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Monte Carlo Estimation of Heteroscedasticity and Periodicity Effects in a Panel Data Regression Model

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***Abstract*—**This research attempts to investigate the effects of heteroscedasticity and periodicity in a Panel Data Regression Model (PDRM) by extending previous works on balanced panel data estimation within the context of fitting PDRM for Banks audit fee. The estimation of such model was achieved through the derivation of Joint Lagrange Multiplier (LM) test for homoscedasticity and zero-serial correlation, a conditional LM test for zero serial correlation given heteroscedasticity of varying degrees as well as conditional LM test for homoscedasticity given first order positive serial correlation via a two-way error component model. Monte Carlo simulations were carried out for 81 different variations, of which its design assumed a uniform distribution under a linear heteroscedasticity function. Each of the variation was iterated 1000 times and the assessment of the three estimators considered are based on Variance, Absolute bias (ABIAS), Mean square error (MSE) and the Root Mean Square (RMSE) of parameters estimates. Eighteen different models at different specified conditions were fitted, and the best-fitted model is that of within estimator when heteroscedasticity is severe at either zero or positive serial correlation value. LM test results showed that the tests have good size and power as all the three tests are significant at 5% for the specified linear form of heteroscedasticity function which established the facts that Banks operations are severely heteroscedastic in nature with little or no periodicity effects.

***Keywords*—**Audit fee, heteroscedasticity, Lagrange multiplier test, periodicity.

I. Introduction

PDRM often suffers from phenomena of heteroscedasticity and periodicity when fitted. This is as a result of the heteroscedastic nature of its individual-specific error μ_i and the serially correlated nature of its time (periods) effect λ_t. The pioneering work of [1] has given rise to further researches on the estimation of heteroscedasticity effects in panel data. Most of the existing literatures were concerned with regression models that have to do with one-way error components model, $u_{it} = \mu_i + v_{it}$, $i = 1,...,T$, where the index i refers to the T time series observations. For instance, both [1] and [6] were concerned with the estimation of a model allowing for heteroscedasticity on the individual-specific error term, i.e. assuming that $\mu_i \sim (0, \sigma^2_{ui})$ while $v_{it} \sim IID(0, \sigma^2_v)$. In contrast, [2], [3], [5], [8] adopted a symmetrically opposite specification allowing for heteroscedasticity on the remainder error term, i.e. assuming that $\mu_i \sim IID\ (0, \sigma^2_\mu)$ while $v_{it} \sim (0, \sigma^2_{vi})$. Other authors who have developed estimation techniques for heteroscedasticity in relations to the aforementioned are [4], [7], [9]-[16]. The authors who have worked on that of serial correlations are among [17]-[20]. Reference [17] extended the error component model to take into account first-order serial correlation in the remainder disturbances of random effects model, while [18] carried out the same work for fixed effects model. References [18] and [19] estimated serial correlations by testing AR (1) against MA (1) disturbances in an error component model. Notably among the works that assume the existence of both problems are [21]-[23]. However, most of these works were concerned with regression models that have to do with one-way error components model. Recently [24] extended [11] and [21] to the two-way error components where heteroscedasticity and spatial correlation are considered in their determination of Joint and Conditional LM tests.

In this paper, focus shall be centered on the estimation of phenomena of heteroscedasticity and periodicity via a PDRM of Banks audit fees by extending the works of [16], [21], [24].

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II. Material and Methods

Monte Carlo simulations were carried out using Uniform distributions for a replicates of 1000 under 81 different variations of space and time via a linear functional form of heteroscedasticity. Three sizes of cross-sectional units (N=20, 40 and 60), three time periods (T = 10, 40, and 100), a homoscedastic situation and two degrees of heteroscedasticity (moderate and severe) and in line with [22], ρ is allowed to vary at three different levels of positive serial correlation (i.e 0, 0.5, 0.9) representing zero, weak and strong positive levels respectively.

A. Model Specification

The two models specified for Fixed and Random effect models are given respectively as

$$\begin{aligned}AF_{it} = {} & \alpha_1 + \alpha_2 D_{2i} + \alpha_3 D_{3i} + \cdots + \alpha_N D_{Ni} + \lambda_0 + \lambda_1 D_1 + \lambda_2 D_2 + \\ & \cdots + \lambda_{T-1} D_{T-1} + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \\ & \gamma_1 (D_{2i} PBT_{it}) + \gamma_2 (D_{2i} TA_{it}) + \gamma_3 (D_{2i} TL_{it}) + \gamma_4 (D_{2i} SHF_{it}) + \\ & \ldots + \gamma_{4(N-1)-3} (D_{Ni} PBT_{it}) + \gamma_{4(N-1)-2} (D_{Ni} TA_{it}) + \\ & \gamma_{4(N-1)-1} (D_{Ni} TL_{it}) + \gamma_{4(N-1)} (D_{Ni} SHF_{it}) + \\ & \psi_1 (D_1 PBT_{it}) + \psi_2 (D_1 TA_{it}) + \psi_3 (D_1 TL_{it}) + \psi_4 (SHF_{it}) + \\ & \ldots + \psi_{4(T-1)-3} (D_{T-1} PBT_{it})\, \psi_{4(T-1)-2} (D_{T-1} TA_{it}) + \\ & \psi_{4(T-1)-1} (D_{T-1} TL_{it}) + \psi_{4(T-1)} (D_{T-1} SHF_{it}) + \varepsilon_{it} \end{aligned} \quad (1)$$

$$AF_{it} = \beta_1 + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \omega_{it} \quad (2)$$

where Audit fees (AF), Profit before Tax (PBT), Total Assets (TA), Total Liability (TL) and Shareholders Fund (SHF) were originated from simulated panel data. β_1, β_2, β_3, β_4 and β_5 are estimable parameters and ω_{it} is a composite error term. α_1 *represents the intercept of the first individual, while* $\alpha_2, \alpha_3 \dots, \alpha_N$ are the differential intercepts coefficients. λ_0 is the intercept of the T^{th} year while $\lambda_1, \lambda_2 \dots, \lambda_{T-1}$ are the remaining years intercepts. $D_{2i}, \dots, D_{16i}$ are the dummy variables of (N-1) individuals, $D_1, \dots, D_{T-1}$ are dummy variables for (T-1) years, $\gamma_1, \gamma_2, \dots, \gamma_{4(N-1)}$ *and* $\psi_1, \psi_2, \dots, \psi_{4(T-1)}$ are the differential slope coefficients for individual and periodic effects respectively.

In the course of this study, it was demonstrated that the conditional variance of AF_{it} increases as each of $PBT_{it}, TA_{it}, TL_{it}$ and SHF_{it} increases.

Model (1) and (2) were estimated using

$$\text{Pooled OLS estimator: } \hat{\beta}_{pooled} = (X'X)^{-1} X' y \quad (3)$$

$$\text{Within estimator: } \hat{\beta} = [X' M_D X]^{-1} [X' M_D \, y] \quad (4)$$

$$\text{GLS estimator: } \hat{\beta}_{RE} = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1} Y^{*}_{it} \quad (5)$$

Assessments of the above estimators were based on Variance, ABIAS, MSE and the RMSE of parameters estimates. After evaluating the above criteria for each of the estimator, their performances were ranked and the best method identified.

B. Model Testing

Here, we shall employ a two-way error component model as earlier emphasized, to test for the violation of homoscedasticity and zero serial correlation assumptions in our researched model.

Considering a two-way error component model stated as:

$$y_{it} = x_{it}\beta + u_{it} \,,; \; i = 1,2, \dots, N \;\; t = 1,2, \dots, T \quad (6)$$

Within the context of two-way error component, the regression disturbances term u_{it} can be described by

$$u_{it} = \mu_i + \lambda_t + v_{it} \quad (7)$$

with μ_i representing individual-specific effect, λ_t representing time-specific effect and v_{it} the idiosyncratic remainder disturbance term, which is usually assumed to be well-behaved and independent from both the regressors x_{it} and μ_i. The two-way error component model can be written in matrix form as

$$y = X\beta + u \quad (8)$$

The disturbance term u in (11) can be written in vector form as

$$u = (I_{NT} \otimes \iota_{NT})v + (I_N \otimes \iota_T)\mu + (I_T \otimes \iota_N)\lambda + V \quad (9)$$

where I_{NT} is an identity matrix of dimension NT, I_N is an identity matrix of dimension N, I_T is an identity matrix of dimension T, ι_{NT} is a vector of ones of dimension NT, ι_T is a vector of ones of dimension T, ι_N is a vector of ones of dimension N, $\mu' = (\mu_1, \dots, \mu_N)$, $\lambda' = (\lambda_1, \dots, \lambda_T), V$ is the AR(1) covariance matrix of dimension T, $\otimes$ denotes the kronecker product and

$$Var(\mu_i) = \sigma^2_{\mu i} = h(f_i'(\alpha)) \;, i = 1, \dots, N \quad (10)$$

According to [25], the function $h(\cdot)$ is an arbitrary strictly positive twice continuously differentiable function, α *is a* $P \, x \, 1$ vector of unrestricted parameters and f_i is a $P \, x \, 1$ vector of strictly exogenous regressors which determine the heteroscedasticity of the individual specific effects and the first element of f_i is one, and without loss of generality, $h(\alpha_1) = \sigma^2_\mu$.

Following [21], the variance-covariance matrix of u can be written as

$$\begin{aligned} E(uu') = \Sigma &= \sigma^2_u(I_N \otimes \iota_T \, \iota_T') + (I_T \otimes \iota_N \iota_N')\sigma^2_\lambda + \sigma^2_v I_{NT} \otimes V \\ &= (I_N \otimes \iota_T) diag[h(f_i'\alpha)](I_N \otimes \iota_T)' + (I_T \otimes \iota_N \iota_N')\sigma^2_\lambda + \sigma^2_v I_{NT} \otimes V \\ &= diag[h(f_i'\alpha)] \otimes J_T + (I_T \otimes \iota_N \iota_N')\sigma^2_\lambda + \sigma^2_v I_{NT} \otimes V \end{aligned} \quad (11)$$

where J_T is a matrix of ones of dimension $T, diag[h(f_i'\alpha)]$ is a diagonal matrix of dimension NxN and V can be expressed as

$$V = E(VV') = \sigma^2_v \left(\frac{1}{1-\rho^2}\right) V_1 \quad (12)$$

where V_1 is a symmetric matrix of order ρ^{T-N}

1. Joint Lagrange Multiplier (JLM) Test

Here, we derived the joint LM test for homoscedasticity and no serial correlation of the first order. As specified in (14), the variance-covariance matrix of the disturbances in (11) is given as

$$\Sigma = diag[h(f_i'\alpha)] \otimes J_T + (I_T \otimes \iota_N \iota_N')\sigma^2_\lambda + \sigma^2_v I_{NT} \otimes V \quad (13)$$

under the null hypothesis, H_0: $\sigma^2_{\mu i} = \sigma^2_\mu$, $\forall_i$ *and* $\sigma^2_{\lambda t} = 0$ *but* $\sigma^2_{v_{it}} \neq 0, \rho = 0$ (such that both individual and time effects are missing), the variance covariance matrix of u reduces to

$$\Sigma = \sigma^2_\mu(I_N \otimes J_T) + (I_T \otimes \iota_N \iota_N')\sigma^2_\lambda + \sigma^2_v(I_N \otimes I_T) \quad (14)$$

And the spectral decomposition according to [26], becomes

$$\Sigma = E_T \otimes (\sigma^2_v I_N + \sigma^2_\lambda \iota_N \iota_N') + J_T \otimes [I_N(\sigma^2_v + T\sigma^2_\mu) + \sigma^2_\lambda \iota_N \iota_N'] \quad (15)$$

In line with (11), the inverse of Σ becomes

$$\Sigma^{-1} = E_T \otimes I_N(\sigma^2_v I_N + \sigma^2_\lambda \iota_N \iota_N')^{-1} + \bar{J}_T \otimes (\sigma^2_1 I_N + \sigma^2_\lambda \iota_N \iota_N')^{-1} \quad (16)$$

where $\sigma^2_1 = \sigma^2_v + T\sigma^2_\mu$.

Under normality of the disturbances, the log-likelihood function, L of a LM follows that of a multivariate normal distribution. Thus,

$$L(\beta,\theta) = \frac{-NT}{2}\ln(2\pi) - \frac{1}{2}ln|\Sigma| - \frac{1}{2}u'^{\Sigma^{-1}}u \qquad (17)$$

where $\theta' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho, \alpha')$ and $u = y - x\beta$.

In order to obtain the JLM statistic, we need to obtain the score statistic $D(\theta) = \frac{\partial L}{\partial\theta}$ and the Information matrix $I(\theta) = -E[\frac{\partial L^2}{\partial\theta\partial\theta'}]$ evaluated at the restricted maximum likelihood (ML) estimator θ. $I(\theta)$ is a block-diagonal between β *and* θ and since $H_0: \theta' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho, \alpha')$ involves only θ, the part of the information due to β is ignored [25]. Following [21], we obtain $D(\theta)$ and $I(\theta)$ as

$$\frac{\partial L}{\partial\theta} = -\frac{1}{2}tr\left[\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\theta}\right)\right] + \frac{1}{2}\left[u'\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\theta}\right)\Sigma^{-1}u\right] \qquad (18)$$

$$-E\left[\frac{\partial L^2}{\partial\theta\partial\theta'}\right] = \frac{1}{2}tr\left[\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\theta}\right)\Sigma^{-1}\frac{\partial\Sigma}{\partial\theta'}\right] \qquad (19)$$

Thus, evaluation of partial derivatives $\frac{\partial L}{\partial\theta}$ at restricted MLE yields

$$\frac{\partial L}{\partial\rho} = D(\tilde{\rho}) = -\frac{1}{2}tr\left[\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\rho}\right)\right] + \frac{1}{2}\left[\tilde{u}'\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\rho}\right)\Sigma^{-1}\tilde{u}\right]$$
$$= -\frac{1}{2}tr\left[\sigma_v^2\left[I_{NT}\otimes\left(\frac{\bar{J}_T Z}{\sigma_1^2} + \frac{\bar{J}_T Z}{\sigma_\lambda^2} + \frac{E_T Z}{\sigma_v^2} + \frac{E_T Z}{\sigma_\lambda^2}\right)\right]\right] + \frac{1}{2}\tilde{u}'\sigma_v^2\left[I_{NT}\otimes\right.$$
$$\left.\left(\frac{\bar{J}_T Z}{\tilde{\sigma}_1^4} + \frac{\bar{J}_T Z}{\sigma_\lambda^4} + \frac{E_T Z}{\sigma_v^4} + \frac{E_T Z}{\sigma_\lambda^4}\right)\right]\tilde{u}$$
$$= -\frac{NT\sigma_v^2}{2}\left[\frac{2(T-1)}{T\tilde{\sigma}_1^2} + \frac{2(T-1)}{T\tilde{\sigma}_\lambda^2} - \frac{2(T-1)}{T\tilde{\sigma}_v^2} - \frac{2(T-1)}{T\tilde{\sigma}_\lambda^2}\right] + \frac{\sigma_v^2}{2}\tilde{u}'\left[I_{NT}\otimes\right.$$
$$\left.\left(\frac{\bar{J}_T Z}{\tilde{\sigma}_1^4} + \frac{\bar{J}_T Z}{\tilde{\sigma}_\lambda^4} + \frac{E_T Z}{\tilde{\sigma}_v^4} + \frac{E_T Z}{\tilde{\sigma}_\lambda^4}\right)\right]\tilde{u}$$

Since $tr(Z) = 0, tr(\bar{J}_T Z) = 2(T-1)/T) = -tr(E_T Z), tr(\bar{J}_T) = 1$ *and* $tr(E_T) = T - 1$ [21]

$$= -\frac{2(T-1)NT\sigma_v^2}{2T}\left[\frac{1}{\tilde{\sigma}_1^2} - \frac{1}{\tilde{\sigma}_v^2}\right] + \frac{\tilde{\sigma}_v^2}{2}\tilde{u}'\left[I_{NT}\otimes\left(\frac{\bar{J}_T Z}{\tilde{\sigma}_1^4} + \frac{\bar{J}_T Z}{\tilde{\sigma}_\lambda^4} + \frac{E_T Z}{\tilde{\sigma}_v^4} + \frac{E_T Z}{\tilde{\sigma}_\lambda^4}\right)\right]\tilde{u}$$
$$= N(T-1)\left[\frac{\tilde{\sigma}_1^2 - \sigma_v^2}{\tilde{\sigma}_1^2}\right] + \frac{\tilde{\sigma}_v^2}{2}\tilde{u}'\left[I_{NT}\otimes\left(\frac{\bar{J}_T Z}{\tilde{\sigma}_1^4} + \frac{\bar{J}_T Z}{\tilde{\sigma}_\lambda^4} + \frac{E_T Z}{\tilde{\sigma}_v^4} + \frac{E_T Z}{\tilde{\sigma}_\lambda^4}\right)\right]\tilde{u}$$
$$= D(\tilde{\rho})$$
$$\frac{\partial L}{\partial\alpha_k} = D(\tilde{\alpha}_1) = -\frac{1}{2}tr\left[\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\alpha_k}\right)\right] + \frac{1}{2}\left[\tilde{u}'\Sigma^{-1}\left(\frac{\partial\Sigma}{\partial\alpha_k}\right)\Sigma^{-1}\tilde{u}\right]$$
$$= -\frac{1}{2}tr\left[\frac{h'(\alpha_1)}{\sigma_1^2}(diag(f_{ik})\otimes J_T)\right] + \frac{1}{2}\tilde{u}'\left[\frac{h'(\alpha_1)}{\sigma_1^4}(diag(f_{ik})\otimes J_T)\right]\tilde{u}$$
$$= -\frac{Th'(\alpha_1)}{2\tilde{\sigma}_1^2}\sum_{i=1}^N f_{ik} + \frac{h'(\alpha_1)}{2\tilde{\sigma}_1^4}\sum_{i=1}^N f_{ik}\,\tilde{u}_i' J_T\tilde{u}_i$$
$$= -\frac{Th'(\alpha_1)}{2\tilde{\sigma}_1^2}\sum_{i=1}^N f_{ik} + \frac{h'(\alpha_1)}{2\tilde{\sigma}_1^4}\sum_{i=1}^N f_{ik}\left(\sum_{t=1}^T\tilde{u}_{it}\right)^2$$
$$= \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^2}\sum_{i=1}^N f_{ik}\left(\frac{\left(\sum_{t=1}^T\tilde{u}_{it}\right)^2}{T\tilde{\sigma}_1^2} - 1\right), k = 1,2,\ldots,p = \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^2}F'g \qquad (20)$$

Equation (20) is the solution obtained after maximization of the first order condition, where $\tilde{u} = y - x\tilde{\beta}_{MLE}$ is the restricted maximum likelihood residuals under H_0, α_1 is the solution of $D(\tilde{\alpha}_1) = 0, \tilde{\sigma}_v^2$ is the solution of $(\tilde{\sigma}_v^2) = 0, F = (f_1, \ldots, f_N)'$ *and* $g = (g_1, \ldots, g_N)'$, *where* $g_i = \frac{\left(\sum_{t=1}^T\tilde{u}_{it}\right)^2}{T\tilde{\sigma}_1^2} - 1$. All the components of the score test statistic $\frac{\partial L}{\partial\theta}(\cdot)$ evaluated at maximization of the first order condition are all equal to zero except $\frac{\partial L}{\partial\rho}$ and $\frac{\partial L}{\partial\alpha}$ [24]. Thus, the partial derivatives under H_0 are rewritten in vector form as

$$D(\tilde{\theta}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ D(\tilde{\rho}) \\ \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^2}F'g \end{pmatrix} \qquad (21)$$

Also, we obtain the information matrix under the null hypothesis as a symmetric matrix of the form

$$\tilde{I}(\theta) = \begin{pmatrix} \sigma_v^4 & \sigma_v^2\sigma_\mu^2 & \sigma_v^2\sigma_\lambda^2 & \sigma_v^2\rho & \sigma_v^2\alpha \\ \sigma_v^2\sigma_\mu^2 & \sigma_\mu^4 & \sigma_\mu^2\sigma_\lambda^2 & \sigma_\mu^2\rho & \sigma_\mu^2\alpha \\ \sigma_v^2\sigma_\lambda^2 & \sigma_\lambda^2\sigma_\mu^2 & \sigma_\lambda^4 & \sigma_\lambda^2\rho & \sigma_\lambda^2\alpha \\ \sigma_v^2\rho & \rho\sigma_\mu^2 & \rho\sigma_\lambda^2 & \rho^2 & \rho\alpha \\ \sigma_v^2\alpha & \alpha\sigma_\mu^2 & \sigma_\lambda^2\alpha & \rho\alpha & \alpha^2 \end{pmatrix} \qquad (22)$$

$$= \begin{pmatrix} 0 & 0 & 0 & N(T-1)\tilde{\sigma}_v^2\left[\frac{1}{\tilde{\sigma}_1^4} - \frac{1}{\tilde{\sigma}_v^4}\right] & \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^4}\iota_N'F \\ 0 & 0 & 0 & 0 & 0 \\ 0 & & & 0 & 0 \\ N(T-1)\tilde{\sigma}_v^2\left[\frac{1}{\tilde{\sigma}_1^4} - \frac{1}{\tilde{\sigma}_v^4}\right] & 0 & 0 & \frac{2N(T-1)^2}{T}\left[\frac{\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4}{\tilde{\sigma}_1^4}\right] & \frac{T(T-1)\tilde{\sigma}_v^2 h'(\tilde{\alpha}_1)}{\tilde{\sigma}_1^4}F \\ & 0 & 0 & & \\ \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^4}\iota_N'F & 0 & 0 & \frac{T(T-1)\tilde{\sigma}_v^2 h'(\tilde{\alpha}_1)}{\tilde{\sigma}_1^4}F & \frac{T^2h'(\tilde{\alpha}_1)^2}{2\tilde{\sigma}_1^4}F'F \end{pmatrix} \qquad (23)$$

Therefore, LM statistic under H_0 is obtained by

$$LM_{\rho,\alpha} = D(\tilde{\theta})'[\tilde{I}(\theta)^{-1}]\,D(\tilde{\theta}) \qquad (24)$$

$$= \begin{pmatrix} D(\hat{\rho}) & \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^2}F'g \end{pmatrix}\begin{pmatrix} \frac{T\tilde{\sigma}_1^4}{2(T-1)^2[(N-T)(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)]} & \frac{-\tilde{\sigma}_1^4\tilde{\sigma}_v^4}{[(N-T)(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)]F'} \\ \frac{-\tilde{\sigma}_1^4\tilde{\sigma}_v^4}{[(N-T)(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)]F'} & \frac{2N\tilde{\sigma}_1^4}{T^2h'(\tilde{\alpha}_1)^2(N-T)} \end{pmatrix}\begin{pmatrix} D(\hat{\rho}) \\ \frac{Th'(\tilde{\alpha}_1)}{2\tilde{\sigma}_1^2}F'g \end{pmatrix}$$
$$= \frac{1}{2[N-T]}\left[\frac{T\tilde{\sigma}_1^4 D(\hat{\rho})^2 - Th'(\tilde{\alpha}_1)2(T-1)^2\tilde{\sigma}_1^4\tilde{\sigma}_v^4 D(\hat{\rho})}{(T-1)^2(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)} + \frac{2N(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)g'\tilde{\sigma}_1^2 F'Fg - Th'(\tilde{\alpha}_1)^2\tilde{\sigma}_1^2\tilde{\sigma}_v^4\,D(\hat{\rho})g}{h'(\tilde{\alpha}_1)(\tilde{\sigma}_v^4 - \tilde{\sigma}_1^4)}\right] \qquad (25)$$

Under the null hypothesis, the LM statistic of (29) is asymptotically distributed as χ_{p+1}^2 *as* $N, T \to \infty$

2. Conditional Lagrange Multiplier (CLM 1) Test

Here, we derive a conditional LM test for $H_0: \sigma_{\mu i}^2 \neq \sigma_\mu^2$, $\forall_i$ *and* $\sigma_{\lambda t}^2 = 0$ *but* $\sigma_{v_{it}}^2 \neq 0, \rho = 0$

Under H_0, the variance covariance matrix of the disturbances as given by (14) becomes

$$\Sigma = diag[h(f_i'\alpha)] \otimes J_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \qquad (26)$$

The spectral decomposition and inverse of Σ respectively becomes

$$\Sigma = diag[Th(f_i'\alpha) + \sigma_v^2] \otimes \bar{J}_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \qquad (27)$$

$$\Sigma^{-1} = diag\left[\frac{1}{\Omega_i^2}\right] \otimes \bar{J}_T + \frac{1}{\sigma_v^2} I_{NT} \otimes E_T + \frac{1}{\sigma_\lambda^2} I_N$$

where

$$\Omega_i^2 = Th(f_i'\alpha) + \sigma_v^2 \qquad (28)$$

Therefore,

$$\frac{\partial L}{\partial \rho} = D(\hat{\rho}) = -\frac{1}{2} tr\left[\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \rho}\right)\right] + \frac{1}{2}\left[\hat{u}'\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \rho}\right)\Sigma^{-1}\hat{u}\right]$$
$$= -\frac{1}{2} tr\left[diag\left[\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right] \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z\right] +$$
$$\frac{1}{2}\left[\hat{u}'\left(diag\left[\frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$
$$= -\frac{1}{2}\left[\frac{2(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - tr\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2}\right] +$$
$$\frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$
$$since\ tr(Z) = 0\ and\ tr(\bar{J}_T Z) = tr(E_T)Z = \frac{2(T-1)}{T}$$
$$= -\frac{1}{2}\left[\frac{2(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - 0\right] + \frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \right.\right.$$
$$\left.\left.\bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right]$$

since there is no serial correlation of which its variance has been expressed as $\hat{\sigma}_\lambda^2$

$$= \frac{(T-1)}{T}\sum_{i=1}^N \sum_{t=1}^T \left(\frac{\hat{\Omega}_i^2 - \hat{\sigma}_v^2}{\hat{\Omega}_i^2}\right) + \frac{\hat{\sigma}_v^2}{2}\left[\hat{u}'\left(diag\left[\frac{1}{\hat{\Omega}_i^4}\right] \otimes \bar{J}_T Z + \right.\right.$$
$$\left.\left.\frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z\right)\hat{u}\right] \qquad (29)$$

Equation (29) is the solution obtained after maximization of the first order condition, where $\hat{u} = y - x\hat{\beta}_{GLS}$ is the generalized least square residuals under $H_0, \hat{\Omega}_i^2 = Th(f_i'\hat{\alpha}) + \hat{\sigma}_v^2, where\ \hat{\alpha}$ is the ML estimator of $\alpha\ under\ H_0, and\ h'(f_i'\hat{\alpha})$ is the evaluated value of $\partial h(f_i'\hat{\alpha})/\partial f_i'\alpha$. All the components of the score test statistic $\frac{\partial L}{\partial \eta_1}(\cdot)$ evaluated at maximization of the first order condition are all equal to zero except $\frac{\partial L}{\partial \rho}$. Thus, the partial derivatives under H_0 are expressed in vector form as

$$D(\hat{\eta}_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D(\hat{\rho}) \end{pmatrix} \qquad (30)$$

Also, we obtain information matrix under the null hypothesis as a symmetric matrix of the form

$$I(\hat{\eta}_1) = \begin{pmatrix} \beta'\beta & \beta\sigma_v^2 & \beta\sigma_\mu^2 & \beta\sigma_\lambda^2 & \beta\rho \\ \beta\sigma_v^2 & \sigma_v^4 & \sigma_v^2\sigma_\mu^2 & \sigma_v^2\sigma_\lambda^2 & \sigma_v^2\rho \\ \beta\sigma_\mu^2 & \sigma_v^2\sigma_\mu^2 & \sigma_\mu^4 & \sigma_\mu^2\sigma_\lambda^2 & \sigma_\mu^2\rho \\ \beta\sigma_\lambda^2 & \sigma_v^2\sigma_\lambda^2 & \sigma_\mu^2\sigma_\lambda^2 & \sigma_\lambda^4 & \sigma_\lambda^2\rho \\ \beta\rho & \sigma_v^2\rho & \sigma_\mu^2\rho & \sigma_\lambda^2\rho & \rho^2 \end{pmatrix} \qquad (31)$$

$$= \begin{pmatrix} I_{\beta\beta}(\hat{\eta}_1) & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(\hat{\sigma})_v^4 & 0 & 0 & \frac{1}{2\hat{\sigma}_v^2} N \xrightarrow{lim} \infty \left[\frac{I_N'Z}{N}\right] \\ 0 & 0 & \frac{1}{2}(\hat{\sigma})_\mu^4 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(\hat{\sigma})_\lambda^4 & 0 \\ 0 & \frac{1}{2\hat{\sigma}_v^2} N \xrightarrow{lim} \infty \left[\frac{I_N'Z}{N}\right] & 0 & 0 & \frac{1}{2} N \xrightarrow{lim} \infty \left[\frac{Z'Z}{N}\right] \end{pmatrix} \qquad (32)$$

Thus, a conditional LM statistic under the specified H_0 is given as

$$LM_{\rho|\alpha} = D(\hat{\rho})'\left[(I_{NT}(\hat{\eta}_1))^{-1}|_{\rho\rho}\right]D(\hat{\rho}) \qquad (33)$$

Setting $H_{NT}^{\rho} = diag\left(\frac{1}{\sqrt{NT}} I_k, \frac{1}{\sqrt{NT}}, \frac{1}{\sqrt{N}}, \frac{1}{\sqrt{T}}, \frac{1}{\sqrt{NT}}\right)$, LM statistic also becomes

$$LM_{\rho|\alpha} = \left[D(\hat{\rho})_{H_{NT}^{\rho}}\right]' \left[H_{NT}^{\rho}(I_{NT}(\hat{\eta}_1))H_{NT}^{\rho})^{-1}|_{\rho\rho}\right] \qquad (34)$$

$$H_{NT}^{\rho}\left(I_{NT}(\hat{\eta}_1)\right)H_{NT}^{\rho} \xrightarrow{N,T\to\infty} I(\hat{\eta}_1)$$

Thus, the LM statistic becomes

$$LM_{\rho|\alpha} = D(\hat{\rho})'\left[(I(\hat{\eta}_1))^{-1}|_{\rho\rho}\right]D(\hat{\rho}) \qquad (35)$$

where $(I(\hat{\eta}_1))^{-1}|_{\rho\rho} = \frac{1}{2} N \xrightarrow{lim} \infty \left[\frac{1}{N} Z'\left(I_N - \frac{I_N I_N'}{N}\right)Z\right]$

Under H_0, LM statistic is asymptotically distributed as $\chi_1^2\ as\ N, T \to \infty$.

3. Conditional Lagrange Multiplier (CLM 2) Test

Here, we derive a conditional LM test for $H_0: \sigma_{\mu i}^2 = \sigma_\mu^2$, $\forall_i\ and\ \sigma_{\lambda t}^2 \neq 0\ but\ \sigma_{vit}^2 \neq 0, \rho > 0$.

Under H_0, the variance covariance matrix of the disturbance term becomes

$$\Sigma = \sigma_\mu^2(I_N \otimes I_T) + \sigma_\lambda^2 I_N I_N' + \sigma_v^2(I_{NT} \otimes)V_\rho) \qquad (36)$$

where $V_\rho = \left(\frac{1}{1-\rho^2}\right)V_1$ and V_1 is the AR(1)correlation matrix.

According to [21], the inverse of Σ under H_0 becomes

$$\Sigma^{-1} = \frac{1}{\sigma_v^2}\left(I_N \otimes V_\rho^{-1}\right) - \left(\frac{\sigma_\mu^2}{\sigma_v^2\sigma_\lambda^2\lambda^2}\right)\left(I_N \otimes V_\rho^{-1} J_T V_\rho^{-1}\right) \qquad (37)$$

Therefore,

$$\frac{\partial L}{\partial \alpha_k} = D(\hat{\alpha}_k) = -\frac{1}{2} tr\left[\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \sigma_\lambda^2}\right)\right] + \frac{1}{2}\left[\hat{u}'\Sigma^{-1}\left(\frac{\partial \Sigma}{\partial \sigma_\lambda^2}\right)\Sigma^{-1}\hat{u}\right]$$

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$$= -\frac{1}{2} tr \left[\frac{h'(\hat{\alpha}_1)}{\hat{\sigma}_v^2}\left\{\left(diag(f_{ik})\otimes\hat{V}_\rho^{-1}J_T\right) - \left(\frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right)\left(diag(f_{ik}) \otimes \hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}J_T\right)\right\}\right] + \frac{1}{2}\hat{u}'\left[\frac{h'(\hat{\alpha}_1)}{\hat{\sigma}_v^4}\left\{\left(diag(f_{ik})\otimes\hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}\right) - 2\left(\frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right)\left(diag(f_{ik}) \otimes \hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}\right)\right\} + \left(\frac{\hat{\sigma}_\mu^4}{\hat{\sigma}_\lambda^4\hat{\lambda}^4}\right)\left(diag(f_{ik}) \otimes \hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}\right)\hat{u}\right]$$

$$= -\frac{h'(\hat{\alpha}_1)}{2\hat{\sigma}_v^2}\left[\varphi^2(1-\hat{\rho})^2\sum_{i=1}^N f_{ik} - \frac{\hat{\sigma}_\mu^2\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\sum_{i=1}^N f_{ik}\right] + \frac{h'(\hat{\alpha}_1)}{2\hat{\sigma}_v^4}\left[\hat{u}'^{\sum_{i=1}^N f_{ik}}\otimes(\varphi^2(1-\hat{\rho})^2\hat{V}_\rho^{-1} - 2\frac{\hat{\sigma}_\mu^2\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\hat{V}_\rho^{-1} + \frac{\hat{\sigma}_\mu^4\varphi^6(1-\hat{\rho})^6}{\hat{\sigma}_\lambda^4\hat{\lambda}^4}\hat{V}_\rho^{-1})\hat{u}\right]$$

$$= -\frac{h'(\hat{\alpha}_1)\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^2}\left[1 - \frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right]\sum_{i=1}^N f_{ik} + \frac{h'(\hat{\alpha}_1)(\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^4}\left[\hat{u}'\hat{V}_\rho^{-1}(1 - 2\frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2} + \frac{\hat{\sigma}_\mu^4\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^4\hat{\lambda}^4})\hat{u}\right]\sum_{i=1}^N f_{ik}$$

$$= \frac{h'(\hat{\alpha}_1)(\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^4}\sum_{i=1}^N f_{ik}\left[\hat{u}'\hat{V}_\rho^{-1}\left(1 - 2\frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2} + \frac{\hat{\sigma}_\mu^4\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^4\hat{\lambda}^4}\right) - \left(1 - \frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right)\right] = \frac{h'(\hat{\alpha}_1)(\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^4}\sum_{i=1}^N f_{ik}\left[\hat{u}'\hat{V}_\rho^{-1}\left(1 - 2\frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2} + \frac{\hat{\sigma}_\mu^4\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^4\hat{\lambda}^4}\right)\hat{u} - 1 + \frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right]$$

$$= \frac{h'(\hat{\alpha}_1)(\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^4}\sum_{i=1}^N f_{ik}\left(\hat{u}_i'\hat{A}\hat{u}_i - 1 + \frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}\right), \quad k = 1, \dots, p \qquad (38)$$

Equation (38) is the solution obtained after maximization of the first order condition, where $\hat{A} = \hat{V}_\rho^{-1}\left(1 - 2\frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2} + \frac{\hat{\sigma}_\mu^4\varphi^4(1-\hat{\rho})^4}{\hat{\sigma}_\lambda^4\hat{\lambda}^4}\right)$, $\hat{u} = y - x\hat{\beta}_{GLS}$ is the maximum likelihood residuals under $H_0, \hat{\beta}, \hat{\sigma}_v^2, \hat{\sigma}_\mu^2$ and $\hat{\alpha}_1$ is the maximum likelihood estimates of $\beta, \sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2$ *and* α_1 respectively. All components of the above score test statistic $\frac{\partial L}{\partial \eta}(\cdot)$ evaluated at $\hat{\eta}$ are equal to zero except $\frac{\partial L}{\partial \alpha}$. Also, $\hat{\sigma}_\mu^2$ is the value of $h(\hat{\alpha}_1)$ and $h'(\hat{\alpha}_1)$ is the evaluated value of $\partial h(f_i'\alpha)/\partial f_i'$ when $\alpha_1 = \alpha_1 = \cdots = \alpha_p = 0$. In addition, $tr(\hat{V}_\rho^{-1}J_T) = \varphi^2(1-\hat{\rho})^2$ and $tr(\hat{V}_\rho^{-1}J_T\hat{V}_\rho^{-1}J_T) = \varphi^4(1-\hat{\rho})^4$. Thus, the partial derivatives under H_0 are expressed in vector form as

$$D(\hat{\eta}_2) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D(\hat{\alpha}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{h'(\hat{\alpha}_1)\varphi^2(1-\hat{\rho})^2}{2\hat{\sigma}_v^4}F'g \end{pmatrix} \qquad (39)$$

where $D(\hat{\alpha}) = \left((\hat{\alpha}_1), D(\hat{\alpha}_2), \dots, D(\hat{\alpha}_p)\right)'$, $F = (f_1, \dots, f_N)'$ *and* $g = (g_1, \dots, g_N)$ where $g_i = \hat{u}_i'\hat{A}\hat{u}_i - 1 + \frac{\hat{\sigma}_\mu^2\varphi^2(1-\hat{\rho})^2}{\hat{\sigma}_\lambda^2\hat{\lambda}^2}$. Also, we obtain information matrix under the null hypothesis as a symmetric matrix of the form.

$$I(\hat{\eta}_2) = \begin{pmatrix} I_{\beta\beta}(\hat{\eta}_2) & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(\hat{\sigma})_v^4 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(\hat{\sigma})_\mu^4 & 0 & \frac{h'(\hat{\alpha}_1)\varphi^4(1-\hat{\rho})^4}{2\hat{\sigma}_v^4} N \xrightarrow{lim} \infty \left[\frac{I_N'F}{N}\right] \\ 0 & 0 & 0 & \frac{1}{2}(\hat{\sigma})_\lambda^4 & 0 \\ 0 & 0 & \frac{h'(\hat{\alpha}_1)\varphi^4(1-\hat{\rho})^4}{2\hat{\sigma}_v^4} N \xrightarrow{lim} \infty \left[\frac{I_N'F}{N}\right] & 0 & \frac{h'(\hat{\alpha}_1)^2\varphi^4(1-\hat{\rho})^4}{2\hat{\sigma}_v^4} N \xrightarrow{lim} \infty \left[\frac{F'F}{N}\right] \end{pmatrix} \qquad (40)$$

Thus, a conditional LM statistic under the specified H_0 is given as

$$LM_{\alpha|\rho} = D(\hat{\alpha})'[(I_{NT}(I(\hat{\eta}_2)))^{-1}|_{\alpha\alpha}]D(\hat{\alpha}) \qquad (41)$$

where $(I(\hat{\eta}))^{-1}|_{\alpha\alpha} = \frac{h'(\hat{\alpha}_1)^2\varphi^4(1-\hat{\rho})^4}{2\hat{\sigma}_v^4} N \xrightarrow{lim} \infty \left[\frac{1}{N}F'\left(I_N - \frac{I_N I_N'}{N}\right)F\right]$. Under H_0, LM statistic is asymptotically distributed as χ_p^2 *as* $N, T \to \infty$ (details of the above mathematical expressions are available in the appendix upon request from the authors).

III. Results and Discussion

The results for the smallest iterated space and time combinations of N=20 and T=10 are hereby presented. Small values of N and T were chosen to demonstrate that the researched model and size of LM tests also work well even for small samples and once the research opinion being investigated worked at that sample level, it would definitely work well asymptotically. This is in line with the opinion expressed by [24].

Scatter Plot of Audit Fees for Replicates of 1000 When N=20 and T=10

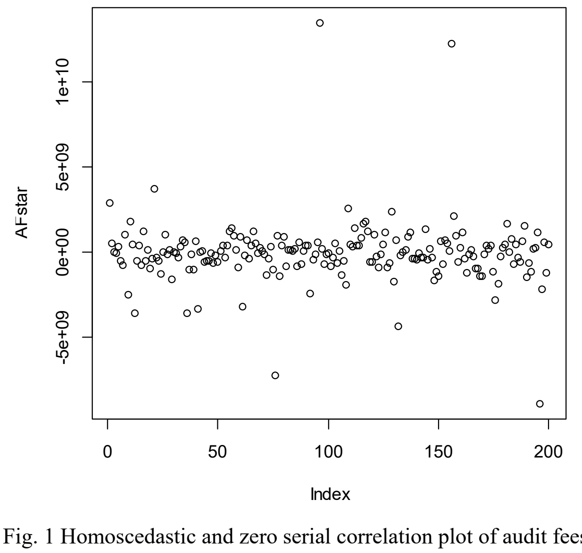

Fig. 1 Homoscedastic and zero serial correlation plot of audit fees

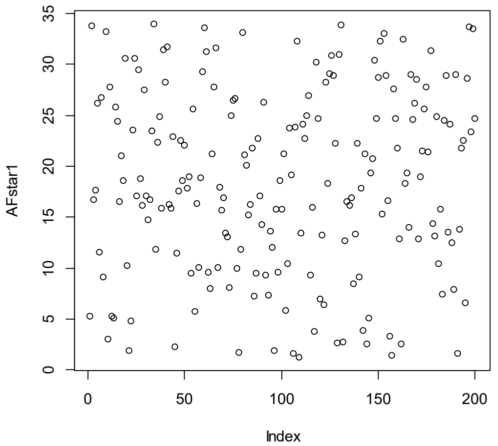

Fig. 2 Moderate Heteroscedastic and zero serial correlation plot of audit fees

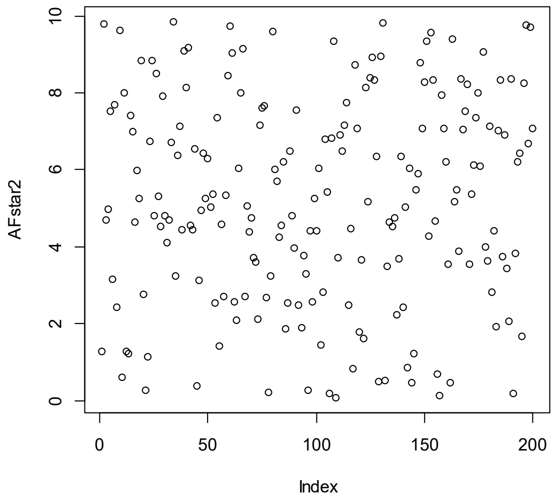

Fig. 3 Severe heteroscedastic and zero serial correlation plot of audit fees

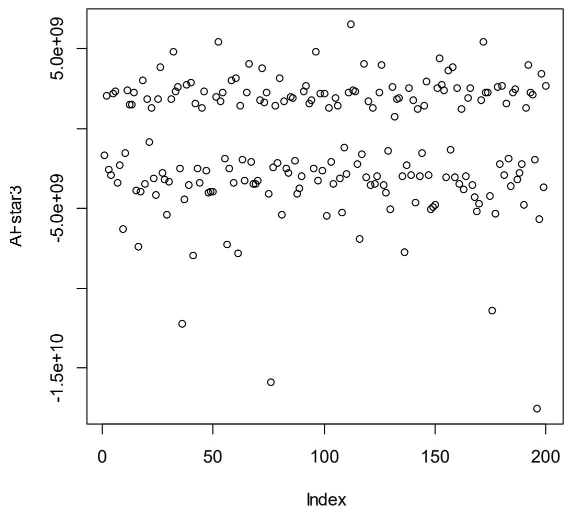

Fig. 4 Homoscedastic and positive serial correlation plot of audit fees

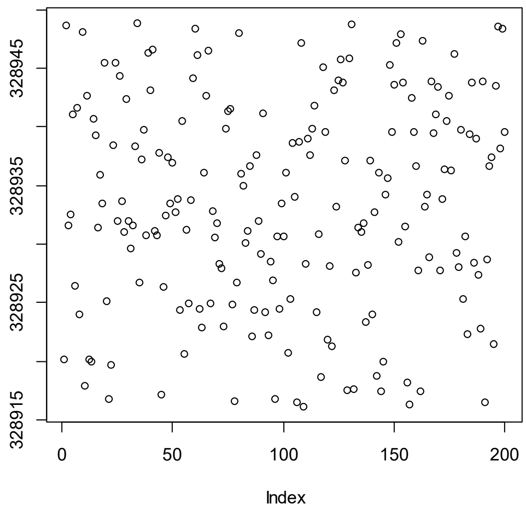

Fig. 5 Moderate heteroscedastic and positive serial correlation plot of audit fees

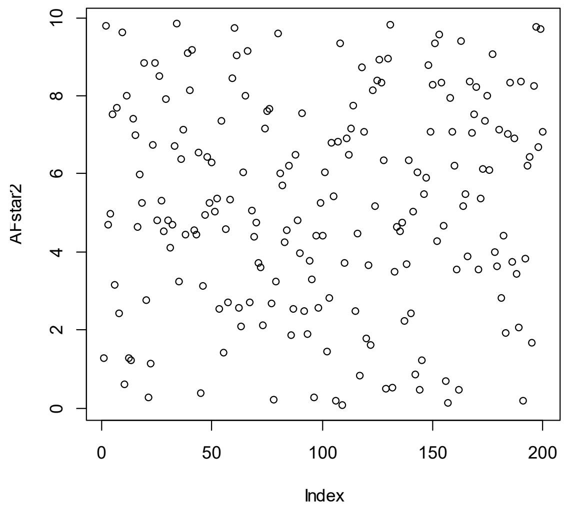

Fig. 6 Severe Heteroscedastic and positive serial correlation plot of audit fees

Figs. 1-6 show the pattern of movement for the individual audit fees of all the banks and it can be observed that the plots are more dispersed when error is heteroscedastic than when it is homoscedastic or serially correlated.

The estimated models from POLS, Within and GLS estimators based on the researched conditions presented in the above figures are given as follows:

$$AF_{POLS} = -210{,}511.8 - 0.001402PBT + 0.0000045TA - 0.000014TL - 0.00044SHF \quad (42a)$$

$$AF_{WITHIN} = -3.6745 - 0.00000002PBT + 0.0000000008TA - 0.0000000002TL - 0.00000008SHF \quad (42b)$$

$$AF_{GLS} = -1.16e-32 - 7.69e-41PBT + 2.49e-42\,TA - 7.68e-43TL - 2.42e-41SHF \quad (42c)$$

$$AF_{POLS} = -210{,}511.88 - 0.001402PBT + 0.0000454TA - 0.000014TL - 0.0004423SHF \quad (43a)$$

$$AF_{WITHIN} = -36{,}745.24 - 0.0002479PBT + 0.0000079TA - 0.0000024TL - 0.00007.72SHF \quad (43b)$$

$$AF_{GLS} = -1.15\mathrm{e}-32 - 7.69\mathrm{e}-41PBT + 2.49\mathrm{e}-42TA - 7.68\mathrm{e}-43TL - 2.42\mathrm{e}-41SHF \quad (43c)$$

$$AF_{POLS} = -210{,}511.8 - 0.001402PBT + 0.0000454TA - 0.000014TL - 0.0004423SHF \quad (44a)$$

$$AF_{WITHIN} = 1{,}116{,}400 + 0.007437PBT - 0.000240TA + 0.000074TL + 0.002346SHF \quad (44b)$$

$$AF_{GLS} = -1.16\mathrm{e}-32 - 7.69\mathrm{e}-41PBT + 2.49\mathrm{e}-42TA - 7.68\mathrm{e}-43TL - 2.42\mathrm{e}-41SHF \quad (44c)$$

$$AF_{POLS} = -210{,}511.80 - 0.001402PBT + 0.000045TA - 0.000014TL - 0.0004423SHF \quad (45a)$$

$$AF_{WITHIN} = -3.674{,}524 - 0.00000002PBT + 0.0000000008TA - 0.0000000002TL - 0.000000007SHF \quad (45b)$$

$$AF_{GLS} = -1.16\mathrm{e}-32 - 7.69\mathrm{e}-41PBT + 2.49\mathrm{e}-42TA - 7.68\mathrm{e}-43TL - 2.42\mathrm{e}-41SHF \quad (45c)$$

$$AF_{POLS} = -210{,}511.80 - 0.001402PBT + 0.0000454TA - 0.000014TL - 0.000442SHF \quad (46a)$$

$$AF_{WITHIN} = -36{,}745.24 - 0.000245PBT + 0.000008TA - 0.000002TL - 0.000077SHF \quad (46b)$$

$$AF_{GLS} = -1.16\mathrm{e}-32 - 7.69\mathrm{e}-41PBT + 2.49\mathrm{e}-42TA - 7.68\mathrm{e}-43TL - 2.43\mathrm{e}-41SHF \quad (46c)$$

$$AF_{POLS} = -210{,}511.80 - 0.00140239PBT + 0.00004541TA - 0.00001400TL - 0.00044234SHF \quad (47a)$$

$$AF_{WITHIN} = 1{,}116{,}400 + 0.0074372PBT - 0.0002408TA + 0.0000743TL + 0.00234585SHF \quad (47b)$$

$$AF_{GLS} = -1.15\mathrm{e}-32 - 7.69\mathrm{e}-41PBT + 2.49\mathrm{e}-42TA - 7.68\mathrm{e}-43TL - 2.43\mathrm{e}-41SHF \quad (47c)$$

Based on the rank results estimated using values of ABIAS presented in Table I, GLS technique ranked highest with a rank sum of 243 compared to that of Within and OLS with a rank sum of 146 and 97, respectively. This implied that GLS technique is expected to have given the best estimate for the specified PDRM and this is in line with the works of [23] where variance was used to rank the results of similar PDRMs. However, the theoretical concept of our researched model does not support the empirical structure of the kind of models fitted through GLS, hence the adoption of a within model which will guarantee a positive value for the banks audit fee in line with prior opinion. This is also in line with [27] that within transformation implements the LSDV model better because the regression on de-meaned data yields the same results as estimating the model from the original data and a set of (N-1) indicator variables for all but one of the panel units. Thus, (44b) and (47b) are the ideal models that best fitted the specified audit fee model with the later explained the variation in audit fees better at 71.92%. This model further exposed the fact that out of the two-way error components considered in this research, the individual specific error term affects PDRM of audit fees more than the time specific error term. This individual specific error term which has been established to be heteroscedastic in nature is seen to be severe across banks as regards their operations with little or no periodicity effects.

Table II presents significance values for the empirical size of the Joint LM test, Conditional LM test for heteroscedasticity and zero serial correlation as well as Conditional LM test for homoscedasticity and serial correlation. This was achieved at 5% level of significance when N = 20, 40, and 60 at T = 10. Adapting [21], both the bi-dimensional remainder error and time specific error terms $(\sigma_v^2, \sigma_\lambda^2)$ take values (2, 2), (2, 6), (6, 2) and (6, 6) in each experiment. These correspond to cases where the percentages of the total variance due to both errors are 25% and 75% accordingly. On the other hand, α is assigned values 0, 1 and 2 with $\alpha = 0$ denoting homoscedastic individual specific error, while $\alpha = 1$ and 2 denote moderate and severe heteroscedastic errors respectively. These results show that the sizes of all the tests are significant at 5% for the specified linear heteroscedasticity function. These results conformed with that of [24] where similar tests have been used to examined heteroscedasticity and spatial correlation in a two-way random effect model.

TABLE I
RANKS OF THE PDRM TECHNIQUES USING ABIAS CRITERION FOR HOMOSCEDASTIC, VARYING DEGREE OF HETEROSCEDASTICITY AND SERIAL CORRELATION LEVELS FOR THE VARIOUS SAMPLE SIZES CONSIDERED

Space	Time	Serial Correlation Level	Homoscedasticity/Heteroscedasticity Degree	POLS	WITHIN	GLS
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
20	10	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3

Space	Time	Serial Correlation Level	Homoscedasticity/Heteroscedasticity Degree	POLS	WITHIN	GLS
			Homoscedastic	2	1	3
		0	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
	40	0.5	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	2	1	3
			Homoscedastic	1	2	3
20	100	0.5	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	2	1	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	2	1	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
	10	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
40			Homoscedastic	2	1	3
		0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
	40	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
40	100	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
		0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
	10	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	2	1	3
60		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	1	2	3
	40		Severe Heteroscedastic	1	2	3
		0.5	Homoscedastic	1	2	3
			Moderate Heteroscedastic	1	2	3

Space	Time	Serial Correlation Level	Homoscedasticity/Heteroscedasticity Degree	POLS	WITHIN	GLS
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
60	100	0.5	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Homoscedastic	1	2	3
		0.9	Moderate Heteroscedastic	1	2	3
			Severe Heteroscedastic	1	2	3
			Sum of the Ranks	**97**	**146**	**243**

TABLE II
ESTIMATED SIZE OF JOINT LM AND CONDITIONAL LM TEST IN LINEAR HETEROSCEDASTICITY WHEN T=10

σ_v^2	σ_λ^2	α	ρ	N=20	N=40	N= 60
Joint LM Test for Homoscedasticity and Zero Serial Correlation						
2	2	0	0	0.00258	0.00257	0.00255
2	6	0	0	0.00260	0.00256	0.00274
6	2	0	0	0.00276	0.00274	0.00271
6	6	0	0	0.00279	0.00282	0.00286
Conditional LM Test for Heteroscedasticity and Zero Serial Correlation						
	2	1	0	0.00461	0.00445	0.00440
2	2	2	0	0.00433	0.00432	0.00430
	6	1	0	0.00402	0.00401	0.00400
	6	2	0	0.00399	0.00389	0.00380
	2	1	0	0.00406	0.00404	0.00401
6	2	2	0	0.00398	0.00396	0.00393
	6	1	0	0.00452	0.00471	0.00490
	6	2	0	0.00499	0.00489	0.00480
Conditional LM Test for Homoscedasticity and Serial Correlation						
2	2	0	0.5	0.0441	0.0432	0.0447
	2	0	0.9	0.0442	0.0437	0.0446
	6	0	0.5	0.0443	0.0449	0.0450
	6	0	0.9	0.0448	0.0443	0.0441
6	2	0	0.5	0.0342	0.0357	0.0359
	2	0	0.9	0.0351	0.0353	0.0360
	6	0	0.5	0.0362	0.0337	0.0346
	6	0	0.9	0.0448	0.0343	0.0341

IV. CONCLUSION

Having used necessary statistical methods, in line with the aim of this research, there is no doubt that the main purpose of this thesis has been fully realized. Therefore, based on the results obtained by the empirical analysis of the simulated data, the following conclusions are arrived at:

- That among the models presented, (44b) and (47b) are the only recommended models that satisfy the purpose for this research, going by the concept of auditors' remuneration which cannot assumed a negative value. These equations are the within models fitted for simulated panel data when heteroscedasticity is severe with and without the presence of positive serial correlation.
- Thus, taking advantage of "Big data" under the condition of full estimation of heteroscedasticity and periodicity effects, (47b) would be preferred over and above that of (44b) and shall become the only ideal model capable of scientifically estimated a non-subjective audit fees for the banks external auditor. This model actually exposed the fact that the heterogeneity situation in banking operations is severe with little or no periodicity effect.
- That OLS completely breaks down and can only give rise to unreliable inference in the presence of severe heteroscedasticity. It only performs better for small samples and as sample size increases, OLS derailed even in the absence of heteroscedasticity.
- That GLS technique ranked highest compared to others. However, the theoretical concept of our researched model does not support the kind of model fitted through GLS, hence the adoption of a within model which guarantees a positive value for the specified audit fees model.
- That Fixed effect model (FEM) fits the proposed audit fees model better than Random effect model (REM).
- That Monte Carlo scheme exposed the fact that out of the two-way error components considered, the individual specific error term affects PDRM of audit fees more than the time specific error term.
- That Monte Carlo results show that both the Joint and the two conditional LM tests have good size and power under the adopted linear functional form of heteroscedasticity.

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