International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:13, No:9, 2019

Multistage Data Envelopment Analysis Model for Malmquist Productivity Index Using Grey's System Theory to Evaluate Performance of Electric Power Supply Chain in Iran

Mesbaholdin Salami, Farzad Movahedi Sobhani, Mohammad Sadegh Ghazizadeh

Abstract**—**Evaluation of organizational performance is among the most important measures that help organizations and entities continuously improve their efficiency. Organizations can use the existing data and results from the comparison of units under investigation to obtain an estimation of their performance. The Malmquist Productivity Index (MPI) is an important index in the evaluation of overall productivity, which considers technological developments and technical efficiency at the same time. This article proposed a model based on the multistage MPI, considering limited data (Grey's theory). This model can evaluate the performance of units using limited and uncertain data in a multistage process. It was applied by the electricity market manager to Iran's electric power supply chain (EPSC), which contains uncertain data, to evaluate the performance of its actors. Results from solving the model showed an improvement in the accuracy of future performance of the units under investigation, using the Grey's system theory. This model can be used in all case studies, in which MPI is used and there are limited or uncertain data.

Keywords**—**Malmquist Index, Grey's Theory, Charnes Cooper & Rhodes (CCR) Model, network data envelopment analysis, Iran electricity power chain.

I. Introduction and Literature Review

THE evaluation of the organizational performance has a significant role in orientation of their future decisions. In this regard, organizational efficiency and productivity should be evaluated to be able to monitor economic growth in future decision making [1]. Productivity improvement is achieved through optimal use of the production elements, and plays a significant role in achieving a continuous economic growth and sustainable production. Today, competition in the arena of global production and trade has been changed with the diminution of economic boundaries and attempt for productivity enhancement, based on economic wisdom, has been emphasized. As a result, productivity requires the operationalization of potential abilities [1], [2]. Therefore, this movement needs a stimulator, and "competition" is the best stimulator in domestic and foreign markets. Productivity enhancement results in progress and development. The majority of the advanced and developing countries have made enormous investments to extend the perception of productivity and generalize the use of productivity improvement techniques. Investigation into the performance of countries with considerable economic growth in recent decades suggests that such achievements were mainly due to productivity enhancement [3].

Productivity is a combination of efficiency and effectiveness. In other words, an organization is productive only if it performs efficiently and effectively at the same time. Organizational efficiency can be evaluated based on appropriate use of inputs to produce outputs. We can assess effectiveness by evaluating the achievements in the outputs. The concept of productivity can be framed by combining effectiveness and efficiency, showing the extent to which organizational goals are achieved using the inputs.

Data Envelopment Analysis (DEA) is a suitable and efficient tool for productivity assessment [4], [5]. It is a nonparametric method to calculate efficiency of decision-making units. Today, the use of DEA in evaluation of different organizations and industries, such as banking, post, hospitals, educational centers, power plants, and refineries, is rapidly growing. DEA models have undergone many theoretical and applied developments; therefore, the identification of different DEA dimensions is essential for more precise application of it. In addition to the determination of relative efficiency, the use of DEA reveals organizational weaknesses in different indices. It then devises the organization's strategy towards efficiency and productivity enhancement by providing optimal values of those indices [5]. Moreover, the efficient models based on which inefficient units are evaluated are introduced to those units. Efficient models are units that produce a greater number of outputs using the same number of inputs used by inefficient units or produce the same number of outputs using fewer inputs. This extensive diversity of results has accelerated the growth of this technique [6]. As a result, the theoretical dimension of this technique also grew significantly and turned into an active branch of operations research.

This study considered the MPI, used to evaluate the overall productivity, within a multistage network model and analyzed it with the Grey's system theory, assuming the presence of uncertain data. There are many articles on the MPI and

Mesbaholdin Salami is with the Department of Industrial Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran
Farzad Movahedi Sobhani is with the Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran (Corresponding author; e-mail: Fmovahedi@iau.ac.ir).
Mohammad Sadegh Ghazizadeh is with the Department of Electrical Engineering, Abbaspour School of Engineering, Shahid Beheshti University, Tehran, Iran.

application of Grey's theory. Sher et al. [7] proposed a model on the Grey Control System. Lin et al. [8] proposed a MPI based on common-weights DEA and studied the application of the Grey's model for prioritization of technical measures for quality improvement. Mao et al. [9] investigated the use of the Grey's model GM (1, 1) to estimate the vehicle breakdown risk. Chen et al. [10] developed a model to use DEA, MPI, and Grey model to determine productivity performance in the wood industry. Deng [24] presented an introduction on the Grey's system theory. Trivedi et al. [11] addressed the use of Grey's system theory in the Development of a Runoff Prediction Model. Liu et al. [12] investigated the application of a relational two-stage DEA model in health care system. Mussard et al. [13] developed the multistage MPI. Fernandes et al. [14] proposed a multi stage model with Application in banking efficiency and financial development. Mavi et al. [15] studied the Joint analysis of eco-efficiency and eco-innovation with common weights in two-stage network DEA. Amani et al. [16] used MPI with carry-overs in power industry. Sakthidharan et al. [17] studied about impact of operating cost components on airline efficiency in India. None of these articles indicated the combination of multistage MPI and Grey's System Theory. As a result, a model capable of combining the network DEA with the Grey's model is needed. In other words, the innovation of this paper is to provide a hybrid network model with the Grey theory and the Malmquist index that is widely used in supply chain models. The case studied in this article was Iran's EPSC. The electricity market manager is responsible for electricity distribution based on a huge amount of uncertain data on adopting a combination of actors in this field at different EPSC levels. After selection a series of combinations, the electricity market manager seeks to evaluate each combination. This evaluation can be effective in future decisions concerning the selection of appropriate combinations.

A. Malmquist Productivity Index (MPI)

In economic analyses, MPI is among the indices always considered in investigating the overall productivity growth. This index addresses technological changes and technical efficiency at the same time. In this index, the calculated overall efficiency is due to technological changes or efficient frontier displacement and distance from efficient frontier, i.e. technical efficiency [18]. Equations (1)-(3) were used to calculate technical efficiency, technological changes, and overall efficiency, respectively.

$$\text{Technical efficiency} = \Theta^{t+1}_{t+1} / \Theta^{t}_{t} \tag{1}$$

$$\text{Technological changes} = ((\Theta^{t}_{t} \times \Theta^{t+1}_{t}) / (\Theta^{t}_{t+1} \times \Theta^{t+1}_{t+1}))^{1/2} \tag{2}$$

$$\text{MPI} = \text{technological changes} * \text{technical efficiency} = (\Theta^{t+1}_{t+1} / \Theta^{t}_{t}) \times ((\Theta^{t}_{t} \times \Theta^{t+1}_{t}) / (\Theta^{t}_{t+1} \times \Theta^{t+1}_{t+1}))^{1/2} \tag{3}$$

Θ^{i}_{j} = efficiency at time i to frontier j.

B. Grey's System Theory

The Grey's system theory is a method to study uncertainty. It is based on mathematical equations and has information and statistical application. In the lack of complete information, the Grey's system theory can be helpful in the study of problems involving small data and poor information [19]. The Grey's system theory is also used for making prediction. In the Grey's system theory, the raw data series can be converted into accumulated generating operation (AGO) series. Consider Raw Dataset 4:

$$X^{(0)} = (X_1^{(0)} X_2^{(0)} X_3^{(0)} \ldots\ldots , X_n^{(0)}) \tag{4}$$

$$AGO\ (1) = X^{(1)} = (\textstyle\sum^{1}_{k=1} X_k^{(0)}, \sum^{2}_{k=1} X_k^{(0)}, \sum^{3}_{k=1} X_k^{(0)} \ldots\ldots , \sum^{n}_{k=1} X_k^{(0)})$$

$$\vdots$$

$$AGO\ (n) = X^{(n)} = (\textstyle\sum^{1}_{k=1} X_k^{(n-1)}, \sum^{2}_{k=1} X_k^{(n-1)}, \sum^{3}_{k=1} X_k^{(n-1)} \ldots\ldots , \sum^{n}_{k=1} X_k^{(n-1)})$$

Data can be easily expressed in a specific category, known as AGO series. Each AGO is obtained from its preceding series and calculates data accumulatively and stage-to-stage (5). Then, the mean weight of two consecutive data is expressed as Z(k). Equation (6) represents the calculation mechanism.

$$Z_K^{(1)} = \alpha X_K^{(1)} + \beta X_{K-1}^{(1)} \quad \alpha+\beta=1$$

$$Z^{(1)} = (Z_1^{(1)}, \ldots\ldots, Z_K^{(1)})) \tag{6}$$

Accordingly, (7) was used to calculate $x^{(1).}$

$$dx^{(1)} / dt + ax^{(1)} = b \tag{7}$$

Based on the Grey's system theory [3], a' =[a, b] was calculated after obtaining above AGO values, using (8):

$$B = \begin{pmatrix} -z(2) & z^{(1)}(2)^2 \\ -z^{(1)}(3) & z^{(1)}(3)^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & z^{(1)}(n)^2 \end{pmatrix}$$

$$Y_N = \begin{pmatrix} X(2)^{(0)} \\ X(3)^{(0)} \\ \vdots \\ X(n)^{(0)} \end{pmatrix} \quad a' = (B^T B)^{-1} B^T Y_n \tag{8}$$

After the calculation of a', based on the Grey's equation developed by [25], next periods were predicted with (9):

$$x^{(1)}{}_{(k+1)} = \frac{a x^{(1)}(1)}{b\, x^{(1)}(1) + (a - a\, x^{(1)}(1))e^{ak}} \quad k=1,2,3,\ldots. \tag{9}$$

C. Multistage CCR Model

The CCR is the first data envelopment model. In this model, the base model was proposed to determine the highest ratio of efficiency, involving the inputs and outputs of decision-making units, and also to determine the optimal weight for investigated units [20]. CCR Fractional programming model is:

$$Max : \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} s.t.$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \ j = 1,2,...,n \ \ u_r \ge 0, v_i \ge 0$$

The above fractional planning model is known as the fractional CCR model where, u_r is the weight of r^th^ output, v_i is the weight of i^th^ input, and o is the index of decision-making unit under investigation ($o \in \{1,2,...,n\}$); y_{ro} and x_{io} are respectively the r^{th} and i^{th} output and input of the unit under investigation (Unit o), respectively; y_{rj} and x_{ij} are respectively the r^{th} and i^{th} output and input of the j^{th} unit; S is the number of outputs, m is the number of inputs, and n is the number of units.

In the input-oriented DEA models, we sought the ratio of technical inefficiency, which should be reduced in the inputs to place the unit at efficient frontier without changing the number of outputs. In the output-oriented DEA models, we intended to make the unit reach efficient frontier by finding the ratio of required increase in the output without changing the number of inputs. Based on the Charnes and Cooper recommendation, the fractional CCR model was converted into the linear planning model by application of the constraint $\sum_{i=1}^{m} V_i x_{i\circ} = 1$ [21]. Multi-axis input-axis model (CCR.I) is:

$$Max \sum_{r=1}^{s} u_r y_{r\circ}$$

$$s.t.: \sum_{i=1}^{m} V_i x_{i\circ} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1,...,n$$

$$u_r \ge 0 \qquad v_i \ge 0$$

This efficiency determination model is known as the input-oriented CCR model (CCR.I). Another technique can be used to convert the fractional CCR into a linear CCR model. In this method, the fractional CCR is converted into the linear CCR model by applying the constraint $\sum_{r=1}^{s} u_r y_{ro} = 1$, which represents the multiple output-oriented CCR (CCR.O) [22]. Multi-axis output-axis model (CCR.O) is:

$$Min \sum_{i=1}^{m} V_i x_{io}$$

$$s.t. \sum_{r=1}^{s} u_r y_{ro} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,...,n$$

$$u_r \ge 0 \quad v_i \ge 0$$

II. Discussion and Modeling

The modeling assumptions are as follows:

- It is a multiple input-oriented CCR model.
- There are n units under evaluation.
- Each unit is comprised of 4 subunits.
- Each subunit can contain many inputs and outputs.

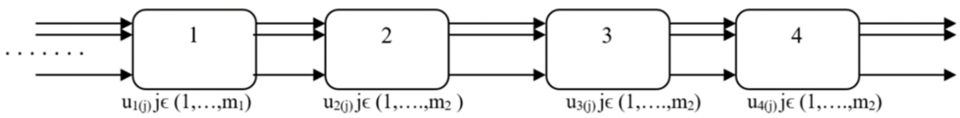

Fig. 1 Assumed multistage DEA diagram with 4 subsystems

Fig. 1 shows a network DEA model. Following equations are used to express the multiple input-oriented CCR model, whose base model was addressed in Section I:

Efficiency notation of the 1st-4th and overall systems is as:

$$e(1) = (\alpha_2 u_2) / (\alpha_1 u_1)$$
$$e(2) = (\alpha_3 u_3) / (\alpha_2 u_2)$$
$$e(3) = (\alpha_4 u_4) / (\alpha_3 u_3)$$
$$e(4) = (\alpha_5 u_5) / (\alpha_4 u_4)$$
$$e(5) = (\alpha_5 u_5) / (\alpha_1 u_1)$$

Fractional multiple input-oriented CCR model is:

$$Max \ \ (\alpha_5 u_{5\,(p)}) / (\alpha_1 u_{1\,(p)})$$

s.t :

$$\sum \forall j \ (\alpha_2 u_{2(j)}) / \sum \forall j \ (\alpha_1 u_{1(j)}) \le 1 \qquad j=1,...,m_1$$
$$\sum \forall j \ (\alpha_3 u_{3(j)}) / \sum \forall j \ (\alpha_2 u_{2(j)}) \le 1 \qquad j=1,...,m_2$$
$$\sum \forall j \ (\alpha_4 u_{4(j)}) / \sum \forall j \ (\alpha_3 u_{3(j)}) \le 1 \qquad j=1,...,m_3$$
$$\sum \forall j \ (\alpha_5 u_{5(j)}) / \sum \forall j \ (\alpha_4 u_{4(j)}) \le 1 \qquad j=1,...,m_4$$
$$\sum \forall j \ (\alpha_5 u_{5(j)}) / \sum \forall j \ (\alpha_1 u_{1(j)}) \le 1 \qquad j=1,...,m_5$$

$$u_1, u_2, u_3, u_4, u_5 \geq 0$$

Linear multiple input-oriented CCR model shows the conversion of Model 4 into a linear model after applying changes:

$$\begin{array}{ll} Max\ (\alpha_5 u_{5(p)}) & \\ (\alpha_1 u_{1(p)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)}) - \sum \forall j\ (\alpha_1 u_{1(j)}) \leq 0 & j=1,...,m_1 \\ \sum \forall j\ (\alpha_3 u_{3(j)}) - \sum \forall j\ (\alpha_2 u_{2(j)}) \leq 0 & j=1,...,m_2 \\ \sum \forall j\ (\alpha_4 u_{4(j)}) - \sum \forall j\ (\alpha_3 u_{3(j)}) \leq 0 & j=1,...,m_3 \\ \sum \forall j\ (\alpha_5 u_{5(j)}) - \sum \forall j\ (\alpha_4 u_{4(j)}) \leq 0 & j=1,...,m_4 \\ \sum \forall j\ (\alpha_5 u_{5(j)}) - \sum \forall j\ (\alpha_1 u_{1(j)}) \leq 0 & j=1,...,m_5 \\ u_{1(j)}, u_{2(j)}, u_{3(j)}, u_{4(j)}, u_{5(j)} \geq 0 & \end{array}$$

To develop the MPI, above input and output variables should be considered in two consecutive time intervals, presented by t and t+1, respectively. The weights of subsystem variables at t are: $u_{1(j)(t)}, u_{2(j)(t)}, u_{3(j)(t)}, u_{4(j)(t)}, u_{5(j)(t)}$. The weights of subsystem variables at t+1 are: $u_{1(j)(t+1)}, u_{2(j)(t+1)}, u_{3(j)(t+1)}, u_{4(j)(t+1)}, u_{5(j)(t+1)}$. Below model was rewritten considering the defined time intervals. In following four models, efficiency at t and t+1 was compared to two efficient frontiers at t and t+1. The ratio of efficiency at t to efficient frontier at t was calculated according to:

$$\begin{array}{ll} \Theta^t{}_{(t)} = Max\ (\alpha_5 u_{5(p)(t)}) & \\ (\alpha_1 u_{1(p)(t)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_5 \\ u_{1(j)(t)}, u_{2(j)(t)}, u_{3(j)(t)}, u_{4(j)(t)}, u_{5(j)(t)} \geq 0 & \end{array}$$

The ratio of efficiency at t to efficient frontier at t+1 was calculated according to:

$$\begin{array}{ll} \Theta^t{}_{(t)} = Max\ (\alpha_5 u_{5(p)(t+1)}) & \\ (\alpha_1 u_{1(p)(t+1)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_5 \\ u_{1(j)(t)}, u_{2(j)(t)}, u_{3(j)(t)}, u_{4(j)(t)}, u_{5(j)(t)}, u_{1(j)(t+1)}, u_{5(j)(t+1)} \geq 0 & \end{array}$$

The ratio of efficiency at t+1 to efficient frontier at t+1 was calculated according to:

$$\begin{array}{ll} \Theta^t{}_{(t)} = Max\ (\alpha_5 u_{5(p)(t+1)}) & \\ (\alpha_1 u_{1(p)(t+1)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t+1)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t+1)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t+1)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t+1)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t+1)}) \leq 0 & j=1,....,m_5 \\ u_{1(j)(t+1)}, u_{2(j)(t+1)}, u_{3(j)(t+1)}, u_{4(j)(t+1)}, u_{5(j)(t+1)} \geq 0 & \end{array}$$

The ratio of efficiency at t+1 to efficient frontier at t was calculated according to:

$$\begin{array}{ll} \Theta^t{}_{(t)} = Max\ (\alpha_5 u_{5(p)(t)}) & \\ (\alpha_1 u_{1(p)(t)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t+1)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t+1)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t+1)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t+1)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t+1)}) \leq 0 & j=1,....,m_5 \\ u_{1(j)(t)}, u_{5(j)(t)}, u_{1(j)(t+1)}, u_{2(j)(t+1)}, u_{3(j)(t+1)}, u_{4(j)(t+1)}, u_{5(j)(t+1)} \geq 0 & \end{array}$$

The MPI criterion with network diagram is as (10):

$$M_p = (\Theta^t{}_{(t+1)} \times \Theta^{t+1}{}_{(t+1)}) / (\Theta^t{}_{(t)} \times \Theta^{t+1}{}_{(t)}) \qquad (10)$$

According to previous sections, the Grey's system theory is applicable when data are not adequately large and/or there are data uncertainties [23]. This theory can predict the following periods by creating a series of cumulative data. To combine the multistage Malmquist model, the next period should be first predicted and the obtained results should be then considered as the development frontier of the new technology, based on the Wang's theory [3]. With the placement of it in the multistage Malmquist model, the Grey's prediction for the next period can be calculated:

$$u'_{n(j)(t+1)} = X^{(1)}{}_{(t+1)} = \frac{a x^{(1)}(1)}{b\, x^{(1)}(1) + \left(a - a\, x^{(1)}(1)\right) e^{a(t)}} \qquad K = 1.2. \dots \quad n = 1.2. \dots .5 \quad (11)$$

The four-fold Malmquist models were then rewritten considering the values predicted by the Grey's theory for the new period (year). Linear multiple input-oriented CCR model between K and K+1 is:

$$\begin{array}{ll} \beta^t{}_{(t)} = Max\ (\alpha_5 u_{5(p)(t)}) & \\ (\alpha_1 u_{1(p)(t)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_5 \\ u_{1(j)(t)}, u_{2(j)(t)}, u_{3(j)(t)}, u_{4(j)(t)}, u_{5(j)(t)} \geq 0 & \end{array}$$

Linear multiple input-oriented CCR model at K+1 relative to efficient frontier at K+1 with Grey's predicted values is:

$$\begin{array}{ll} \beta^t{}_{(t)} = Max\ (\alpha_5 u'_{5(p)(t+1)}) & \\ (\alpha_1 u'_{1(p)(t+1)}) = 1 & \\ \sum \forall j\ (\alpha_2 u_{2(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_1 \\ \sum \forall j\ (\alpha_2 u_{3(j)(t)}) - \sum \forall j\ (\alpha_1 u_{2(j)(t)}) \leq 0 & j=1,....,m_2 \\ \sum \forall j\ (\alpha_2 u_{4(j)(t)}) - \sum \forall j\ (\alpha_1 u_{3(j)(t)}) \leq 0 & j=1,....,m_3 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{4(j)(t)}) \leq 0 & j=1,....,m_4 \\ \sum \forall j\ (\alpha_2 u_{5(j)(t)}) - \sum \forall j\ (\alpha_1 u_{1(j)(t)}) \leq 0 & j=1,....,m_5 \end{array}$$

$$u_{1(j)(t)},\ u_{2(j)(t)},\ u_{3(j)(t)},\ u_{4(j)(t)},\ u_{5(j)(t)},\ u_{1(j)(t+1)},\ u_{5(j)(t+1)} \geq 0$$

Linear multiple input-oriented CCR model at K+1 relative to efficient frontier at K with Grey's predicted values is:

$$\begin{array}{ll} \beta^{t}{}_{(t)} = Max\ \ (\alpha_5\, u'_{5(p)(t+1)}) & \\ (\alpha_1\, u'_{1(p)(t+1)}) = 1 & \\ \sum \forall j\ (\alpha_2\, u'_{2(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{1(j)(t+1)}) \leq 0 & j=1,\ldots,m_1 \\ \sum \forall j\ (\alpha_2\, u'_{3(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{2(j)(t+1)}) \leq 0 & j=1,\ldots,m_2 \\ \sum \forall j\ (\alpha_2\, u'_{4(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{3(j)(t+1)}) \leq 0 & j=1,\ldots,m_3 \\ \sum \forall j\ (\alpha_2\, u'_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{4(j)(t+1)}) \leq 0 & j=1,\ldots,m_4 \\ \sum \forall j\ (\alpha_2\, u'_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{1(j)(t+1)}) \leq 0 & j=1,\ldots,m_5 \\ u'_{1(j)(t+1)},\ u'_{2(j)(t+1)},\ u'_{3(j)(t+1)},\ u'_{4(j)(t+1)},\ u'_{5(j)(t+1)} \geq 0 & \end{array}$$

and Linear multiple input-oriented CCR model at K relative to efficient frontier at K with Grey's predicted values is:

$$\begin{array}{ll} \beta^{t}{}_{(t)} = Max\ \ (\alpha_5\, u_{5(p)(t)}) & \\ (\alpha_1\, u_{1(p)(t)}) = 1 & \\ \sum \forall j\ (\alpha_2\, u'_{2(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{1(j)(t+1)}) \leq 0 & j=1,\ldots,m_1 \\ \sum \forall j\ (\alpha_2\, u'_{3(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{2(j)(t+1)}) \leq 0 & j=1,\ldots,m_2 \\ \sum \forall j\ (\alpha_2\, u'_{4(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{3(j)(t+1)}) \leq 0 & j=1,\ldots,m_3 \\ \sum \forall j\ (\alpha_2\, u'_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{4(j)(t+1)}) \leq 0 & j=1,\ldots,m_4 \\ \sum \forall j\ (\alpha_2\, u'_{5(j)(t+1)}) - \sum \forall j\ (\alpha_1\, u'_{1(j)(t+1)}) \leq 0 & j=1,\ldots,m_5 \\ u_{1(j)(t)}, u_{5(j)(t)},\ u'_{1(j)(t+1)},\ u'_{2(j)(t+1)},\ u'_{3(j)(t+1)},\ u'_{4(j)(t+1)},\ u'_{5(j)(t+1)} \geq 0 & \end{array}$$

Considering the Grey's theory prediction for k following periods, the MPI is as (12):

$$M'_p = \frac{\beta^{t+1}(t) \times \beta^{t}(t)}{\beta^{t+1}(t+1) \times \beta^{t}(t+1)}\ \forall p \epsilon 1,\ldots,5 \qquad (12)$$

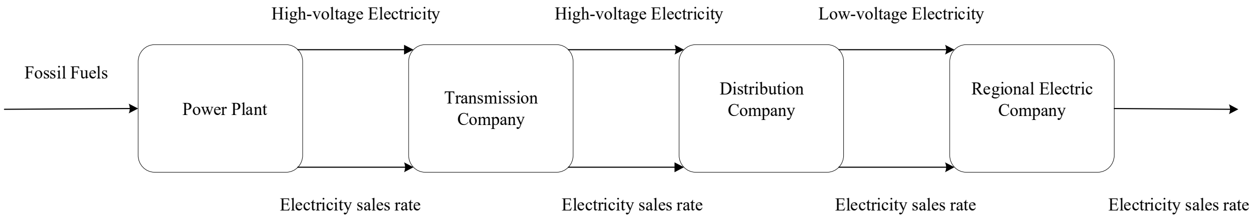

Fig. 2 (a) EPSC diagram

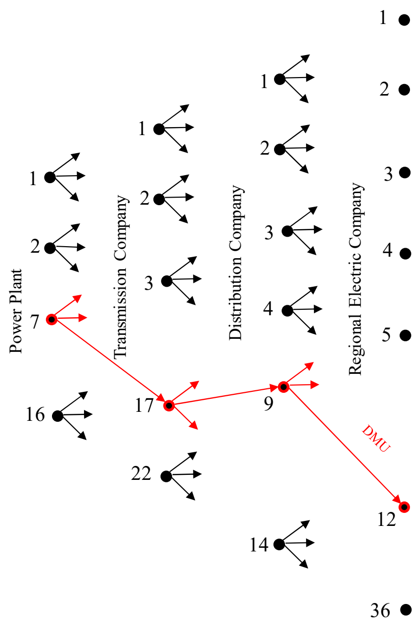

Fig. 2 (b) DMU selection

III. Case Study: Electricity Power Supply Chain

The Iran electricity market has started operating since 2002. It is responsible for the management of electricity generation, distribution and sale within the EPSC, which resulted in the emergence of an independent entity, called electricity market management or the Independent System Operator (ISO), in the new structure of the electric power industry to control the interactions between purchasers and sellers. The prerequisite of a competitive market is the separation of the network ownership from network management. As a result, a new entity independent of producers, distributors, and owners is needed for pricing, maintaining the system safety, planning for maintenance, and monitoring the performance data of each actor in the electricity market. The market management regulates the mechanism of the selection of market actors based on cost minimization. Each set of actors selected by the electricity market manager is laid out as the electricity transmission criterion. In this study, each combination of electricity generation, transition, and distribution was considered as a DMU (Fig. 2) and then evaluated. These evaluations were based on previous data and predicted for future periods, and may be associated with error. As a result, the Grey's theory was used because of data uncertainty. These evaluations can be effective in setting restraints and/or providing facilities for electricity transmission using combinations with the highest efficiency. Fig. 2 presents the EPSC diagram and mechanism of DMU selection.

Using the Grey's system theory and equations addressed in Section III, we predicted the next period (2017) through the accumulation of three consecutive years. The raw data are summarized in Table I.

The new prediction was considered as the new efficient frontier using the Grey's theory. The technological changes were made with predicted values for the new year. The calculated MPI values from solving the model using the metaheuristic genetic method in MATLAB 2016 for 365 units under investigation are presented in Fig. 3. The mean, minimum and maximum MPI were 0.79, 0.48, and 1 in 2017, respectively. Among the investigated units, Units 23, 104, 121, 142, 181, and 227 had the highest efficiency with the MPI of 1 (Fig. 3). These predictions were for 2014, 2015, and 2016, using data of preceding years. Fig. 4 presents their box diagram.

TABLE I
RAW INPUT AND OUTPUT DATA OF EPSC IN DIFFERENT YEARS

DMU	Year	Unit 1(input & output)			Unit 2(output)		Unit 3 (output)		Unit 4 (output)
		Fossil Fuels	High-voltage electricity (MWH)	Electricity sales rate (dollar)	High-voltage electricity (MWH)	Electricity sales rate (dollar)	High-voltage electricity (MWH)	Electricity sales rate (dollar)	Amount of consumption MWH)
	2014	88000	2306	12.7	1890	14.3	250	23.2	198.9
1	2015	86990	2660	18.7	19970	13.8	234	24.5	203.6
	2016	85980	2456	14.3	2309	13.9	231	25.6	216.6
Gray Forcast	2017	84679	25679	15.3	22340	13.5	242	23.8	217.4
	2014	76780	3290	15.4	3129	14.5	270	25.2	256.6
2	2015	84500	4350	18.2	4270	16.8	239	27.3	230.7
	2016	75349	4560	16.7	4500	18.8	245	28.3	241.4
Gray Forcast	2017	80430	46539	17.3	4650	19.2	248	29.3	245.6
	2014	83450	3456	16.7	3467	19.3	230	27.5	228.8
3	2015	82560	4007	15.8	4000	17.8	260	26.6	256.6
	2016	86540	3877	16.7	86500	20.5	210	25.6	207.8
Gray Forcast	2017	85340	3978	16.9	48600	19.3	250	26.6	232.2
	2014	80540	3400	14.3	3380	23.2	240	26.5	238.8
4	2015	89970	3280	17.5	32100	24.4	240	24.4	238.9
	2016	87770	3456	18.9	3450	23.2	280	24.8	276.6
Gray Forcast	2017	88670	34222	18.6	3500	23.7	291	25.4	279.5
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
	2014	85540	4378	12.4	4230	24.3	310	29.1	307.8
356	2015	86780	4320	16.5	4300	28.3	320	29.4	315.6
	2016	79880	4367	17.7	3980	26.3	310	25.5	308,6
Gray Forcast	2017	82340	43560	17.7	4090	26.6	315	27.6	309.8

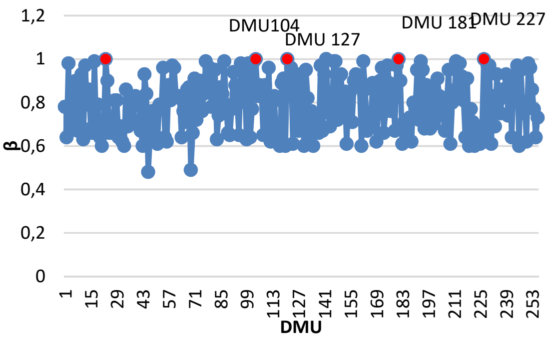

Fig. 3 Prediction of efficiency changes for 356 units under investigation in four periods

IV. CONCLUSION AND RECOMMENDATIONS

a. Conclusion

There are different methods for prediction of future data, out of which the Grey's system theory can provide better predictions using fewer data [9]. The combination of the Grey's system theory and MPI can calculate one of the most important productivity indices and specify the future position of the units based on their previous performance by predicting the future changes. In this article, the MPI was rewritten using the Grey's theory. In addition, an applied example of the status of the Iran's EPSC and different combinations of its actors in the electricity transmission network was evaluated using this theory (Fig. 5).

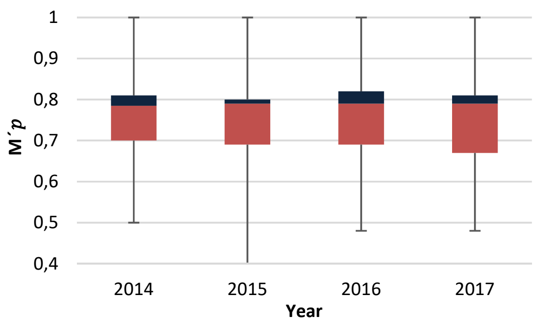

Fig. 4 Prediction of efficiency changes for 356 units under investigation in four periods

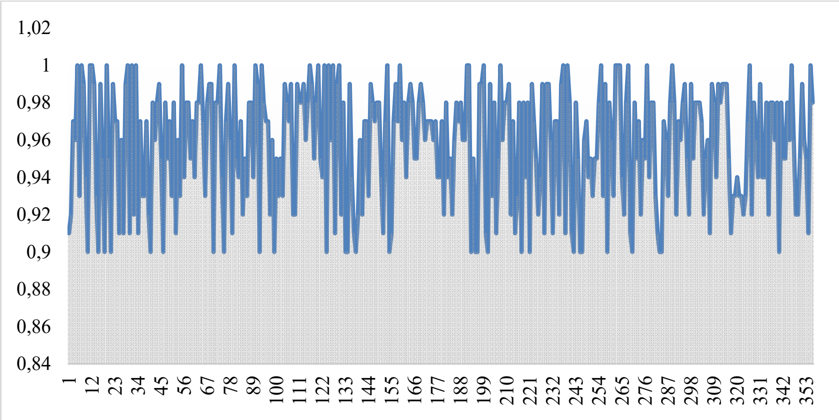

Fig. 5 Technical efficiency in 2014

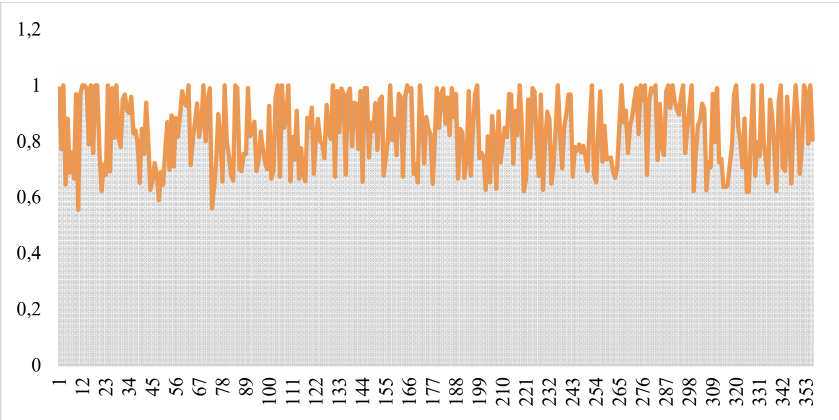

Fig. 6 Technological changes in 2014

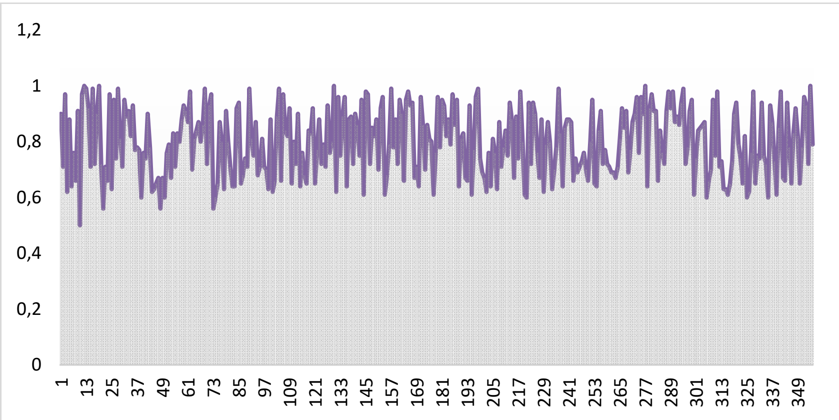

Fig. 7 MPI in 2014

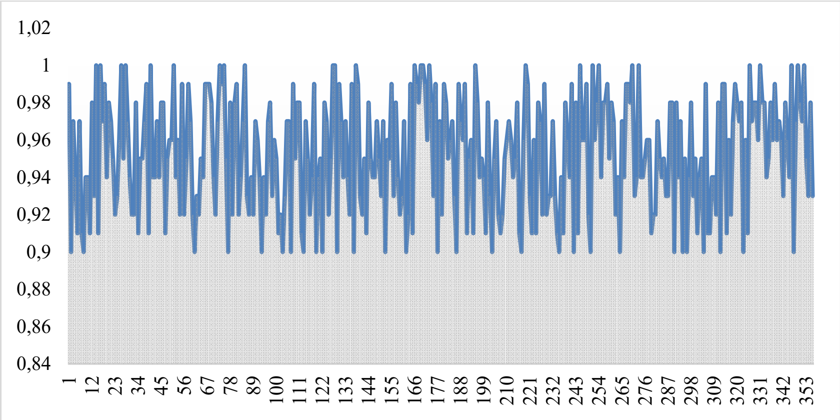

Fig. 8 Technical efficiency in 2015

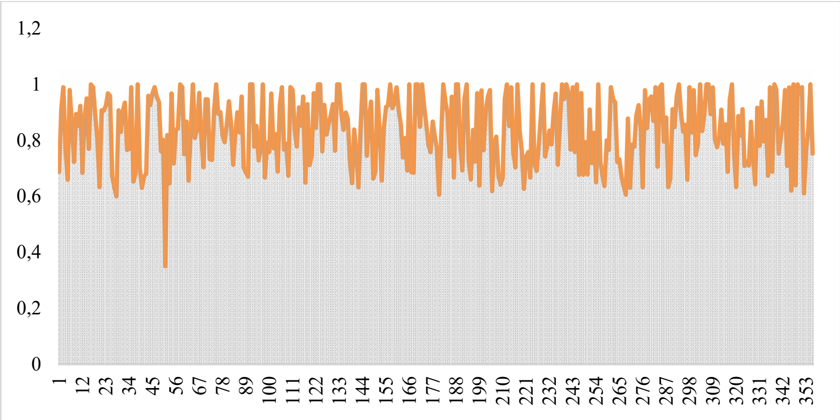

Fig. 9 Technological changes in 2015

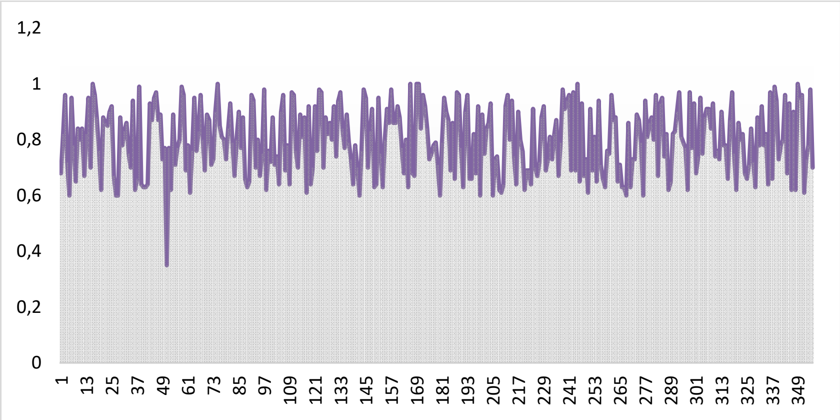

Fig. 10 MPI in 2015

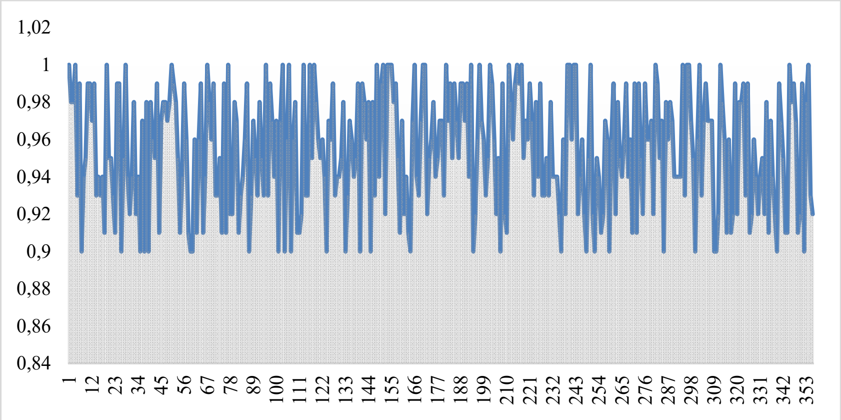

Fig. 11 Technical efficiency in 2016

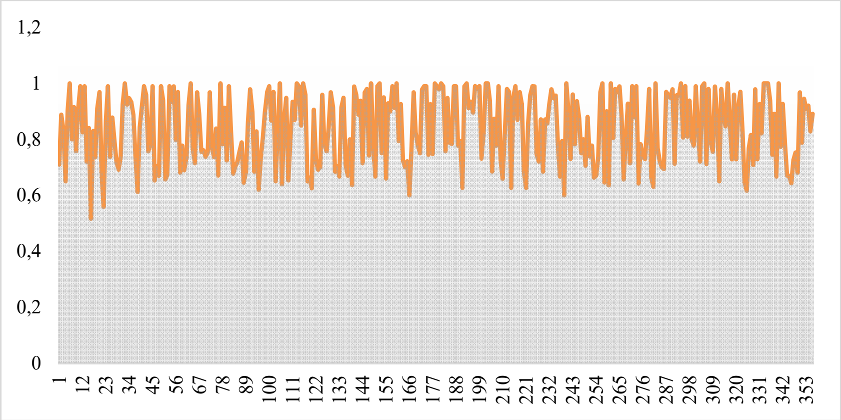

Fig. 12 Technological changes in 2106

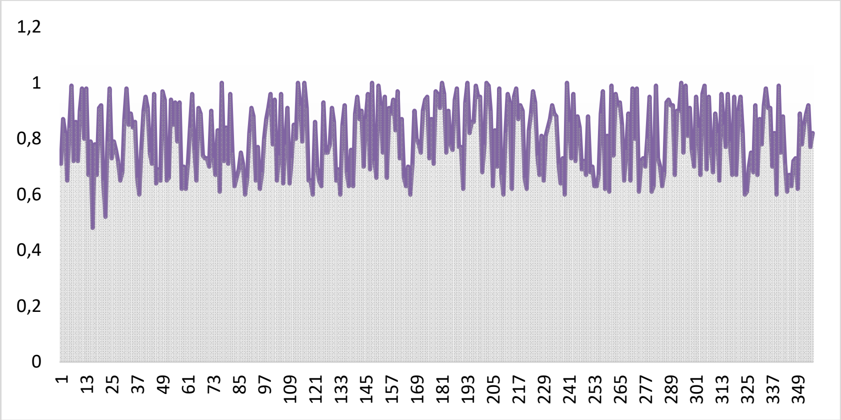

Fig. 13 MPI in 2016

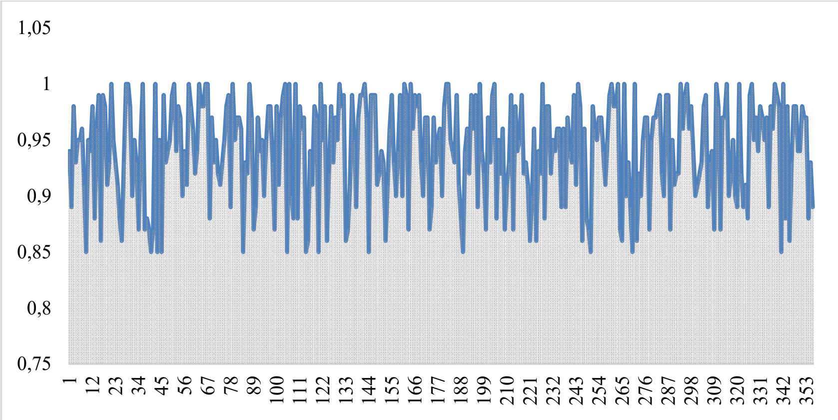

Fig. 14 Technical efficiency in 2017

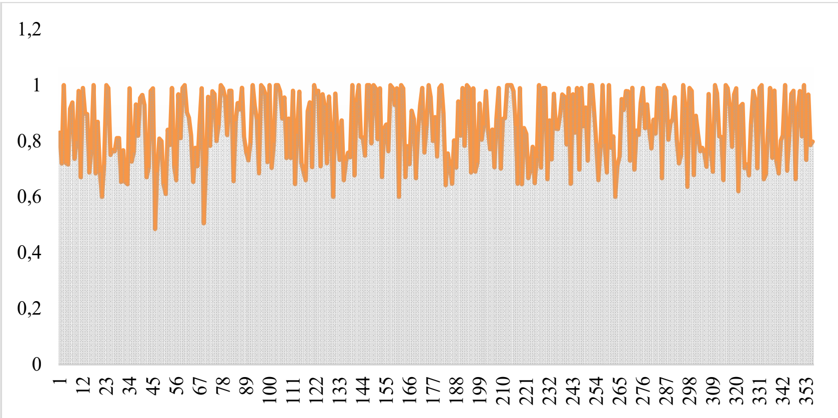

Fig. 15 Technological changes in 2017

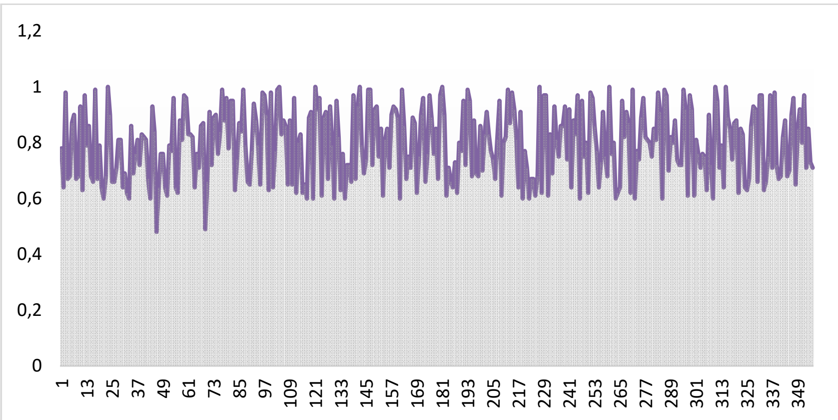

Fig. 16MPI in 2017

Prediction of productivity changes, as a system feedback, can inform managers for making appropriate changes to obtain better results in the future. Despite the presence of uncertain data and the use of Malmquist method, the calculated MPI values were compared to the results from another prediction technique (moving average). Results from comparing actual data and prediction data of MPI showed that despite limited data, an improvement was observed in data predicted by the Grey's theory. According to Figs. 17-20, the MPI prediction improved by 1.9% on average in four consecutive years.

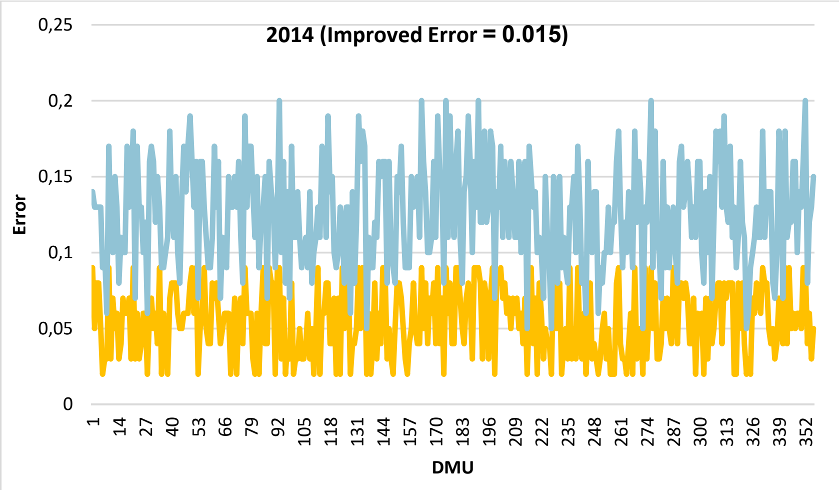

Fig. 17 Prediction of efficiency changes for 356 units under investigation in 2014

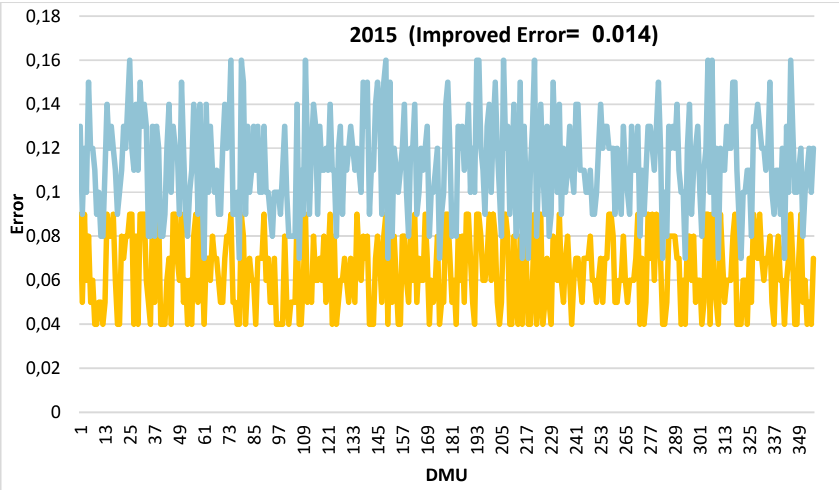

Fig. 18 Prediction of efficiency changes for 356 units under investigation in 2015

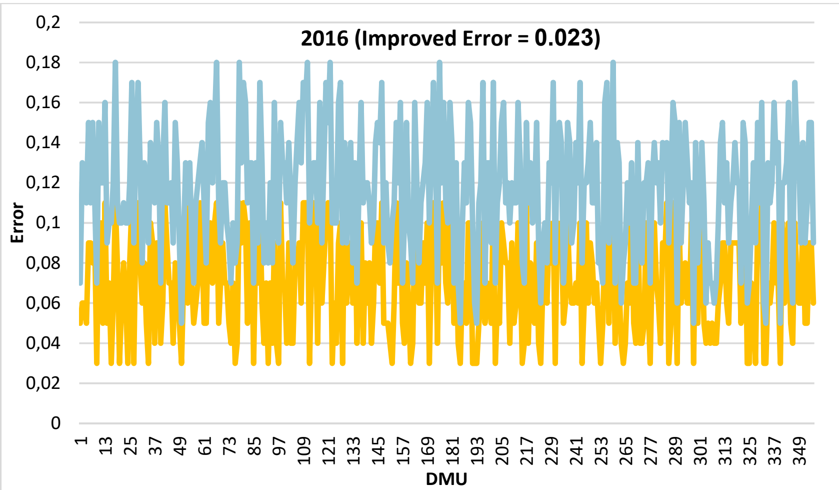

Fig. 19 Prediction of efficiency changes for 356 units under investigation in 2016

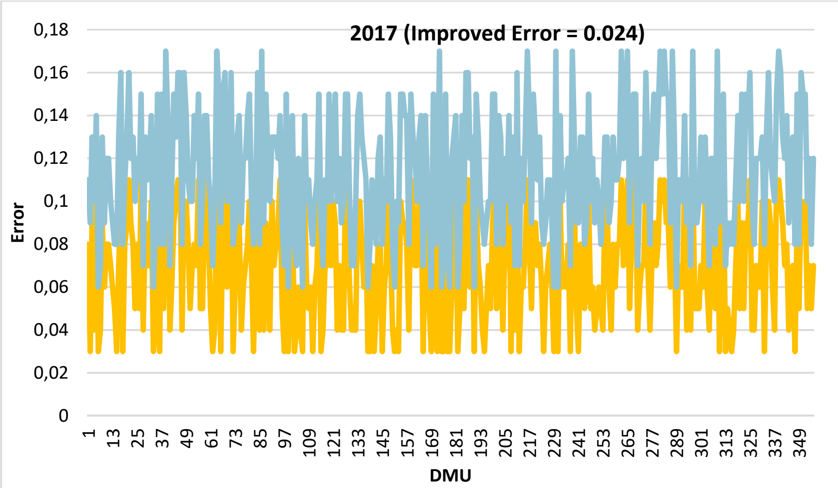

Fig. 20 Prediction of efficiency changes for 356 units under investigation in 2017

b. Recommendations

Following recommendations are made for the model development:

1. Applying the Grey's theory to other network DEA models
2. Using statistical methods to set upper and lower limits for predicted values (distance estimation)

Acknowledgment

Thanks to Hesam al-Din Salami for all the contributions that were made to advance the paper's goal.

References

[1] J. Johnes, Data envelopment analysis and its application to the measurement of efficiency in higher education, Econ. Educ. Rev. 5 (3) (2006) 273–288.
[2] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, Manag. Sci. 30 (9) (1984)
[3] M.L. Song, S.H. Wang, DEA decomposition of China’s environmental efficiency based on search algorithm, Applied Mathematics and Computation, 247(2014) 562-572
[4] M. Jim´enez, Carlos R. G. Alonso, L.S. Carulla, V.F. Rodr´ıguez, Evaluation of system efficiency using the Monte Carlo DEA: the case of Small Health Areas, European Journal of Operational Research
[5] H. Leleua, J. Moisesc, V.G. Valdmanisd, How do payer mix and technical inefficiency affect hospital profit? A weighted DEA approach, Operations Research for Health Care, 3(4) 2014 231-237
[6] G.Vlontzosa, S.Niavisa, B.Manosb, A DEA approach for estimating the agricultural energy and environmental efficiency of EU countries, Renewable and Sustainable Energy Reviews, 40 (2014) 91-96
[7] P.J. Sher, P.Y. Yang, The effects of innovative capabilities and R&D clustering on firm performance, the evidence of Taiwan's semiconductor industry, Technovation 25 (2005) 33–43.
[8] W.T. Lin, H.J. Yang, M.Y. Lin, The implementation of enterprise resource planning and product data management in semiconductor related industries, an empirical study in Taiwan, Int. J. Manage. 23 (1) (2006) 117–131.
[9] Mingzhi Mao, E.C. Chirwa, Application of grey model GM(1, 1) to vehicle fatality risk estimation, Technological Forecasting and Social Change, Volume 73, Issue 5, 2006, Pages 588-605.
[10] Yu-Shan Chen, Bi-Yu Chen, Applying DEA, MPI, and grey model to explore the operation performance of the Taiwanese wafer fabrication industry, Technological Forecasting and Social Change, Volume 78, Issue 3, 2011, Pages 536-546.
[11] H.V. Trivedi, J.K. Singh, Application of Grey System Theory in the Development of a Runoff Prediction Model, Biosystems Engineering, Volume 92, Issue 4, 2005, Pages 521-526.
[12] Wenbin Liu, Zhongbao Zhou, Chaoqun Ma, Debin Liu, Wanfang Shen, Two-stage DEA models with undesirable input-intermediate-outputs, Omega, Volume 56, 2015, Pages 74-87.
[13] Stephane Mussard, Nicolas Peypoch, On multi-decomposition of the aggregate Malmquist productivity index, Economics Letters, Volume 91, Issue 3, 2006,Pages 436-443.
[14] Filipa Da Silva Fernandes, Charalampos Stasinakis, Valeriya Bardarova, Two-stage DEA-Truncated Regression: Application in banking efficiency and financial development, Expert Systems with Applications, Volume 96, 2018, Pages 284-301.
[15] Reza Kiani Mavi, Reza Farzipoor Saen, Mark Goh, Joint analysis of eco-efficiency and eco-innovation with common weights in two-stage network DEA: A big data approach, Technological Forecasting and Social Change, 2018.
[16] Naser Amani, Hadi Bagherzadeh Valami, Ali Ebrahimnejad Application of Malmquist productivity index with carry-overs in power industry, Alexandria Engineering Journal, 2018.
[17] Visakh Sakthidharan, Sunitha Sivaraman, Impact of operating cost components on airline efficiency in India: A DEA approach, Asia Pacific Management Review, 2018.
[18] C.A. Ptak, E. Schragenheim, ERP Tools, Techniques, and Applications for Integrating the Supply Chain, St. Lucie Press, Florida, 2000.
[19] E. Thanassoulis, M.C.A.S. Portela, M. Graveney, Using DEA to estimate potential savings at GP units at medical specialty level, Socio-Economic Planning Sciences, 48(1) (2014) 38-48
[20] C.J. McDonald, New tools for yield improvement in integrated circuit manufacturing: can they be applied to reliability? Microelectron. Reliab. 39 (6–7) (1999) 731–739.
[21] R.M. Dabbas, H.N. Chen, Mining semiconductor manufacturing data for productivity improvement — an integrated relational database approach, Comput. Ind. 45 (1) (2001) 29–44.
[22] C.A. Bode, B.S. Ko, T.F. Edgar, Run-to-run control and performance monitoring of overlay in semiconductor manufacturing, Control Eng. Pract. 12 (2004) 893–900.
[23] Chiang Kao, Malmquist productivity index based on common-weights DEA: The case of Taiwan forests after reorganization, Omega, Volume 38, Issue 6, 2010, Pages 484-491.