{aligned} \quad (21)$$

where

$$\begin{aligned} f &= Y(1), \\ f' &= Y(2) = \frac{dY(1)}{d\eta} = F(1), \\ f'' &= Y(3) = \frac{dY(2)}{d\eta} = F(2), \\ f''' &= \frac{dY(3)}{d\eta} = F(3), \\ S' &= Y(9), \\ \theta &= Y(4), \\ \phi &= Y(6), \\ F' &= Y(11) \end{aligned} \quad (22)$$

Values of the local Nusselt number  $-\theta'(0)$  and skin friction coefficient  $f''(0)$  for some parameters at  $Gm = 6, G^* = 0.2, Re = 0.4, Fs = 0.5, Ec = 2, Sc = 5, \alpha_d = 0.1, \lambda = 1, \Gamma = 0.1$  have been estimated in Table I.

#### IV. RESULTS AND DISCUSSION

Herein we examine the induced results from the impact of dimensionless governing parameters on different physical quantities in the dimensionless form such the velocity and temperature for fluid phase and particle phase, too on species concentration function and angular velocity (micro-rotation) function. Our parameters values range as follows magnetic parameter ( $0 \leq M \leq 6$ ), mass concentration of the dust particles ( $0 \leq D_\rho \leq 10$ ), fluid particle interaction parameter ( $0.01 \leq \alpha_d \leq 0.1$ ), Forchheimer number ( $0.5 \leq Fs \leq 5$ ), Darcy number ( $0.3 \leq Da \leq 1.1$ ), convective parameter ( $1 \leq Bi \leq \infty$ ), local Grashof number ( $5 \leq Gr \leq 8$ ), modified Grashof number ( $1 \leq Gm \leq 6$ ), Prandtl number ( $0.7 \leq Pr \leq 1$ ), constant parameter (microrotation parameter) ( $0 \leq n \leq 1$ ) and coupling constant parameter ( $0.5 \leq B \leq 9$ ). But the other parameters remain constant in our numerical computations as follows the micropolar material parameters  $\lambda = 1$  and  $G^* = 0.2$ , local Reynolds number  $Re = 0.4$ , Eckert number  $Ec = 2$ , Schmidt number  $Sc = 5$  and the specific heat ratio of the mixture  $\Gamma = 0.1$ .

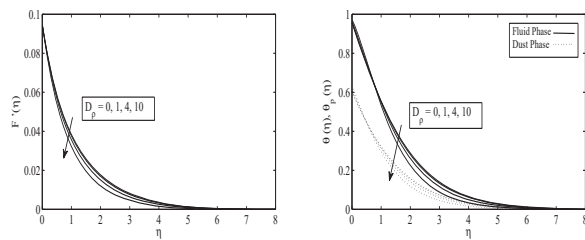
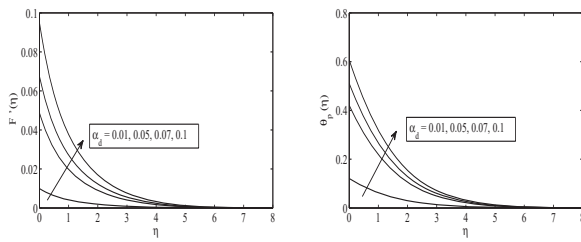
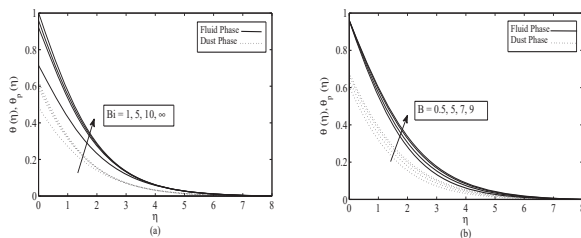
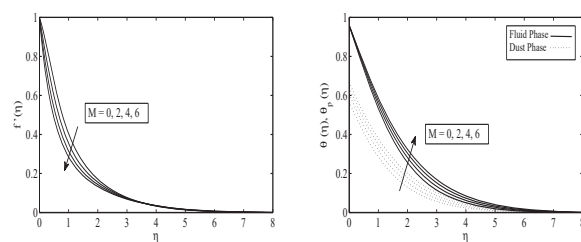


Fig. 2 Impact of  $D_\rho$  on velocity profile  $F'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$

Foremost we display the effecting of dust parameters on the velocity and temperature profiles. Fig. 2 represents the extent of influence of mass concentration of the dust particles  $D_\rho$  versus  $\eta$  on the magnitude of the dimensionless velocity and temperature distribution for fluid phase and particle phase. It is

Fig. 3 Impact of  $\alpha_d$  on velocity profile  $F'(\eta)$  and temperature profile  $\theta_p(\eta)$ 

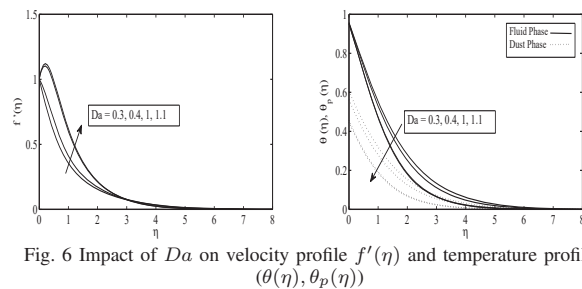
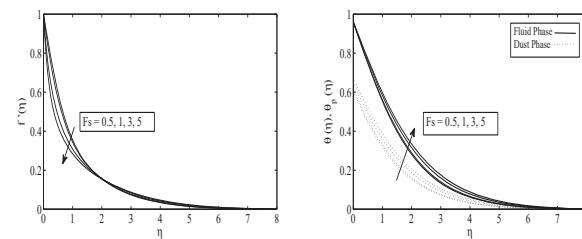
observed that the velocity ( $F'$ ) and temperatures ( $\theta, \theta_p$ ) reduce with an increase in values of  $D_p$ . In other words, increasing the mass concentration of the dust particles leads to the lack of movement and increase the rate of heat transfer as a result of improved thermal conductivity of the particle phase. It is interesting to observe that putting  $D_p = 0$  represents viscous micropolar fluid without dust particles. Fig. 3 refers that the velocity and temperature of the dust particles enhance as fluid-particle interaction parameter  $\alpha_d$  increases. This behavior may be attributed to the fact that the interaction between fluid and particles is large and hegemony of thermal conductivity of the particle phase then the particle phase reduces the velocity of the fluid till it reaches the same fluid velocity. This means a decrease in fluid velocity and an increment in dust particles velocity.

Fig. 4 (a-b) Impact of  $Bi$  and  $B$  on temperature profile  $(\theta(\eta), \theta_p(\eta))$ Fig. 5 Impact of  $M$  on velocity profile  $f'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$ 

As shown in Fig. 4 (a), Biot number  $Bi$  improves the temperature profiles in two phases fluid and dust ( $\theta, \theta_p$ ). This shows that increasing the convective parameter leads to that fluid temperature approaching from the isothermal case. Hence the thickness of the thermal boundary layer rises due to interchange in convective heat. The temperature distributions ( $\theta, \theta_p$ ) for various values of coupling constant parameter  $B$  are presented in Fig. 4 (b). From this depiction, it is clearly shown that there is progress in the temperatures under the

influence of  $B$ . The dimensionless velocity and temperature profiles in both phases fluid and dust for different values of the magnetic parameter  $M$  are displayed in Fig. 5. It is obvious that there is shrinking in the dimensionless velocity ( $f'$ ) with rising values of  $M$ . While the dimensionless temperatures ( $\theta$ ) and ( $\theta_p$ ) growing with  $M$ . Physically this behavior is due to that an application of a magnetic field produce Lorentz force which represents the opposite force to the flow and this force increases with the enhancement of magnetic field parameter which leads to diminishing motion and improves the thermal boundary layer thickness.

When studying the Darcy number  $Da$  and Forchheimer number  $Fs$  on the velocity and temperature profiles for fluid and particle phases in Figs. 6 and 7, it is found that the dimensionless velocity ( $f'$ ) rises with the enhancement of Darcy number but reduces with Forchheimer number. Also, it is noted from these figures Darcy number minimizes the dimensionless temperatures while Forchheimer number supports the temperature profiles. This behavior can be indicated by the use of Forchheimer number, which represents inertial drag and its rise leads to increase the flow resistance making the movement in a low state, as for the increase in the temperatures is due to that the energy deployed in the medium represented in the form of heat.

Fig. 6 Impact of  $Da$  on velocity profile  $f'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$ Fig. 7 Impact of  $Fs$  on velocity profile  $f'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$ 

With the help of Grashof number  $Gr$  in Fig. 8, it is reached that the Grashof number supports the velocity profiles ( $f'$ ) and leads to shrinking the temperature profiles ( $\theta, \theta_p$ ). The temperature behavior can be attributed to the fact that temperatures drop rapidly near the surface of the plate because high values of  $Gr$  transfers heat away from the plate. In Fig. 9, the velocity of dust particles ( $F'$ ) and temperatures ( $\theta, \theta_p$ ) decrement with an increase of modified Grashof number  $Gm$ .



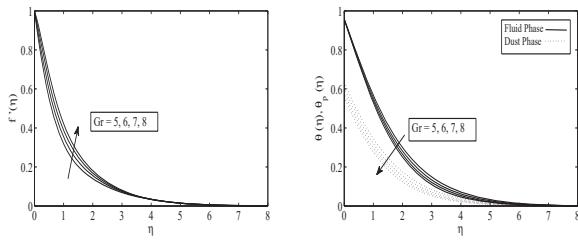


Fig. 8 Impact of  $Gr$  on velocity profile  $f'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$

For different values of Prandtl number  $Pr$  as shown in Fig. 10, we recognize that the excess in  $Pr$  leads to a fall in the velocity profiles ( $f'$ ) and the temperature profiles ( $\theta, \theta_p$ ). Deflation behavior in the velocity and temperature profiles is due to the thickness of the large fluid that tends to the contraction of velocity due to the viscosity of the large fluids and the thermal boundary layer thickness reduced (small thermal conductivity) by increasing  $Pr$ , respectively.

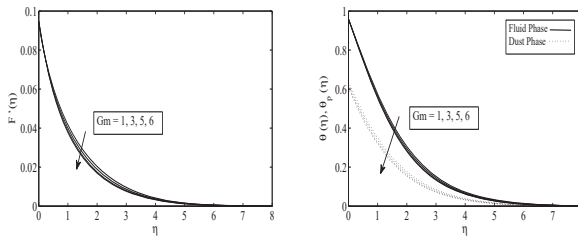


Fig. 9 Impact of  $Gm$  on velocity profile  $F'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$

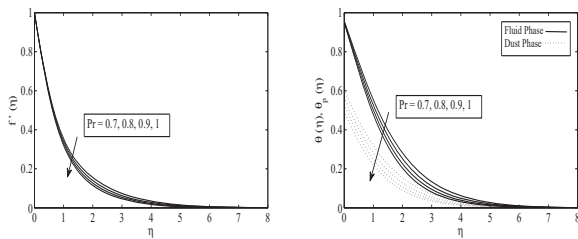


Fig. 10 Impact of  $Pr$  on velocity profile  $f'(\eta)$  and temperature profile  $(\theta(\eta), \theta_p(\eta))$

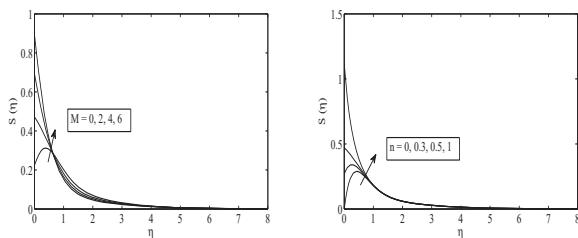


Fig. 11 Impact of  $M$  and  $n$  on angular velocity  $S$ .

Finally, we present the graphs that explain the behavior of the angular velocity when studying some parameters such as magnetic parameter  $M$ , constant parameter (microrotation

parameter)  $n$ , and Biot number  $Bi$ . In Fig. 11 the values of the angular velocity mount with high values of  $M$  and  $n$ . Taking  $n = 0$  leads to  $S = 0$  (no-spin condition) according to the condition at the wall  $S(0) = -nf''(0)$  indicates that the microelements in the flow of concentrated particles near the wall surface can not able to rotate. Taking  $n = 0.5$  ( $S$  can not equal zero when  $f''(0) \neq 0$ ) refers that the part antisymmetric from the stress tensor disappears and represents a weak concentration. The particle spin for fine particle suspensions is to be equal to the velocity of the fluid at the wall. Using  $n = 0.5$  it is potential to clarify the possibility of reducing the ruling equations to the equations of the Newtonian fluid ( $k^* = 0$ ). Taking  $n = 1$  for the purpose of modeling the flows of the turbulent boundary layer. The results obtained in Fig. 12 shows that the effect of  $Bi$  tends to dwindle the values of angular velocity.

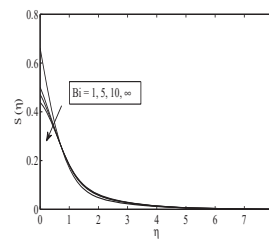


Fig. 12 Impact of  $Bi$  on angular velocity  $S$

## V. CONCLUSION

The problem of MHD natural convection heat and mass transfer of a dusty micropolar fluid in a non-Darcy porous medium with a convective boundary condition has been studied in the present paper. To solve the dimensionless boundary layer equations we used the Runge-Kutta method of fourth-order. A parametric study of our problem was conducted to elucidate and discuss the effects of some parameters on the magnitude of the velocity and temperature distribution for the fluid phase and particle phase and on the angular velocity. Important findings of this study are displayed as follow:

- 1) The profiles of velocity for fluid phase  $f'$  reduce with increasing magnetic parameter, Forchheimer number, and Prandtl number. While an enhancement in values of Darcy number and Grashof number increases the dimensionless velocity.
- 2) The profiles of velocity for dust phase  $F'$  increment with rising values of fluid-particle interaction parameter and deteriorates by an enhancement in both the mass concentration of the dust particles and modified Grashof number.
- 3) The increment in magnetic parameter, Biot number, Forchheimer number, and coupling constant parameter lead to improving the profiles of temperature for both fluid and dust phase  $(\theta, \theta_p)$ . But a reduction takes place with higher values of mass concentration of the dust particles, Darcy number, Grashof number, modified Grashof number, and Prandtl number.

- 4) There is an elevation in the behavior of  $\theta_p$  at studying fluid-particle interaction parameter.
- 5) An increase in Biot number tends to diminish the dimensionless angular velocity while this velocity progress by augmentation the magnetic parameter and microrotation parameter.

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