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Peridynamic Modeling of an Isotropic Plate under Tensile and Flexural Loading

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Abstract**—**Peridynamics is a new modeling concept of non-local interactions for solid structures. The formulations of Peridynamic (PD) theory are based on integral equations rather than differential equations. Through, undefined equations of associated problems are avoided. PD theory might be defined as continuum version of molecular dynamics. The medium is usually modeled with mass particles bonded together. Particles interact with each other directly across finite distances through central forces named as bonds. The main assumption of this theory is that the body is composed of material points which interact with other material points within a finite distance. Although, PD theory developed for discontinuities, it gives good results for structures which have no discontinuities. In this paper, displacement control of the isotropic plate under the effect of tensile and bending loading has been investigated by means of PD theory. A MATLAB code is generated to create PD bonds and corresponding surface correction factors. Using generated MATLAB code the geometry of the specimen is generated, and the code is implemented in Finite Element Software. The results obtained from non-local continuum theory are compared with the Finite Element Analysis results and analytical solution. The results show good agreement.

Keywords**—**Flexural loading, non-local continuum mechanics, Peridynamic theory, solid structures, tensile loading.

I. Introduction

In recent years, designing lightweight structural parts especially in the areas of aerospace, defense, and automotive industries has become important and high strength steels and aluminum alloys are strong candidates for lightweight structures. Therefore, numerical models enabling accurate results for deformations, damage initiation and propagation inside solid medium provide flexibility in design process.

Classical Continuum Mechanics (CCM) formulations are derived from Partial Differential Equations (PDEs). However, PDEs become undefined when the equation of motion derived based on CCM are applied on a region including discontinuities. Instead of using PDEs, a non-local particle-based approach named as PD approach has been introduced by Silling [1]. The main difference between CCM and PD theory is their formulations. PD theory solves discontinuities by replacing PDEs with integrals of interaction forces between grid points named as material points. The first formulation of PD is known as bond-based PD. This assumption leads to fixed Poisson ratio of 1/3 in 2D and 1/4 in 3D [2]. This limitation is removed with the state based formulation PD theory presented in [3] and [4]. In the original formulation, PD has been applied to dynamic problems dealing with impact loadings. However, because of its flexibility in describing crack propagation, it might be applied to static problems as well [5].

PD has been applied different types of static and quasi static problems. Huang et al. [6] developed a formulation to investigate the quasi-static response and crack propagation of cantilevered beam. Zaccariotto et al. [5] applied PD for the solution of static equilibrium problems using implicit Newton Rapson method. Additionally, one another study by means of non-ordinary state based approach in case of quasi-static loading condition has been proposed for linear elastic materials in [7].

PD theory assumes that particles in a continuum interact with each other across a finite distance. The interaction might be constructed with using truss or spring elements. Macek and Silling developed a PD approach using truss elements and PD is implemented in finite element code [8]. Kılıç et al. [9] presented PD model using explicit modeling. Explicit solution provides more detailed solution, but it has higher computational cost. In order to solve this issue Dipasquale and Zaccariotto [10] solved this problem using adaptive meshing approach.

In this study, two-dimensional PD is implemented in FEA code and deformation of an isotropic plate under plane stress condition under the effect of tensile and flexural loading has been performed via analytical and finite element methods. Then, obtained results from PD have been compared with analytical and FEA results.

II. PD Theory

Original formulation of PD theory is first introduced by Silling. PD theory is a nonlocal continuum theory with integro-differential equations. It may be considered as continuum version of molecular dynamics [1]. In this study bond-based PD theory, which considers the behavior of a bond completely independent of all the others, is implemented in Finite Element Analysis (FEA) software using MATLAB pre-processing code. While implementing MATLAB code T3D2 truss elements were used. Numerical implementation of PD theory is given in Appendix. Equation of motion in PD theory is given as in (1):

$$\rho(x)\ddot{u}(x,t) = \int_{H_x} f(u^{'} - u, x^{'} - x)dH + b(x,t) \tag{1}$$

where ρ is mass density, u is the displacement of material

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point at x, b is the body force, and $f(u^{'}-u, x^{'}-x)$ is the force density vector function between the material points x' and x. The direct physical interaction between the particles at x and $\mathrm{x}^{'}$ will be called as bond.

In PD theory, any material point x interacts with other material points within a distance δ, named as horizon and H_x denotes the material points within a distance of δ as illustrated in Fig. 1 [11]. Boundary conditions and loads should be applied to internal nodes within a horizon thickness of the external surface [1]. Thereby, as expressed in PD equations, force per unit volume is conserved.

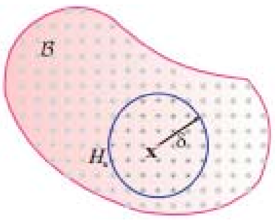

Fig. 1 Relationship of the bonds in PD approach [11]

PD force density is defined as [2],

$$f(\eta,\xi)=\frac{\xi+\eta}{|\xi+\eta|}cs \tag{2}$$

where η is the relative displacement, ξ is the relative position between the material points, c is the bond constant and s is the stretch. For an isotropic material the bond constant is defined as [1].

$$c=\frac{18K}{\pi\delta^4} \tag{3}$$

where K is the Bulk Modulus of the material, δ is the horizon. According to (3), in bond-based PD formulation only one PD parameter is used as opposed to two independent Lame parameters of CCM for isotropic material behavior. For the isotropic materials Elastic modulus of the truss elements (E_t) and cross sectional area of the truss elements (A_t) might be calculated as [8].

$$E_t=c\Delta\mathrm{x}^4 \tag{4}$$

$$A_t=\Delta\mathrm{x}^2 \tag{5}$$

where $\Delta\mathrm{x}$ indicates the grid size.

III. Analytical and Numerical Solution of a Plate under Tensile Loading

Tensile loading of the plate is analyzed using PD, FEA and numerical methods.

The length (L), and the width (w) of the specimen are 150 and 20 mm, respectively and thickness (t) is 1.55 mm. The representation of the applied problem is shown in Fig. 2.

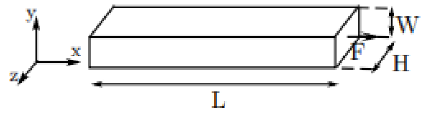

Fig. 2 Representation of tensile loading

In Fig. 2 F represents the applied load to the plate. Steel is used for this work and material properties of the specimen are shown in Table I.

TABLE I
MECHANICAL PROPERTIES OF THE SPECIMEN

Elastic Modulus (E)	Poisson's ratio (ν)	Density (ρ)
200 GPa	0.33	7850 kg/m^3

FEA of the tensile loading is generated using C3D8R elements having a mesh size of 0.32 mm.

PD model is generated using T3D2 truss elements and stiffness of the truss elements is calculated using (4). In FEA and PD theory one end of the plate is fixed, and 550 N force is applied in $+x$ direction.

In the analytical solution of the problem described above, since applied force does not affect the object along its axis, plane stress assumption is made. Thereby, strain values in all directions might be obtained by using generalized Hook's law formulas. Considering the forces acting on the plate, stress values are obtained as $\sigma_x=F$, $\sigma_y=0$ and $\sigma_z=0$. With the help of these equations strain values might be obtained as given in (6)-(8):

$$\varepsilon_x=\frac{1}{E}(\sigma_x-\upsilon\sigma_y)=\frac{F}{E} \tag{6}$$

$$\varepsilon_y=\frac{1}{E}(\sigma_y-\upsilon\sigma_x)=-\frac{\upsilon F}{E} \tag{7}$$

$$\varepsilon_z=-\frac{\upsilon}{E}(\sigma_x+\sigma_y)=-\frac{\upsilon F}{E} \tag{8}$$

By using the generalized Hook's law equations, strain displacement equations may be calculated by using partial derivatives and the displacement values in all directions using (9)-(11):

$$\varepsilon_x=\frac{\partial u}{x}\rightarrow u_x=\varepsilon_x x=\frac{F}{E}x \tag{9}$$

$$\varepsilon_y=\frac{\partial u}{y}\rightarrow u_y=\varepsilon_y y=-\frac{\nu F}{E}y \tag{10}$$

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$$\varepsilon_z = \frac{\partial u}{z} \rightarrow u_z = \varepsilon_z z = -\frac{\nu F}{E} z \tag{11}$$

In order to compare the obtained displacement results from different methods, two different material paths have been defined. For the displacement values in x direction (U_x) a path is defined for $y = 0.775$ mm and $z = 10$ mm. In Fig. 3, displacement values of a plate under tensile load obtained by PD theory on the determined material point have been compared with analytical solution and FEA results.

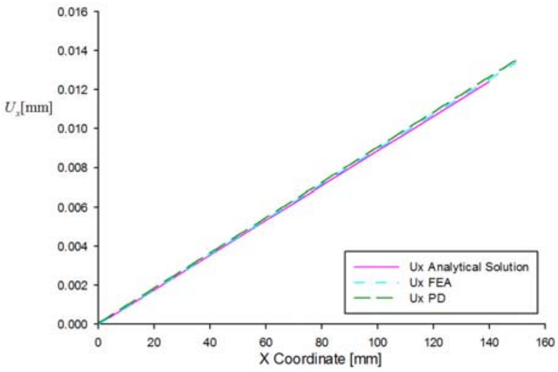

Fig. 3 Comparison of displacement results in x direction by different methods

For the displacement values in y direction (U_y) a path is defined for $x = 150$ mm and $z = 10$ mm. Comparison of displacement results obtained from path is shown in in Fig. 4.

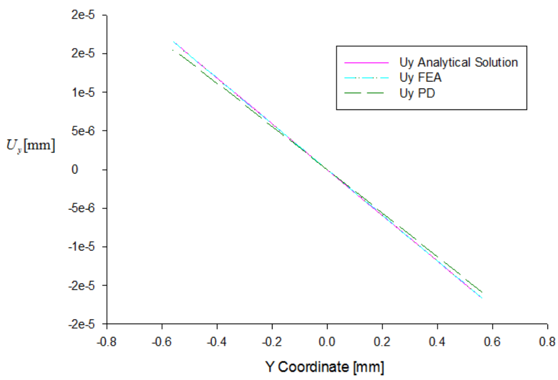

Fig. 4 Comparison of displacement results in y direction by different methods

IV. Analytical and Numerical Solution of a Plate under Three Point Bending Loading

Three point loading of a plate is solved using PD, FEA and numerical methods and the obtained results compared with each other. Representation of applied problem is shown in Fig. 5.

The same specimen geometry and material properties applied for flexural loading case and PD model are generated using the same strategy as in the previous case.

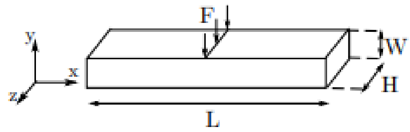

Fig. 5 Representation of the problem

In FEA and PD theory, the translation movement of one end of the specimen and the rotation and translation movement of the other end are restricted in all directions as the boundary condition. Load was applied from middle section of the specimen.

While applying PD theory, the solution is applied with four different Δx values. These values are 0.22 mm, 0.119 mm, 0.103 mm, and 0.062 mm. Generated PD mesh is displayed in Fig. 6.

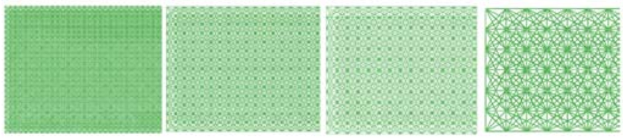

Fig. 6 PD mesh obtained with different Δx values

The investigation with varied grid size has been examined but constant m ratio is considered between the distance δ and the grid size Δx as shown in Fig. 5. For this analysis m ratio is applied as 3.015.

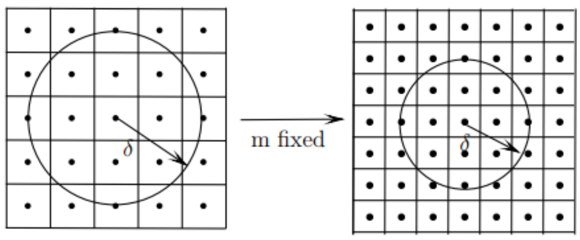

Fig. 7 Convergence of horizon for different grid sizes

Analytical solution of three point bending problem is calculated using the formulation in [12].

$$u = -\frac{Fx}{48EI}(3L^2 - 4x^2) \qquad (0 \le x \le L/2) \tag{12}$$

where F, E and I denote applied force, elastic modulus and inertia respectively.

Comparison of PD, FEA and analytical solutions in terms of displacement fields in y direction along the horizontal mid line is given in Fig. 8.

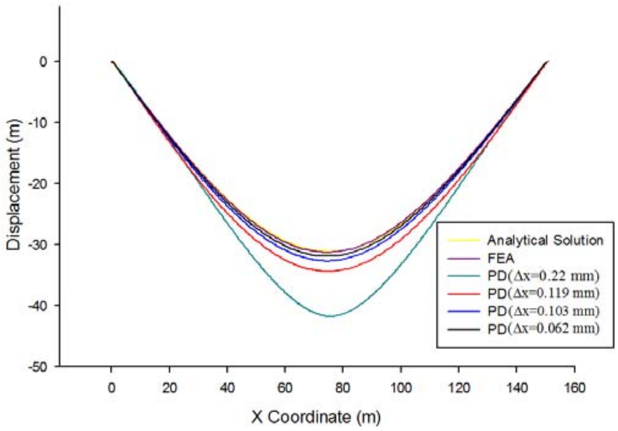

Fig. 8 Comparison of results with different mesh sizes for different PD, FEA and analytical solution

V. CONCLUSION

Even though PD theory fundamentally has been developed for modeling of discontinuities its formulations might be implemented to simulate deformation fields without any discontinuity field.

In this study, displacement field of isotropic medium is modeled using PD theory. Obtained results have consistency compared to outputs of FEA and numerical results. Although, the mesh size does not affect the results of tensile loading, PD theory gives better results with smaller mesh sizes under flexural loading.

APPENDIX

A general solution procedure for PD theory is described by Madenci and Oterkus [11]. This theory is solved through meshfree approach. For this work the applied solution procedure is shown in Fig. 9.

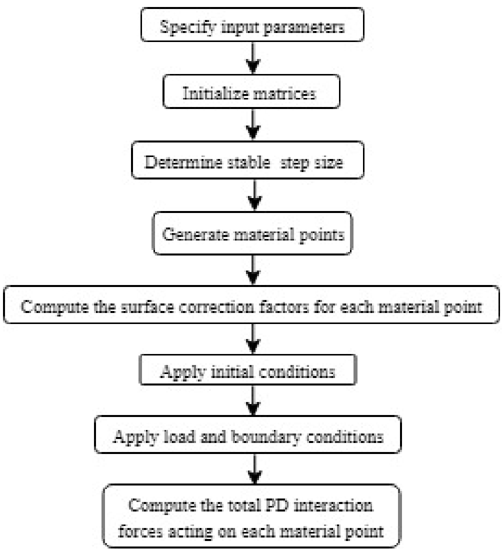

Fig. 9 PD implementation flowchart

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