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Discrete Estimation of Spectral Density for Alpha Stable Signals Observed with an Additive Error

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***Abstract*—**This paper is interested in two difficulties encountered in practice when observing a continuous time process. The first is that we cannot observe a process over a time interval; we only take discrete observations. The second is the process frequently observed with a constant additive error. It is important to give an estimator of the spectral density of such a process taking into account the additive observation error and the choice of the discrete observation times. In this work, we propose an estimator based on the spectral smoothing of the periodogram by the polynomial Jackson kernel reducing the additive error. In order to solve the aliasing phenomenon, this estimator is constructed from observations taken at well-chosen times so as to reduce the estimator to the field where the spectral density is not zero. We show that the proposed estimator is asymptotically unbiased and consistent. Thus we obtain an estimate solving the two difficulties concerning the choice of the instants of observations of a continuous time process and the observations affected by a constant error.

***Keywords*—**Spectral density, stable processes, aliasing, periodogram.

I. Introduction

THIS paper considers a class of symmetric alpha stable processes particular family of processes with infinite energy. The harmonizable process symmetric α-stable process and its proprieties have been considered by numerous authors like [1]-[10] to name a few.

Symmetric alpha processes are considerably accurate models for many phenomena in several fields such as: physics, biology, electronics and electrical, hydrology, economics, communications and radar applications... see [11]-[23].

As in [4], we consider the harmonizable symmetric α-stable process: $Z = \{Z(t), -\infty < t < +\infty\}$:

$$Z(t) = \int_{-\infty}^{\infty} e^{itu} d\xi(u) \qquad (1)$$

where ξ is a complex-valued SαS process with independent isotropic increments. The existence of the stochastic integral (1) is given [1], [5].

The spectral measure: $\mu(]s,t])=|\ \xi(s)-\xi\ (t)|_\alpha^{\alpha}$ is Lebesgue-Stieljs measure defined in [1] and [4]. If μ is absolutely continuous $d\mu(x)=f(x)\ dx$, the function f is called the spectral density of the process *X.* The paper [4] gives an estimate of the spectral density function when the process is continuous-time. The spectral density is estimated in [24] when the process is discrete-time and in [25] when the process is p-adic-time.

In this paper we consider a case frequently encountered during observation of the process: it is a question of observing the process with a constant additive unknown error. The process $X_t = a + Z_t$ is observed instead of the process Z alone. The paper [26] gives an estimate of the constant error a when the process is discrete-time. This error is also estimated from the discrete observations of the continuous time process in [27].

Our goal is to give an estimate of the spectral density ϕ, from sample of the process $X(t_n)$ at discrete instants t_n, where the sampling instants t_n are equally distant, i.e., $t_n = n\tau$, $\tau > 0$.

To avoid the aliasing phenomenon, we suppose that the spectral density ϕ is vanishing for $|\lambda| > \Omega$ and $\phi(0) \neq 0$, where Ω is a nonnegative real number. The value of τ is chosen such that $\tau\Omega < \pi$. For more details about the aliasing phenomenon see [28].

This paper is organized as follows: The second section gives some definitions and proprieties Jackson polynomial kernel and a periodogram as an estimator of the spectral density depending of τ. We show that this estimate is unbiased. In Section III, in order to give unbiased and consistent estimator we smooth this periodogram by a spectral window. Section IV is reserved for the numerical studies and simulation.

II. Periodogram Construction

As in [20], [24], we give the definition of the Jackson polynomial kernel: Let $Z_1, \ldots, Z_N$ observations of the process Z: $\left(Z_{(n)}\right)_{0 \le n \le N-1}$, where N satisfies:

$$N - 1 = 2k(n-1) \qquad with \quad n \in N \quad k \in N \cup \{1/2\}$$

if $k = 1/2$ then $n = 2n_1 - 1, n_1 \in N$.

The Jackson's polynomial kernel is defined by:

$$|H_N(\lambda)|^\alpha = \left|A_N H^{(N)}(\lambda)\right|^\alpha$$

where

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$$H^{(N)}(\lambda)=\frac{1}{q_{k,n}}\left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})}\right)^{2k} \quad with \quad q_{k,n}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})}\right)^{2k} d\lambda$$

In addition, we have $A_N=(B_{\alpha,N})^{-\frac{1}{\alpha}}$ with

$$B_{\alpha,N}=\int_{-\pi}^{\pi}\left|H^{(N)}(\lambda)\right|^{\alpha} d\lambda.$$

The following lemmas are used in the reminder of this paper. Their proofs are given in [24].

Lemma1. There is a non-negative function h_k such as:

$$H^{(N)}(\lambda)=\sum_{m=-k(n-1)}^{k(n-1)} h_k\left(\frac{m}{n}\right)\cos(m\lambda)$$

Lemma2. Let

$$B'_{\alpha,N}=\int_{-\pi}^{\pi}\left|\frac{\sin\frac{n\lambda}{2}}{\sin\frac{\lambda}{2}}\right|^{2k\alpha} d\lambda \quad and \quad J_{N,\alpha}=\int_{-\pi}^{\pi}|u|^{\gamma}|H_N(\lambda)|^{\alpha} d\lambda,$$

where $\gamma\in]0,2]$. Then

$$B'_{\alpha,N}\begin{cases}\geq 2\pi\left(\frac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha-1} if\ 0<\alpha<2\\ \leq \frac{4\pi k\alpha}{2k\alpha-1} n^{2k\alpha-1} if\ \frac{1}{2k}<\alpha<2\end{cases} \quad and$$

$$J_{N,\alpha}\leq\begin{cases}\frac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma-2k\alpha+1)}\frac{1}{n^{2k\alpha-1}} & if \quad \frac{1}{2k}<\alpha<\frac{\gamma+1}{2k}\\ \frac{2k\alpha\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma+1)(2k\alpha-\gamma-1)}\frac{1}{n^{\gamma}} & if \quad \frac{\gamma+1}{2k}<\alpha<2\end{cases}$$

In this paper we assume that the spectral density is uniformly continuous and we choose τ such that $\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}$ converges to zero for example $\tau=\frac{1}{n^2}$

We propose the following periodogram defined by

$$d_N(\lambda)=\tau^{\frac{1}{\alpha}}\frac{A_N}{2}\sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(n'/n\right)\left(e^{-in'\tau\lambda}-\frac{H_N(\lambda)}{H_N(\tau)}e^{-i\tau n'}\right)X(\tau n'+\tau k(n-1)).$$

By using Lemma 2, we show that

$$Eexp\left(irRed_N(\lambda)\right)=exp\left(-C_{\alpha}|r|^{\alpha}\psi_N(\lambda)\right)$$

where

$$\psi_N(\lambda)=\int_{-\infty}^{\infty}\tau\left|\frac{A_N}{2}\sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right)\left(e^{-in'\tau\lambda}-\frac{H_N(\lambda)}{H_N(\tau)}e^{-i\tau n'}\right)e^{\tau n'u}\right|^{\alpha}\phi(u)du$$

Thus,

$$\psi_N(\lambda)=\frac{\tau}{2^{\alpha}}\int_{-\infty}^{\infty}\left|H_N(\tau\lambda-\tau u)-\frac{H_N(\lambda)}{H_N(\tau)}H_N(\tau-\tau u)\right|^{\alpha}\phi(u)du$$

$\psi_N(\lambda)\leq\psi_{N,1}(\lambda)+\psi_{N,2}(\lambda)$, where

$$\psi_{N,1}(\lambda)=\frac{1}{2^{\alpha}}2^{\alpha}\int_{-\infty}^{\infty}\left|H_N(\tau\lambda-v)\right|^{\alpha}\phi\left(\frac{v}{\tau}\right)dv \quad and$$

$$\psi_{N,2}(\lambda)=\frac{1}{2^{\alpha}}2^{\alpha}\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}\int_{-\infty}^{\infty}\left|H_N(\tau-v)\right|^{\alpha}\phi\left(\frac{v}{\tau}\right)dv$$

Thus $\psi_{N,1}(\lambda)=\sum_{j\in Z}\int_{(2j-1)\pi}^{(2j+1)\pi}\left|H_N(\tau\lambda-v)\right|^{\alpha}\phi\left(\frac{v}{\tau}\right)dv$ and

$$\psi_{N,2}(\lambda)=\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}\sum_{j\in Z}\int_{(2j-1)\pi}^{(2j+1)\pi}\left|H_N(\tau-v)\right|^{\alpha}\phi\left(\frac{v}{\tau}\right)dv$$

Putting $v=y-2\pi j$ and using the fact that H_N is 2π-periodic, we obtain

$$\psi_{N,1}(\lambda)=\sum_{j\in Z}\int_{-\pi}^{\pi}\left|H_N(\tau\lambda-y)\right|^{\alpha}\phi\left(\frac{y}{\tau}-\frac{2\pi j}{\tau}\right)dy$$

and

$$\psi_{N,2}(\lambda)=\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}\sum_{j\in Z}\int_{-\pi}^{\pi}\left|H_N(\tau-y)\right|^{\alpha}\phi\left(\frac{y}{\tau}-\frac{2\pi j}{\tau}\right)dv$$

Let j be an integer such that $-\Omega<\frac{y-2\pi j}{\tau}<\Omega$. Using the fact that $\tau\Omega<\pi$ and $|y|<\pi$, we get $|j|<\frac{\tau\Omega}{2\pi}+\frac{1}{2}<1$ and then $j=0$. As ϕ is vanishing for $|\lambda|>\Omega$ we obtain

$$\psi_{N,1}(\lambda)=\int_{-\pi}^{\pi}\left|H_N(\tau\lambda-y)\right|^{\alpha}\phi\left(\frac{y}{\tau}\right)dy \quad and$$

$$\psi_{N,2}(\lambda)=\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}\int_{-\pi}^{\pi}\left|H_N(\tau-y)\right|^{\alpha}\phi\left(\frac{y}{\tau}\right)dv$$

Since $\left|H_N(u)\right|^{\alpha}$ is a kernel, $\phi(0)\neq 0$ and $\left|\frac{H_N(\lambda)}{H_N(\tau)}\right|^{\alpha}$ converges to zero, then $\psi_N(\lambda)$ converges to $\phi(\lambda)$. We modify this periodogram by taking the power $p, 0<p<\frac{\alpha}{2}$, and multiplying by a normalization constant:

$$\hat{I}_N(\lambda)=C_{(p,\alpha)}|d_N(\lambda)|^{p}$$

The normalization constant is given by

$$C_{(p,\alpha)} = \frac{D_p}{F_{p,\alpha} C_\alpha^{p/\alpha}}$$

where $D_p = \int \frac{1-cos(u)}{|u|^{1+p}} du$ and $F_{p,\alpha} = \int \frac{1-e^{-|u|^\alpha}}{|u|^{1+p}} du$. As in [4] and [24], we show that

$$E\hat{I}_N(\lambda) = (\psi_N(\lambda))^{\frac{p}{\alpha}} \text{ and } Var(\hat{I}_N(\lambda)) = V_{\alpha,p}\psi_N(\lambda)^{\frac{2p}{\alpha}}$$

III. Smoothed Periodogram

In order to give an unbiased consistent estimate of ϕ, we smooth $\hat{I}_N$ by a the following spectral window: $f_N(\lambda) = \int_{-\pi}^{\pi} W_N(\lambda - u)\hat{I}_N(u)du$ where the spectral window is defined by $W_N(x) = M_N W(M_N x)$ where W is a non-negative even continuous function which is vanishing for $|x|>1$ with $\int_{-1}^{1} W(x)dx = 1$ and M_N is a sequence converging to infinity such that $\frac{M_N}{N} \to 0.$

As in [20], to give the best rate of convergence of this estimator, we introduce two hypothesis (h_1) and (h_2), that are called regularity hypothesis, on ϕ :

(h_1) : $|\phi(\lambda-u)-\phi(\lambda)| \le C_1 |u|^\gamma$ where $0<\gamma\le 1$

(h_2) : $|\phi(\lambda-u)-\phi(\lambda)-u\phi'(\lambda)| \le C_2 |u|^\gamma$

where $1 \le \gamma \le 2$, C_1 and C_2 are non-negative constants.

Theorem 1. Let λ be a real number. Then

(i) $f_N(\lambda)$ is an asymptotically unbiased estimator of the $\phi(\lambda)^{\frac{p}{\alpha}}$

(ii) Taking k integer number such that $1<2k\alpha n$, where k and n are given in the definition of Jackson's polynomial kernel, we have

$$Ef_N(\lambda) - \phi(\lambda)^{\frac{p}{\alpha}} = \begin{cases} O\left(\frac{1}{M_N{}^\gamma}\right) & \text{if } \phi \text{ satisfies } (h_1) \\ O\left(\frac{1}{M_N}\right) & \text{if } \phi \text{ satisfies } (h_2) \end{cases}$$

(iii) $Var(f_N(\lambda))$ converges to zero. If ϕ satisfies (h_1) or (h_2) and $M_N = n^c$ where c is a real number less than 1/2 and $c \le \inf\left(\frac{2k^2\alpha^2+1}{6\alpha^2k^2}, \frac{k\alpha+2}{3(k\alpha+1)}\right)$ *then*

$$Var(f_N(\lambda)) = O\left(\frac{1}{n^{2(1-2c)}}\right)$$

Proof.

i) From the definition of the estimator

$$f_N(\lambda) = \int M_n W(M_n(x-\lambda))I(x)dx$$

Let $M_n(x-\lambda) = v$

$$E(f_N(\lambda)) = \int W(v)\left(\psi_N(\lambda - \frac{v}{M_n})\right)^p dv$$

As $p < \frac{\alpha}{2}$, then we obtain

$$Bias(f_N(\lambda)) \le \int W(v)\left|\psi_N(\lambda - \frac{v}{M_n}) - \phi(\lambda)\right|^{p/\alpha} dv$$

where $Bias(f_N) = Ef_N(\lambda) - \phi(\lambda)^{\frac{p}{\alpha}}$. Since ϕ is uniformly continuous, we obtain $Bias(f_N(\lambda)) = o(1)$.

ii) Choosing k so large that $+1<2k\alpha n$, we get

$$Bias(f_N(\lambda)) = \begin{cases} O\left(\frac{1}{M_N{}^\gamma}\right) & \text{if } \phi \text{ satisfies } (h_1) \\ O\left(\frac{1}{M_N}\right) & \text{if } \phi \text{ satisfies } (h_2) \end{cases}$$

iii) From the definition of the variance, we have:

$$Var(f_N(\lambda)) = \iint W(x)W(y)\,\mathrm{cov}(\lambda - \frac{x}{M_n}, \lambda - \frac{y}{M_n})dxdy$$

We split the integral as follows:

$$Var(f_N(\lambda)) = \iint_{|x-y|<\varepsilon_n} \iint_{|x-y|>\varepsilon_n}$$

where ε_n is a positive real converging to zero. As in [24], we get the result of this theorem.

Theorem 2. Let λ be a real number such that $\phi(\lambda)>0$. Then $(f_N(\lambda))^{\frac{\alpha}{p}}$ converges in probability to $\phi(\lambda)$.

Proof. We show that $f_N(\lambda)$ converges in mean quadratic to $\phi(\lambda)^{\frac{p}{\alpha}}$. Indeed,

$$E\left|f_N(\lambda) - \phi(\lambda)^{\frac{p}{\alpha}}\right|^2 = \left(Ef_N(\lambda) - \phi(\lambda)^{\frac{p}{\alpha}}\right)^2 + Var(f_N(\lambda))$$

Then from theorem 1, $E\left|f_N(\lambda)-\phi(\lambda)^{\frac{p}{\alpha}}\right|^2$ converges to 0.

Thus, $\left(f_N(\lambda)\right)^{\frac{\alpha}{p}}$ converges in probability to $\phi(\lambda)$.

IV. Numerical Studies

We give the simulation of the studied process:

$$Z_N=\int_{-\pi}^{\pi}e^{iN\lambda}d\xi(\lambda),$$

where $1<\alpha<2$ and ξ is a complex symmetric α-stable with independent and isotropic increments. The control measure *m* such that: *mdx* = *ϕ(x)dx*. Indeed, we use the series representations given by [29] where the authors have shown that the process *Z* can be expressed as follows:

$$Z_n=C_\alpha\left(\int\phi(x)dx\right)^{\frac{1}{\alpha}}\sum_{k=1}^{\infty}\varepsilon_k\Gamma_k^{-\frac{1}{\alpha}}e^{inV_k}e^{i\theta_k},$$

where ε_k is a sequence of i.i.d. random variables such as $P[\varepsilon_k=0]=P[\varepsilon_k=1]=\frac{1}{2}$, Γ_k is a sequence of arrival times of Poisson process, V_k is a sequence of i.i.d. random variables independent of ε_k and of Γ_k having the same distribution of control measure *m*, which has probability density ϕ, θ_k is a sequence of i.i.d. random variables that has the uniform distribution on $[-\pi,\pi]$, independent of ε_k, of Γ_k and of V_k.

To generate *N* values (*N* = 1001) of the process Z_n, we use the following steps:

- generate 2000 values of ε_k,
- generate 2000 values of V_k
- generate 2000 values of Γ_k
- generate 2000 values of θ_k

Then we calculate for all $0\le n\le N$

$$Z_n=C_\alpha\left(\int\phi(x)dx\right)^{\frac{1}{\alpha}}\sum_{k=1}^{1001}\varepsilon_k\Gamma_k^{-\frac{1}{\alpha}}e^{inV_k}e^{i\theta_k},\ \text{with } \alpha=1.7$$

where the spectral density is chosen as

$$\phi(x)=e^{-x^{(1.7)}}\text{if } x\in[-2.5,2.5]\quad\text{and}\quad\phi(x)=0\quad\text{if}\quad x\notin[-2.5,2.5]$$

thus we take $\Omega=2.5$ and $\tau=\frac{\pi}{5}$. We generate $X_n=a+Z_n$ where *a* is chosen equal to 15.

We calculate the estimator $f_N(\lambda)$. Fig. 1 gives the graphic of the estimator $f_N(\lambda)$ and the graphic of the spectral density $\phi(\lambda)$. The curve of the estimator well approximates the curve of the spectral density. The result is satisfied.

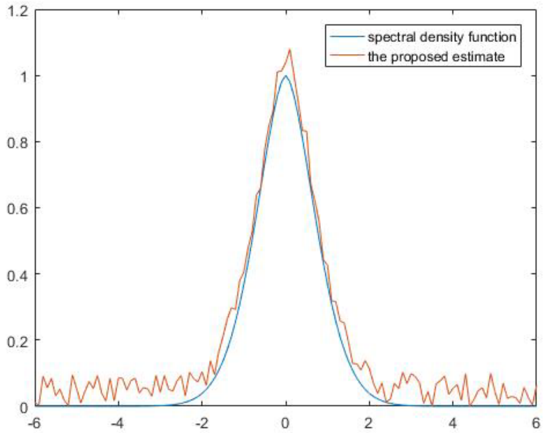

Fig. 1 The blue curve is the spectral ϕ and the red curve is the estimator $f_N(\lambda)$

V. Conclusion

In this paper, we estimate the spectral density of (SαS) process with continuous time when the process is observed with an additive error. The aliasing phenomenon is avoided by assuming that the spectral is a support compact. This work could be applied to several cases when processes have an infinite variance and the observation of these processes is perturbed by a constant noise. For example:

- The decomposition of audio signals with background noise by separating the different musical instruments.
- The denoising of a degraded historical record. The signal is considered infinitely variable.

The perspective of this work is to optimize the smoothing parameters to have a better speed of convergence. For this purpose, the cross-validation method will be the most appropriate tool.

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