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Aliasing Free and Additive Error in Spectra for Alpha Stable Signals

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***Abstract*—**This work focuses on the symmetric alpha stable process with continuous time frequently used in modeling the signal with indefinitely growing variance, often observed with an unknown additive error. The objective of this paper is to estimate this error from discrete observations of the signal. For that, we propose a method based on the smoothing of the observations via Jackson polynomial kernel and taking into account the width of the interval where the spectral density is non-zero. This technique allows avoiding the "Aliasing phenomenon" encountered when the estimation is made from the discrete observations of a process with continuous time. We have studied the convergence rate of the estimator and have shown that the convergence rate improves in the case where the spectral density is zero at the origin. Thus, we set up an estimator of the additive error that can be subtracted for approaching the original signal without error.

***Keywords*—**Spectral density, stable processes, aliasing, p-adic.

I. Introduction

RECENT years have shown an increasing interest in the study of stationary stable processes and in general stationary processes with infinite variance. The harmonizable process is an important example of a symmetric α-stable stationary process, and its proprieties have been considered by numerous authors like [1]-[10] to name a few.

Stable symmetric processes find applications in various fields including: signal and image processing, hydrology, economies, electronic and electric, communications and radar applications ... see [11]-[23].

In this work, we consider, as in [4], a complex stationary symmetric α-stable (SαS) process: $X=\{X(t),-\infty<t<+\infty\}$, with $\alpha\in(0,2)$; more specifically, X is a complex-valued stochastic process for which the finite dimensional characteristic function is:

$$E\exp\left\{i\Re e\sum_{j=1}^{n} z_j X(t_j)\right\}=\exp\left\{-C_\alpha\int_{-\infty}^{+\infty}\left|\sum_{j=1}^{n} z_j e^{it_j u}\right|^\alpha \phi(u)du\right\}$$

with $z_j=x_j-iy_j$ and $C_\alpha=(\alpha\pi)^{-1}\int_0^\pi|\cos(\theta)|^\alpha d\theta$, where ϕ is a nonnegative integrable function called the spectral density of X. This spectral density plays a role analogous to that played by the usual power spectral density function of a second order stationary process. It is clear that the spectral density ϕ fully describes the distribution of the process X. Alternatively X has the integral representation

$$X(t)=\int_{-\infty}^{\infty}e^{itu}d\xi(u) \tag{1}$$

where ξ is a complex-valued SαS process with independent isotropic increments. The stochastic integral in (1) is defined by means of convergence in probability, for details see [1], [5].

The spectral density function was already estimated by [4], when the process is continuous-time, by [24] when the process is discrete-time and by [25] when the process is p-adic-time.

This paper deals with the fairly common situation in practice, namely that the observations of the process are tainted by an unknown and constant error. The process $X_t=a+Z_t$ is observed instead of the process Z alone. The constant a is estimated in [26] when the process is discrete-time.

Our goal is to establish nonparametric estimate of a, from sample of the process $X(t_n)$ at instants t_n, where the sampling instants t_n are equally distant, i.e., $t_n=n\tau$, $\tau>0$. It is known that aliasing occurs. For more details about aliasing phenomenon, see [27], [28]. To avoid this difficulty, we suppose that the spectral density ϕ is vanishing for $|\lambda|>\Omega$ where Ω is a nonnegative real number. The value of τ is chosen such that $\tau\Omega<\pi$.

In this paper we particularly study some cases where the spectral density is zero at the origin: $\phi(\lambda)=\sin^{2k\alpha}(\lambda/2)g(\lambda)$ and $\phi(\lambda)=|\lambda|^\beta g(\lambda)$. We show that the rate of convergence of the estimator of the constant "a" is improved.

This paper is organized as follows: The second section gives some definitions and proprieties of Jackson polynomial kernel and an estimator of the constant a is defined. We show that this estimate converges in probability to a. Then we show that the estimate converges to a in L_p $(p<\alpha)$. Since the second moment of this process does not exist, the convergence in Lp substitutes the quadratic. In third section, the spectral density of Z is assumed vanishing at origin precisely: $\phi(\lambda)=|\lambda|^\beta g(\lambda)$. We improve the rate of convergence depending to the value of β. The fourth section is reserved to numerical studies.

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II. Estimation of the Constant Error

As in [20], [24], we give the definition of the Jackson polynomial kernel: Let $Z_1,\ldots,Z_N$ observations of the process $Z:\ \left(Z_{(n)}\right)_{0\le n\le N-1}$, where N satisfies:

$$N-1=2k(n-1) \qquad with \quad n\in N \quad k\in N\cup\{1/2\}$$

if $k=1/2$ then $n=2n_1-1, n_1\in N$.

The Jackson's polynomial kernel is defined by:

$$\left|H_N(\lambda)\right|^{\alpha}=\left|A_N H^{(N)}(\lambda)\right|^{\alpha}$$

where

$$H^{(N)}(\lambda)=\frac{1}{q_{k,n}}\left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})}\right)^{2k} \qquad with$$

$$q_{k,n}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})}\right)^{2k} d\lambda.$$

In addition, we have $A_N=(B_{\alpha,N})^{-\frac{1}{\alpha}}$ with

$$B_{\alpha,N}=\int_{-\pi}^{\pi}\left|H^{(N)}(\lambda)\right|^{\alpha} d\lambda.$$

We give the following lemmas which are used in the reminder of this paper. Their proofs are given in [24].

Lemma1. There is a non negative function h_k such as:

$$H^{(N)}(\lambda)=\sum_{m=-k(n-1)}^{k(n-1)} h_k\left(\frac{m}{n}\right)\cos(m\lambda)$$

Lemma2. Let

$$B'_{\alpha,N}=\int_{-\pi}^{\pi}\left|\frac{\sin\frac{n\lambda}{2}}{\sin\frac{\lambda}{2}}\right|^{2k\alpha} d\lambda \quad and$$

$$J_{N,\alpha}=\int_{-\pi}^{\pi}|u|^{\gamma}|H_N(\lambda)|^{\alpha}\, d\lambda,$$

where $\gamma\in]0,2]$. Then

$$B'_{\alpha,N}\begin{cases}\ge 2\pi\left(\frac{2}{\pi}\right)^{2k\alpha} n^{2k\alpha-1} if\ 0<\alpha<2\\ \le\frac{4\pi k\alpha}{2k\alpha-1}n^{2k\alpha-1} if\ \frac{1}{2k}<\alpha<2\end{cases} \quad and$$

$$J_{N,\alpha}\le\begin{cases}\frac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma-2k\alpha+1)}\frac{1}{n^{2k\alpha-1}} & if \quad \frac{1}{2k}<\alpha<\frac{\gamma+1}{2k}\\ \frac{2k\alpha\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma+1)(2k\alpha-\gamma-1)}\frac{1}{n^{\gamma}} & if \quad \frac{\gamma+1}{2k}<\alpha<2\end{cases}$$

The estimator of the a is constructed as follows:

$$\hat{a}=\frac{\tau^{\frac{1}{\alpha}}A_N}{H_N(0)}\sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right)X(\tau n'+\tau k(n-1)). \qquad (2)$$

Theorem1. Let p a real number such that $0<p<\alpha$. Then

$$|\hat{a}-a|^{p}=O\left(\frac{1}{n^{p\alpha}}\right)$$

Proof. From (1) and (2), we have

$$\hat{a}=\frac{\tau^{\frac{1}{\alpha}}A_N}{H_N(0)}\sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right)\int_{-\infty}^{\infty}exp\left[i([\tau n'+\tau k(n-1)]\lambda)\right]d\xi(\lambda)$$
$$+a.$$

As in [1], the characteristic function of $(\hat{a}-a)$ can be written as:

$$E\exp\left[i\Re e\bar{r}(\hat{a}-a)\right]=$$
$$\exp[-C_{\alpha}|r|^{\alpha}\int_{-\infty}^{\infty}\tau\,|\frac{A_N}{H_N(0)}\sum_{n'=-k(n-1)}^{k((n-1)} h_k\left(\frac{n'}{n}\right)e^{in'\tau\lambda}|^{\alpha}\, d\xi(\lambda)].$$

where $r=r_1+ir_2$. It is easy to show that:

$$E\exp\left[i\Re e\bar{r}(\hat{a}-a)\right]=\exp\left(-C_{\alpha}|r|^{\alpha}\psi_N\right),$$

where

$$\psi_N=\int_{-\infty}^{\infty}\frac{\left|H_N(\lambda)\right|^{\alpha}}{\left|H_N(0)\right|^{\alpha}}\phi\left(\frac{\lambda}{\tau}\right)d\lambda=\sum_{j\in Z}\int_{(2j-1)\pi}^{(2j+1)\pi}\frac{\left|H_N(\lambda)\right|^{\alpha}}{\left|H_N(0)\right|^{\alpha}}\phi\left(\frac{\lambda}{\tau}\right)d\lambda.$$

Putting $\lambda=y-2\pi j$ and using the fact that H_N is 2π-periodic, we obtain

$$\psi_N=\sum_{j\in Z}\int_{-\pi}^{\pi}\frac{\left|H_N(\lambda)\right|^{\alpha\, a}}{\left|H_N(0)\right|^{\alpha}}\phi_j(y)dy,$$

where $\phi_j(y)=\phi\left(\frac{y}{\tau}-\frac{2\pi}{\tau}j\right)$. Let j be an integer such that

$-\Omega < \frac{y-2\pi j}{\tau} < \Omega$. Using the fact that $\tau\Omega < \pi$ and $|y| < \pi$, we get $|j| < \frac{\tau\Omega}{2\pi} + \frac{1}{2} < 1$ and then $j = 0$. Consequently:

$$\psi_N = \int_{-\pi}^{\pi} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi\left(\frac{y}{\tau}\right) dy.$$

The function ϕ being bounded on $[-\pi,\pi]$ and $|H_N(.)|^\alpha$ being a kernel, it can be shown that $\int_{-\pi}^{\pi} |H_N(\lambda)|^\alpha \phi(\lambda) d\lambda$ is converging to $\phi(0)$. On the other hand, from lemma 2, we have:

$$\frac{1}{|H_N(0)|^\alpha} = \frac{B'_{\alpha,N}}{n^{2k\alpha}} = O(1/n). \quad (3)$$

Therefore ψ_N converges to 0. Consequently, the characteristic function of $\hat{a} - a$ converges to 1 when N approaches infinity. Hence we have the convergence in probability of $\hat{a}$ to a.

We study now the convergence of $\hat{a}$ to a in L_p where $0 < p < \alpha$, which replaces the convergence in mean square, because the second order moment of X is infinity. Let

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - e^{ir\cos\theta}}{|r|^{1+p}} dr d\theta.$$

Let $t = r'e^{i\theta'}$ and $x = \rho e^{i\tau_0}$. Assuming now $r = \varepsilon r'$, $\tau = \tau' - \tau_0$

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\varepsilon r'\cos(\tau'-\tau_0)}}{|\varepsilon|^{1+p} |r'|^{1+p}} \varepsilon dr' d\theta'$$

$$D_p |x|^p = \Re e \int_{0}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t| d\theta'$$
$$- \Re e \int_{-\infty}^{0} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t| d\theta'.$$

Replacing in this formula x by $\hat{a} - a$, from the previous result we have

$$D_p E|x|^p = \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} d|t| d\theta'$$
$$= \frac{\pi}{2} \int_{-\infty}^{\infty} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} dt.$$

Let $u = t[\psi_N]^{\frac{1}{\alpha}}$ and using (3), we obtain

$$\frac{2}{\pi} C_{p,\alpha} E|\hat{a} - a|^p = (\psi_N)^{\frac{p}{\alpha}} = O\left(\frac{1}{n^{p/\alpha}}\right)$$

where $C_{p,\alpha} = R_p F_{p,\alpha}^{-1} (C_\alpha)^{-\frac{p}{\alpha}}$ with $R_p = \int \frac{1-\cos(u)}{|u|^{1+p}} du$ and $F_{p,\alpha} = \int \frac{1 - e^{-|u|^\alpha}}{|u|^{1+p\alpha}} du.$

III. The Rate of Convergence for Particular Cases

Suppose that the spectral density is zero at the origin. This assumption gives our estimator a better convergence rate.

Theorem2. Assume that the spectral density is satisfying: $\phi(\lambda) = |\lambda|^\beta g(\lambda)$ where $\beta \in]0, 2k\alpha - 1[$, $\lambda \in [-\pi,\pi]$ and $g(\lambda)$ is a bounded function on $[-\pi,\pi]$ and vanishing for $|\lambda| > \Omega$, continuous in neighborhood of 0 and $g(0) \neq 0$. Then

$$2^{4kp} L \tau^{2k\alpha-\beta} \leq \lim_{N\to\infty} n^{\frac{p(\beta+1)}{\alpha}} E|\hat{a} - a|^p \leq \tau^\beta \pi^{4kp} L,$$

where L is the following constant:

$$L = \frac{\pi}{2C_{p,\alpha}} \left[g(0) \int_{-\infty}^{\infty} \frac{|\sin u2|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right]^{\frac{p}{\alpha}}.$$

Proof. The function ψ_N can be written as:

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} \left|\frac{\lambda}{\tau}\right|^\beta g\left(\frac{\lambda}{\tau}\right) d\lambda$$

Using the inequality: $\left|\sin \frac{x}{2}\right| \geq \frac{x}{\pi}$ $\quad 0 \leq x \leq \pi,$ we maximize ψ_N as:

$$\psi_N \leq \pi^{4k\alpha} n^{-2k\alpha} \tau^{-\beta} \int_{-\pi}^{\pi} \frac{\left|\sin \frac{n\lambda}{2}\right|^{2k\alpha}}{|\lambda|^{2k\alpha-\beta}} g\left(\frac{\lambda}{\tau}\right) d\lambda.$$

Putting $n\lambda = u$, we have

$$\psi_N \leq \pi^{4k\alpha} \tau^{-\beta} n^{-1-\beta} \int_{-\infty}^{\infty} \frac{\left|\sin \frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g\left(\frac{u}{n\tau}\right) du$$

The Lebesgue's dominated convergence theorem gives the following convergence:

$$\lim_{N\to\infty}\int_{-\infty}^{\infty}\frac{\left|\sin\frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}}g\left(\frac{u}{n\tau}\right)du=g(0)\int_{-\infty}^{\infty}\frac{\left|\sin\frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}}du.$$

Lemma 2 gives:

$$\lim_{N\to\infty}n^{\frac{p(\beta+1)}{\alpha}}\left(\psi_N\right)^{\frac{p}{\alpha}}\leq\pi^{4kp}\left(g(0)\int_{-\infty}^{+\infty}\frac{\left|\sin\frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}}du\right)^{\frac{p}{\alpha}}. \quad (4)$$

Thus ψ_N converges to zero. Using the following inequality $|\sin x|\leq|x| \quad \forall x\in[-\pi,\pi]$, we obtain:

$$\psi_N\geq 2^{4k\alpha}n^{-2k\alpha}\tau^{2k\alpha-\beta}\int_{-\pi}^{\pi}\frac{\left|\sin\frac{n\lambda}{2}\right|^{2k\alpha}}{|\lambda|^{2k\alpha-\beta}}g\left(\frac{\lambda}{\tau}\right)d\lambda.$$

Therefore

$$\psi_N\geq 2^{4k\alpha}n^{-\beta-1}\tau^{2k\alpha-\beta}\int_{-\pi}^{\pi}\frac{\left|\sin\frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}}g\left(\frac{u}{n\tau}\right)du$$

$$\lim_{N\to\infty}n^{\frac{p(\beta+1)}{\alpha}}\left(\psi_N\right)^{\frac{p}{\alpha}}\geq 2^{4kp}\tau^{2k\alpha-\beta}\left(g(0)\int_{-\infty}^{+\infty}\frac{\left|\sin\frac{u}{2}\right|^{2k\alpha}}{|u|^{2k\alpha-\beta}}du\right)^{\frac{p}{\alpha}}.$$

The first equality of (4) reaches the result of this theorem.

Theorem3. Assuming that the spectral density satisfies:

$$\phi(\lambda)=\sin^{2k\alpha}\left(\frac{\lambda}{2}\right)g(\lambda)$$

where the function g is integrable on $[-\pi,\pi]$, $g(0)\neq 0$ and vanishing for $|\lambda[>\Omega$. Then

$$cte\pi 2C_{p,\alpha}\tau^{p\alpha}\left(\int_{-\pi}^{\pi}g(\lambda)d\lambda\right)^{\frac{p}{\alpha}}\leq\lim_{N\to\infty}n^{2pk}E\,|\hat{a}-a|^{p}$$
$$\leq\frac{\pi}{2C_{p,\alpha}}\tau^{\frac{p}{\alpha}}\left(\int_{-\pi}^{\pi}g(\lambda)d\lambda\right)^{\frac{p}{\alpha}}$$

Proof. From the definition of ψ_N, we have

$$\psi_N=n^{-2k\alpha}\int_{-\pi}^{\pi}\left|\sin\frac{n\lambda}{2\tau}\right|^{2k\alpha}g\left(\frac{\lambda}{\tau}\right)d\lambda.$$

As $1\leq k\alpha$ and the sinus function is smaller than 1, the next expression is connected to

$$\psi_N\leq n^{-2k\alpha}\int_{-\pi}^{\pi}\left|\sin\frac{n\lambda}{2\tau}\right|^{2k\alpha}g\left(\frac{\lambda}{\tau}\right)d\lambda$$

$$\psi_N\leq n^{-2k\alpha}\left[\int_{-\pi}^{\pi}g\left(\frac{\lambda}{\tau}\right)d\lambda\right].$$

So, from lemma 2, we have:

$$\lim_{N\to\infty}\psi_N n^{2k\alpha}\leq\tau\int_{-\pi}^{\pi}g(\lambda)d\lambda.$$

Using the fact that the sinus function is between -1 and 1 and $k\alpha<[k\alpha]+1$ where $[k\alpha]$ represents the integer part of $k\alpha$, we obtain

$$\psi_N\geq n^{-2k\alpha}\int_{-\pi}^{\pi}\left[\left(\sin\frac{n\lambda}{2\tau}\right)^{2}\right]^{[k\alpha]+1}g\left((\frac{\lambda}{\tau}\right)d\lambda$$

$$\psi_N\geq n^{-2k\alpha}\frac{n^{-1}}{2B'_{\alpha,N}}\int_{-\pi}^{\pi}\left[\frac{1-\cos\left(\frac{n\lambda}{\tau}\right)}{2}\right]^{[k\alpha]+1}g\left(\frac{\lambda}{\tau}\right)d\lambda.$$

The binomial formula gives:

$$2^{[k\alpha]+1}\psi_N\geq n^{-2k\alpha}\tau\int_{-\pi}^{\pi}g(\lambda)d\lambda$$

IV. Simulation

The proposed estimator can be applied to concrete situations. For example, the arrival time of a signal sent by Wi-Fi using new wireless transmission technologies can be modeled by alpha stable process. Indeed, [29] proposed an arrival time model based on Poisson distributions. Reference [15] provided a model based on stable alpha distributions. The sum of arrival times modeled by independent and isotropic Poisson distributions can be represented by a stable harmonizable process like that given in (1), see [30]. When these signals are collected in an aquatic environment with a constant disturbance, the process will be added with a constant error ($X_n=Z_n+a$).

Throughout this section, we give the simulation of the studied process:

$$Z_t = \int_{-\infty}^{\infty} e^{it\lambda} d\xi(\lambda),$$

where $1 < \alpha < 2$ and ξ is a complex symmetric SαS measure on R with independent and isotropic increments and with control measure m such that $mdx = \phi(x)dx$. In order to achieve this, we use the series representation defined in [26]. Therefore, the process Z given in (8) can be expressed as:

$$Z_t = C_\alpha \left(\int \phi(x)dx\right)^{1\alpha} \sum_{j=1}^{\infty} \varepsilon_j \Gamma_j^{-1\alpha} e^{inV_j} e^{i\theta_j}$$

where ε_j is a sequence of i.i.d. random variables such as $P[\varepsilon_j = 0] = P[\varepsilon_j = 1] = 12$, Γ_k is a sequence of arrival times of Poisson process, V_j is a sequence of i.i.d. random variables independent of ε_k and of Γ_k having the same distribution of control measure m, which has probability density ϕ, θ_j are independent random variables, having the uniform distribution on $[-\pi, \pi]$, independent of ε_j, Γ_j and V_j.

To generate N values $(N = 101, 501, 1001)$ of the process Z_{t_n}, we use the following steps:

- generate 2000 values of ε_j
- generate 2000 values of Γ_j
- generate 2000 values of V_j
- generate 2000 values of θ_j

Then we calculate for all $0 \le n \le N$: $Z_{t_n} = C_\alpha \left(\int \phi(x)dx\right)^{\frac{1}{\alpha}} \sum_{j=1}^{2000} \varepsilon_j \Gamma_j^{-1\alpha} e^{it_n V_j} e^{i\theta_j}$ where the spectral density is chosen as $\phi(x) = |x|^\beta e^{-|x|}$ for $x \in [-\pi, \pi]$ and $\phi(x) = 0$ otherwise and $\alpha = 1,7$ and $k = 4$. The β is taken as 2, between 0 and $2k\alpha - 1$ g Afterwards, we generate $X_{t_n} = a + Z_{t_n}$ where a is chosen equal to 40.

We calculate the estimator $\hat{a}$ given in (1) for different sizes of sample $N = 101, 501, 1001$. The result is given in Table I.

TABLE I
THE VALUE OF THE ERROR ESTIMATOR FOR DIFFERENT SIZES OF SAMPLES

$2k\alpha - 1 = 12.6$	$\beta = 2$
N=101	$\hat{a} = 31,5$
N=501	$\hat{a} = 36,3$
N=1001	$\hat{a} = 41,05$

Comparing $\hat{a}$ to 40 (value of the constant a), we find that the estimator $\hat{a}$ increasingly approaches the constant error a when the sample size is large.

V. CONCLUSIONS

We give an estimator of the constant additive error in spectral representation of SαS process. This work could be applied to several cases when processes have an infinite variance and the observation of these processes is perturbed by a constant noise. For example:

- the decomposition of audio signals with background noise by separating the different musical instruments.
- the denoising of a degraded historical record. The signal is considered infinitely variable.

The perspective of this work is to optimize the smoothing parameters to have a better speed of convergence. For this purpose, the cross-validation method will be the most appropriate tool.

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