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# The Non-Stationary BINARMA(1,1) Process with Poisson Innovations: An Application on Accident Data

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*Abstract*—This paper considers the modelling of a non-stationary bivariate integer-valued autoregressive moving average of order one (BINARMA(1,1)) with correlated Poisson innovations. The BINARMA(1,1) model is specified using the binomial thinning operator and by assuming that the cross-correlation between the two series is induced by the innovation terms only. Based on these assumptions, the non-stationary marginal and joint moments of the BINARMA(1,1) are derived iteratively by using some initial stationary moments. As regards to the estimation of parameters of the proposed model, the conditional maximum likelihood (CML) estimation method is derived based on thinning and convolution properties. The forecasting equations of the BINARMA(1,1) model are also derived. A simulation study is also proposed where BINARMA(1,1) count data are generated using a multivariate Poisson R code for the innovation terms. The performance of the BINARMA(1,1) model is then assessed through a simulation experiment and the mean estimates of the model parameters obtained are all efficient, based on their standard errors. The proposed model is then used to analyse a real-life accident data on the motorway in Mauritius, based on some covariates: policemen, daily patrol, speed cameras, traffic lights and roundabouts. The BINARMA(1,1) model is applied on the accident data and the CML estimates clearly indicate a significant impact of the covariates on the number of accidents on the motorway in Mauritius. The forecasting equations also provide reliable one-step ahead forecasts.

*Keywords*—Non-stationary, BINARMA(1,1) model, Poisson Innovations, CML.

## I. Introduction

Time series of counts have commonly been modelled using integer-valued autoregressive (INAR) and integer-valued moving average (INMA) models, compared to its INARMA counterpart. The simplest family of stationary first order INAR (INAR(1)) models were initially developed by McKenzie [1] and Al-Osh and Alzaid [2] and thereon, several INAR(1) models under different distributional assumptions have been developed in literature ([3], [4], [5]). While INAR(1) models have gained lots of attention in literature, some researchers have also concentrated on the development of INMA(1) models ([1], [4], [6], [7]). However, these models were limited to analysing univariate time series only. Hence, many researchers considered the extension of these univariate models to a bivariate set-up in order to analyse bivariate count observation.

Bivariate INAR(1) (BINAR(1)) ([8], [9], [10], [11], [12], [13]) and bivariate INMA(1) (BINMA(1)) ([14], [15], [16]) have been considered and applied extensively in literature under different distributional assumptions of the innovation terms and thinning operations. However, count models that include both AR and MA components have received less attention in literature and have rarely been applied in practice. The first INARMA was developed by McKenzie [1] under stationarity condition, but the construction of such models is not appealing to researchers due to the complication of including both the AR and MA component. Recently, Weib et al. [17] considered INARMA modelling of count time series under stationarity assumption. However, the INARMA models developed so far are not appropriate to analyse real-life data which exhibit non-stationarity moments. Hence, this paper proposes a BINARMA(1,1) model with correlated Poisson innovations which can analyse non-stationary real-life bivariate counts. As regards to the estimation of the model parameters, the conditional maximum likelihood (CML) method will be used due to the complicated nature of the BINARMA(1,1) model.

The organisation of the paper is as follows: In Section II, the BINARMA(1,1) model with correlated Poisson innovations is developed by deriving the moments. Section III presents the CML method for estimating the unknown parameters. The forecasting equations for the BINARMA(1,1) model are developed in Section IV. In Section V, a simulation study is conducted in order to assess the BINARMA(1,1) model. A real-life application on accident data in Mauritius is also considered in Section VI. The conclusion is presented in Section VII.

## II. BINARMA(1,1) Model

The BINARMA(1,1) model is specified as

$$Y_t^{[1]} = \rho_1 * Y_{t-1}^{[1]} + \rho_2 * R_{t-1}^{[1]} + R_t^{[1]}, \qquad (1)$$

$$Y_t^{[2]} = \rho_3 * Y_{t-1}^{[2]} + \rho_4 * R_{t-1}^{[2]} + R_t^{[2]}, \qquad (2)$$

where $Y_t^{[k]}$ is the counting random observation for the $k^{th}$ series at the $t^{th}$ time point with corresponding innovation terms $R_t^{[k]}$, for $k = 1, 2$ and $t = 1, 2, \ldots, T$. The other assumptions of the BINARMA(1,1) model are:

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(a) The pair of innovations $\{[R_t^{[1]}, R_t^{[2]}]\}$ follows the bivariate Poisson distribution [18], where

$$\text{Corr}(R_t^{[1]}, R_{t+h}^{[2]}) = \begin{cases} \rho_{12} & h = 0, \\ 0 & h \neq 0. \end{cases} \tag{3}$$

(b) $R_t^{[k]}$ is an independent and identically distributed sequence of Poisson counts, i.e, $R_t^{[k]} \sim \text{Poisson}(\lambda_t^{[k]})$ where $\lambda_t^{[k]} = \exp(\boldsymbol{x}_t' \boldsymbol{\beta}^{[k]})$ with $\boldsymbol{x}_t = [x_{t1}, x_{t2}, \ldots, x_{tj}, \ldots, x_{tp}]'$ and $\boldsymbol{\beta}^{[k]} = [\beta_1^{[k]}, \beta_2^{[k]}, \ldots, \beta_j^{[k]}, \ldots, \beta_p^{[k]}]'$ for $p$ covariates.

(c) $*$ is the binomial thinning operator [19], i.e.,

$$\rho * V = \begin{cases} \sum_{l=1}^{V} b_l(\rho), & V > 0, \\ 0 & V = 0, \end{cases}$$

where the counting series $\{b_l(\rho)\}$ is a sequence of independent and identically distributed Bernoulli random variables with $\rho * V | V \sim \text{Binomial}(V, \rho)$. Thus, $\text{E}(\rho * V) = \rho E(V)$ and $\text{Var}(\rho * V) = \rho(1-\rho)E(V) + \rho^2 \text{Var(V)}$.

(d)

$$\text{Cov}(Y_t^{[k]}, R_{t+h}^{[k]}) = \begin{cases} Var(R_t^{[k]}) & h = 0, \\ 0 & h > 0. \end{cases} \tag{4}$$

and

$$\text{Cov}(Y_t^{[k]}, R_{t+h}^{[j]}) = \begin{cases} Cov(R_t^{[k]}, R_t^{[j]}) & h = 0, \\ 0 & h > 0. \end{cases} \tag{5}$$

Using the above assumptions, we obtain

$$\mu_t^{[1]} \equiv E(Y_t^{[1]}) = \rho_1 \mu_{t-1}^{[1]} + \rho_2 \lambda_{t-1}^{[1]} + \lambda_t^{[1]}, \tag{6}$$

$$\mu_t^{[2]} \equiv E(Y_t^{[2]}) = \rho_3 \mu_{t-1}^{[2]} + \rho_4 \lambda_{t-1}^{[2]} + \lambda_t^{[2]}. \tag{7}$$

As for the marginal variances,

$$\begin{aligned} \text{Var}(Y_t^{[1]}) &= \text{Var}(\rho_1 * Y_{t-1}^{[1]}) + \text{Var}(\rho_2 * R_{t-1}^{[1]}) + \text{Var}(R_t^{[1]}) \\ &+ 2\text{Cov}(\rho_1 * Y_{t-1}^{[1]}, \rho_2 * R_{t-1}^{[1]}) \\ &= E[\text{Var}(\rho_1 * Y_{t-1}^{[1]} | Y_{t-1}^{[1]})] + \text{Var}[E(\rho_1 * Y_{t-1}^{[1]} | Y_{t-1}^{[1]})] \\ &+ E[\text{Var}(\rho_2 * R_{t-1}^{[1]} | R_{t-1}^{[1]})] + \text{Var}[E(\rho_2 * R_{t-1}^{[1]} | R_{t-1}^{[1]})] \\ &+ \text{Var}(R_t^{[1]}) + 2\text{Cov}(\rho_1 * Y_{t-1}^{[1]}, \rho_2 * R_{t-1}^{[1]}) \\ &= \rho_1(1-\rho_1)\mu_{t-1}^{[1]} + \rho_1^2 \text{Var}(Y_{t-1}^{[1]}) + \rho_2(1-\rho_2)\lambda_{t-1}^{[1]} \\ &+ \rho_2^2 \lambda_{t-1}^{[1]} + 2\rho_1\rho_2 \text{Cov}(Y_{t-1}^{[1]}, R_{t-1}^{[1]}) + \lambda_t^{[1]} \\ &= \rho_1(1-\rho_1)\mu_{t-1}^{[1]} + \rho_1^2 \text{Var}(Y_{t-1}^{[1]}) + (\rho_2 + 2\rho_1\rho_2)\lambda_{t-1}^{[1]} \\ &+ \lambda_t^{[1]}. \end{aligned} \tag{8}$$

Similarly,

$$\begin{aligned} \text{Var}(Y_t^{[2]}) &= \rho_3(1-\rho_3)\mu_{t-1}^{[2]} + \rho_3^2 \text{Var}(Y_{t-1}^{[2]}) + (\rho_4 + 2\rho_3\rho_4)\lambda_{t-1}^{[2]} \\ &+ \lambda_t^{[2]}. \end{aligned} \tag{9}$$

The cross-covariances between the two series is given by

$$\begin{aligned} \text{Cov}(Y_t^{[1]}, Y_t^{[2]}) &= \rho_1\rho_3 \text{Cov}(Y_{t-1}^{[1]}, Y_{t-1}^{[2]}) \\ &+ (\rho_1\rho_4 + \rho_2\rho_3 + \rho_2\rho_4 + 1)\rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}. \end{aligned} \tag{10}$$

The lag-covariances for the same series for $h \geq 1$ are

$$\text{Cov}(Y_t^{[1]}, Y_{t+h}^{[1]}) = \rho_1^h \text{Var}(Y_t^{[1]}) + \rho_1^{h-1}\rho_2\lambda_t^{[1]}, \tag{11}$$

$$\text{Cov}(Y_t^{[2]}, Y_{t+h}^{[2]}) = \rho_3^h \text{Var}(Y_t^{[2]}) + \rho_3^{h-1}\rho_4\lambda_t^{[2]}, \tag{12}$$

while the cross-covariances are

$$\begin{aligned} \text{Cov}(Y_t^{[1]}, Y_t^{[2]}) &= \rho_1\rho_3 \text{Cov}(Y_{t-1}^{[1]}, Y_{t-1}^{[2]}) \\ &+ (\rho_1\rho_4 + \rho_2\rho_3 + \rho_2\rho_4 + 1)\rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}, \end{aligned} \tag{13}$$

$$\text{Cov}(Y_t^{[1]}, Y_{t+h}^{[2]}) = \rho_3^h \text{Cov}(Y_t^{[1]}, Y_t^{[2]}) + \rho_3^{h-1}\rho_4\rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}. \tag{14}$$

**Remark 1.** The moments in (6)-(10) are obtained iteratively for $t = 2, \ldots, T$ using the following initial means, variances and cross-covariances:

$$\mu_1^{[1]} = \left(\frac{1+\rho_2}{1-\rho_1}\right)\lambda_1^{[1]}, \tag{15}$$

$$\mu_1^{[2]} = \left(\frac{1+\rho_4}{1-\rho_3}\right)\lambda_1^{[2]}, \tag{16}$$

$$\text{Var}(Y_1^{[1]}) = \frac{\rho_1(1-\rho_1)\mu_1^{[1]} + (1 + 2\rho_1\rho_2 + \rho_2)\lambda_1^{[1]}}{(1-\rho_1^2)}, \tag{17}$$

$$\text{Var}(Y_1^{[2]}) = \frac{\rho_3(1-\rho_3)\mu_1^{[2]} + (1 + 2\rho_3\rho_4 + \rho_4)\lambda_1^{[2]}}{(1-\rho_3^2)}, \tag{18}$$

$$\text{Cov}(Y_1^{[1]}, Y_1^{[2]}) = \frac{(1 + \rho_1\rho_4 + \rho_2\rho_3 + \rho_2\rho_4)\rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}}{(1-\rho_1\rho_3)}. \tag{19}$$

## III. Estimation of Parameters

This section describes the CML estimation method for estimating the regression and correlation parameters based on thinning and convolution properties following Pedeli and Karlis [10]. Thus, the conditional density of the BINARMA(1,1) model is given by

$$\begin{aligned} f_1(k) = \sum_{j_1=0}^{k} & \begin{pmatrix} y_{t-1}^{[1]} \\ j_1 \end{pmatrix} \begin{pmatrix} r_t^{t-1} = y_{t-1}^{[1]} - k \\ k - j_1 \end{pmatrix} \\ & \rho_1^{j_1}(1-\rho_1)^{y_{t-1}^{[1]} - j_1}\rho_2^{k-j_1}(1-\rho_2)^{y_{t-1}^{[1]} - 2k + j_1}, \end{aligned} \tag{20}$$

$$\begin{aligned} f_2(s) = \sum_{j_2=0}^{s} & \begin{pmatrix} y_{t-1}^{[2]} \\ j_2 \end{pmatrix} \begin{pmatrix} r_{t-1}^{[2]} = y_{t-1}^{[2]} - s \\ s - j_2 \end{pmatrix} \\ & \rho_3^{j_2}(1-\rho_3)^{y_{t-1}^{[2]} - j_2}\rho_4^{s-j_2}(1-\rho_4)^{y_{t-1}^{[2]} - 2s + j_2}, \end{aligned} \tag{21}$$

and a bivariate distribution of the innovation terms $f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s) = P_{(R_t^{[1]}=r_t^{[1]}, R_t^{[2]} r_t^{[2]})}$, where

$$f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s)$$
$$= e^{-(\lambda_t^{[1]} + \lambda_t^{[2]} - \rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}})}$$
$$\sum_{m=0}^{\min(k,s)} \{[\lambda_t^{[1]} - \rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}]^{y_{t-1}^{[1]}-k-m}$$
$$\times [\lambda_t^{[2]} - \rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}]^{y_{t-1}^{[2]}-s-m}$$
$$\times [\rho_{12}\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}]^m\}/[(y_{t-1}^{[1]} - k - m)!(y_{t-1}^{[2]} - s - m)!m!]. \quad (22)$$

The conditional density is written as $f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta})$ $=\sum_{k=0}^{g_1}\sum_{s=0}^{g_2} f_1(k) f_2(s) f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s)$, where $\boldsymbol{\theta} = [\rho_1, \rho_2, \rho_3\rho_4, \rho_{12}, \boldsymbol{\beta}^{[k]}]$ is the vector of unknown parameters, $g_1 = \min(y_t^{[1]}, y_{t-1}^{[1]})$ and $g_2 = \min(y_t^{[2]}, y_{t-1}^{[2]})$.

The conditional likelihood function is given by

$$L(\boldsymbol{\theta}|\boldsymbol{y}) = \prod_{t=1}^{T} f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta}) \quad (23)$$

and the maximum likelihood estimators of $\boldsymbol{\theta}$ is obtained by maximizing

$$\log[L(\boldsymbol{\theta}|\boldsymbol{y})] = \log\left[\sum_{t=1}^{T} f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta})\right] \quad (24)$$

for some initial value of $\boldsymbol{y_0}$

## IV. Forecasting Equations

The conditional expectation and variance of the one-step ahead forecast $Y_{t+1}^{[k]}$ given $Y_t^{[k]}, R_t^{[k]}$ are expressed as follows:

$$E(Y_{t+1}^{[1]}|Y_t^{[1]}, R_t^{[1]}) = \hat{\lambda}_{t+1}^{[1]} + \hat{\rho}_1 Y_t^{[1]} + \hat{\rho}_2 R_t^{[1]}, \quad (25)$$
$$E(Y_{t+1}^{[2]}|Y_t^{[2]}, R_t^{[2]}) = \hat{\lambda}_{t+1}^{[2]} + \hat{\rho}_3 Y_t^{[2]} + \hat{\rho}_4 R_t^{[2]} \quad (26)$$

and

$$\text{Var}(Y_{t+1}^{[1]}|Y_t^{[1]}, R_t^{[1]}) = \hat{\rho}_1(1-\hat{\rho}_1)Y_t^{[1]} + \hat{\rho}_2(1-\hat{\rho}_2)R_t^{[1]} + \hat{\lambda}_{t+1}^{[1]}, \quad (27)$$
$$\text{Var}(Y_{t+1}^{[2]}|Y_t^{[2]}, R_t^{[2]}) = \hat{\rho}_3(1-\hat{\rho}_3)Y_t^{[2]} + \hat{\rho}_4(1-\hat{\rho}_4)R_t^{[2]} + \hat{\lambda}_{t+1}^{[2]}. \quad (28)$$

where $R_t^{[k]}$ is approximated by $\lambda_t^{[k]}$.

## V. Simulation Study

In this section, we generate BINARMA(1,1) count data using (1)-(2) and present a simulation study to assess the performance of the proposed model. Hence, the first step is to generate the bivariate innovation terms using the multivariate Poisson R code developed by Yahav and Shmueli [20]. Thereon, by assuming $Y_1^{[k]} = R_1^{[k]}$, we generate $Y_t^{[1]}$ and $Y_t^{[2]}$ for $t = 2, \ldots, T$ with $\lambda_t^{[k]} = \exp(\beta_1^{[k]} x_{t1} + \beta_2^{[k]} x_{t2})$, where

$$x_{t1} = \begin{cases} 1 & (t = 1, \ldots, T/4), \\ 2t & (t = (T/4)+1, \ldots, 3T/4), \\ cos(\frac{2\pi t}{6}) & (t = (3T/4)+1, \ldots, T), \end{cases}$$

$$x_{t2} = \begin{cases} sin(\frac{3\pi t}{12}) & (t = 1, \ldots, T/4), \\ cos(\frac{\pi t}{6}) & (t = (T/4)+1, \ldots, 3T/4), \\ sin(\frac{2\pi t}{6}) & (t = (3T/4)+1, \ldots, T). \end{cases}$$

Assuming $T = 60, 300, 600$, we conduct 5000 Monte Carlo replications using $\rho_1, \rho_2, \rho_3, \rho_4$ are combinations of [0.3,0.4], $\beta_1^{[k]} = \beta_2^{[k]} = 1$ and $\rho_{12} = [0.1, 0.5]$ and the results are shown below:

TABLE I
Estimates of the Regression Parameters and Standard Errors under Non-Stationary BINARMA(1,1) Process

| $\rho_{12}$ | $T$ | $\hat{\beta}_1^{[1]} = 1$ | $\hat{\beta}_2^{[1]} = 1$ | $\hat{\beta}_1^{[2]} = 1$ | $\hat{\beta}_2^{[2]} = 1$ |
|---|---|---|---|---|---|
| 0.1 | 60 | 0.9825 | 0.9802 | 0.9842 | 0.9879 |
| | | (0.1807) | (0.1814) | (0.1790) | (0.1703) |
| | 300 | 0.9921 | 0.9938 | 0.9904 | 0.9910 |
| | | (0.1394) | (0.1415) | (0.1302) | (0.1354) |
| | 600 | 0.9953 | 1.0025 | 0.9995 | 0.9976 |
| | | (0.0514) | (0.0444) | (0.0302) | (0.0418) |
| 0.5 | 60 | 0.9830 | 0.9831 | 0.9818 | 0.9835 |
| | | (0.1718) | (0.1891) | (0.1878) | (0.1767) |
| | 300 | 0.9925 | 0.9912 | 0.9940 | 0.9926 |
| | | (0.1477) | (0.1480) | (0.1488) | (0.1381) |
| | 600 | 0.9964 | 0.9954 | 0.9966 | 1.0022 |
| | | (0.0462) | (0.0413) | (0.0492) | (0.0320) |

TABLE II
Estimates of the Correlation Parameters and Standard Errors under Non-Stationary BINARMA(1,1) Process

| $T$ | $\rho_1 = 0.3$ | $\rho_2 = 0.4$ | $\rho_3 = 0.3$ | $\rho_4 = 0.4$ | $\rho_{12} = 0.1$ |
|---|---|---|---|---|---|
| 60 | 0.2823 | 0.3801 | 0.2850 | 0.3851 | 0.0888 |
| | (0.1752) | (0.1857) | (0.1710) | (0.1888) | (0.1947) |
| 300 | 0.2902 | 0.3947 | 0.2911 | 0.3917 | 0.0945 |
| | (0.1370) | (0.1330) | (0.1320) | (0.1338) | (0.1459) |
| 600 | 0.3011 | 0.3950 | 0.2971 | 0.3993 | 0.0986 |
| | (0.0426) | (0.0521) | (0.0417) | (0.0440) | (0.0522) |
| $T$ | $\rho_1 = 0.3$ | $\rho_2 = 0.4$ | $\rho_3 = 0.3$ | $\rho_4 = 0.4$ | $\rho_{12} = 0.5$ |
| 60 | 0.2844 | 0.3839 | 0.2818 | 0.3828 | 0.5826 |
| | (0.1733) | (0.18) | (0.1861) | (0.1874) | (0.2061) |
| 300 | 0.2927 | 0.3932 | 0.2926 | 0.3910 | 0.5920 |
| | (0.1394) | (0.1321) | (0.1361) | (0.1270) | (0.1418) |
| 600 | 0.2980 | 0.4019 | 0.2983 | 0.3979 | 0.5965 |
| | (0.0459) | (0.0440) | (0.0548) | (0.0410) | (0.0511) |

From Tables I, II, it can be concluded that the mean estimates of the model parameters are efficient for the different combinations. As the time points increase, we observe a decrease in the standard errors throughout.

## VI. Application

The BINARMA(1,1) model is applied on daytime ($Y_t^{[1]}$) and nighttime ($Y_t^{[2]}$) accidents that occurred on the motorway from International Airport of Mauritius to Reduit from $1^{st}$ January 2017 to $31^{st}$ May 2017, comprising of 151 paired observations. The following explanatory variables were also collected: number of policemen (NP) deployed along the motorway daily for patrol, number of speed cameras (NSC), number of traffic lights (NTL) and number of roundabouts (NRA). Table III presents the summary statistics of the accident data:

TABLE III
SUMMARY STATISTICS FOR THE NUMBER OF DAYTIME AND NIGHTTIME ACCIDENTS WITH THE EMPIRICAL SERIAL- AND CROSS-CORRELATION COEFFICIENTS

| Series | Mean | Variance | Lag-1 | Cross |
|---|---|---|---|---|
| Day Accident | 1.1394 | 1.5489 | 0.1716 | 0.1076 |
| Night Accident | 1.2841 | 1.6122 | 0.2524 | |

The BINARMA(1,1) model is used to analyse the in-sample accident data from $1^{st}$ January 2017 to $15^{th}$ May 2017 by assuming $\lambda_t^{[k]} = \exp(\hat{\beta}_0^{[k]} + \hat{\beta}_1^{[k]} NTL + \hat{\beta}_2^{[k]} NSC + \hat{\beta}_3^{[k]} NP + \hat{\beta}_4^{[k]} NRA)$. The CML estimates are presented in Tables IV, V.

TABLE IV
DAYTIME AND NIGHTTIME ACCIDENTS: ESTIMATES OF THE REGRESSION PARAMETERS

| Series | $\hat{\boldsymbol{\beta}}_0$ | $\hat{\boldsymbol{\beta}}_1$ | $\hat{\boldsymbol{\beta}}_2$ | $\hat{\boldsymbol{\beta}}_3$ | $\hat{\boldsymbol{\beta}}_4$ |
|---|---|---|---|---|---|
| $Y_t^{[1]}$ | 0.1743 | -0.0497 | -0.0981 | -0.1033 | 0.0428 |
| s.e | (0.2089) | (0.0108) | (0.0403) | (0.0451) | (0.0141) |
| $Y_t^{[2]}$ | 0.1614 | -0.0344 | -0.0716 | -0.0926 | 0.1060 |
| s.e | (0.2113) | (0.0129) | (0.0344) | (0.0408) | (0.0393) |

TABLE V
DAYTIME AND NIGHTTIME ACCIDENTS: ESTIMATES OF THE DEPENDENCE PARAMETERS

| Series | $\hat{\rho}_1$ | $\hat{\rho}_2$ | $\hat{\rho}_3$ | $\hat{\rho}_4$ | $\hat{\rho}_{12}$ |
|---|---|---|---|---|---|
| $Y_t^{[1]}$ | 0.1267 | 0.0938 | | | 0.1788 |
| s.e | (0.0468) | (0.0647) | | | (0.0696) |
| $Y_t^{[2]}$ | | | 0.1647 | 0.1052 | |
| s.e | | | (0.0540) | (0.0538) | |

From Tables III, IV, it is observed that all the covariates are significant and there is the existence of dependence between daytime and nighttime accidents. Using the forecasting equations (25)-(26), we compute the one-step ahead forecast for the out-sample observations $16^{th}$ May 2017 to $31^{st}$ May 2017. Hence, the root mean square errors (RMSEs) for daytime and nighttime accidents are 0.1855 and 0.2117.

## VII. CONCLUSION

This paper introduces a non-stationary BINARMA(1,1) model with correlated Poisson innovations. The mean, variance and covariance expressions are derived under the assumption of non-stationarity. The model parameters are estimated using the CML method through a simulation study. These estimates prove to be efficient and reliable. The BINARMA(1,1) model is applied on a bivariate accident data. The estimates and the RMSEs are both reliable.

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