International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:13, No:5, 2019

Mixed Mode Fracture Analyses Using Finite Element Method of Edge Cracked Heavy Spinning Annulus Pulley

Bijit Kalita, K. V. N. Surendra

Abstract**—**Rotating disk is one of the most indispensable parts of a rotating machine. Rotating disk has found many applications in the diverging field of science and technology. In this paper, we have taken into consideration the problem of a heavy spinning disk mounted on a rotor system acted upon by boundary traction. Finite element modelling is used at various loading condition to determine the mixed mode stress intensity factors. The effect of combined shear and normal traction on the boundary is incorporated in the analysis under the action of gravity. The variation near the crack tip is characterized in terms of the stress intensity factor (SIF) with an aim to find the SIF for a wide range of parameters. The results of the finite element analyses carried out on the compressed disk of a belt pulley arrangement using fracture mechanics concepts are shown. A total of hundred cases of the problem are solved for each of the variations in loading arc parameter and crack orientation using finite element models of the disc under compression. All models were prepared and analyzed for the uncracked disk, disk with a single crack at different orientation emanating from shaft hole as well as for a disc with pair of cracks emerging from the same center hole. Curves are plotted for various loading conditions. Finally, crack propagation paths are determined using kink angle concepts.

Keywords**—**Crack-tip deformations, static loading, stress concentration, stress intensity factor.

I. Introduction

THIS study is mainly focused on the pulleys of the transmission system. Pulleys are one of the main components used in a belt conveyor, supporting the loads generated from the belt tension according to its wrapping angle. The pulley works under both the fatigue compressive loading due to belt tension and the torque due to the rotation. In most of the heavy-duty applications, belt drives are used as an imperative mechanism for transmitting power from one machine shaft to the other over a large center distance. Usually, very heavy circular disks are used as pulleys, which is characterized by the use of a continuous belt to convey power between the shafts. Besides its self-weight, as the pulley rotates at a very high speed, pulley experiences centrifugal force along with the normal and shear traction on its periphery. As a result of the complicated loading conditions created by the tractions, centrifugal loading, the high-stress concentration may arise due to the repetition of which pulley may lead to fatigue failure. Hence, a detailed fracture analysis is quite necessary while designing the drive optimally to avoid unexpected failures. The model of the pulley is represented as a two-dimensional heavy circular disk which is supported at its center where the transmission of motion takes place through friction. To assure the safety of the belt drive system is very important to investigate the lifetime and optimization of the pulley. A rotating compound disk containing a radial crack was studied, and the SIFs of the crack were calculated by Xu [1]. He presented extensive numerical results for both plane strain and plane stress cases in tabular and graphical forms. Bowie and Neal [2] and Isida [3] also utilized numerical methods for calculation of SIFs of cracked disks. Despite other studies that used two-dimensional (2D) models, Pook et al. [4] used a three-dimensional (3D) finite element model for investigation of fracture parameters in a cracked disk under anti-plane loading. The applications of pulley drive in the automotive industry range from crankshafts, water pumps, and air conditioners to power steering pumps. Due to the complexities of loading conditions and installation environment, the evaluation of the design of the pulley becomes difficult. The research work of Euler [5] in the year 1979, considered to be the first work on belt drive mechanics. He showed the relationship between belt tension and coefficient of friction. Grubler investigated the rotating disk systems in the year 1906. His work was further followed by Donatch in 1912. Beomkeun et al. [6] showed that pulleys rotating at high speeds develop high-stress concentration due to complex loading conditions. This leads to fatigue failure of the pulley. In the works of Arnold et al. [7], the author tried to put emphasis on the problem of the single rotating disk as well as a number of concentric disks incorporating two major features of fracture static and cyclic limit (burst) speeds. The axisymmetric deformations of a rotating disc with variable thickness were explored [8]. Blauel et al. [9] determined stress-intensity factors both analytically and by using a photo-elastic method for the simple case of a rotating solid disk containing radial cracks. Therefore, in this research, an extensive study is performed for calculation of SIFs in a pulley which is a more practical scenario of a rotating disk. Further study will be extended for the various application of rotating machinery such as flywheel, jet engine compressor disk, roller disk cutter, etc. where SIF calculation plays a significant role in the accuracy and reliability of a safe design.

Bijit Kalita is M. S. (R) Student, Department of Mechanical Engineering, Indian Institute of Technology, Palakkad, Kerala- 678557, India (phone: 08486421570, e-mail: 131803101@smail.iitpkd.ac.in).
K. V. N. Surendra is Assistant Professor, Dept of Mechanical Engineering, Indian Institute of Technology, Palakkad, Kerala-678557, India (e-mail: kvns@iitpkd.ac.in).

II. Objectives

A. Definition and Methodology

In the present scope of work, the pulley, which is subjected to asymmetric loading conditions, will be used to predict contact stresses and its effect on SIF at the crack tip. Finite Element Method (FEM) is used to analyze the stresses and stress concentration zones. Distributed stresses are represented as consistence nodal forces using FEM concepts. The crack path is also predicted for different cases using fracture mechanics concepts.

B. Assumption

The material of pulley is considered as elastic, homogeneous and isotropic. There is no effective temperature change due to operation and model is under the consideration of the plan state of stress.

C. Geometrical Dimension and Nomenclature

TABLE I
Geometrical Dimension and Nomenclature

Symbol	Quantity	Units
R_o	outer radius of pulley	meter
R_i	inner radius of pulley	meter
β	arc loading angle	degree
h	thickness	meter
P_x , P_y	the reaction at the centre along iX, iY direction	Newton
σ_o , σ_i	Outside and inside normal traction	N/m^2
τ_i, τ_o	Inside and outside shear traction	N/m^2
α	crack orientation	degree
a	crack length	meter
M	total moment	Nm
P	total force	N
w	Pulley width (R_o-R_i)	meter

III. Finite Element Analyses

Asymmetric loading on the pulley due to belt tension and torque due to the rotation along with the centrifugal force generated has been represented in the CAD diagram. Finite element modelling is used for various loading conditions. Shaft reactions and arc loadings are represented by consisting nodal forces using FEM concept.

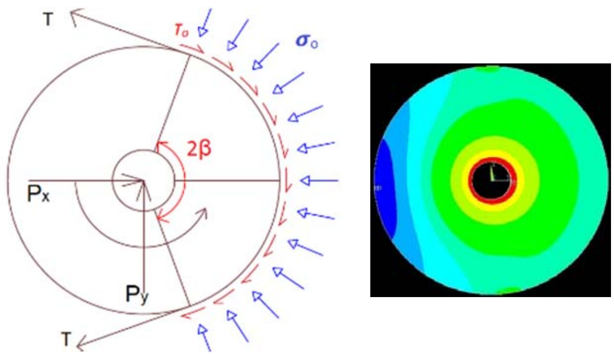

Fig. 1 Pulley under asymmetric loading (a) CAD design, (b) FEM modelling

From Fig. 1 (a)

$$P_x = \int_{-\beta}^{\beta} \sigma_o Cos\theta.R_o.d\theta.h = 2\sigma_o Sin\beta.R_o.h \quad (1)$$

$$P_y = \int_{-\beta}^{\beta} \tau_o Cos\theta.R_o.d\theta.h = 2\tau_o Sin\beta.R_o.h \quad (2)$$

$$P = \int_{-\beta}^{\beta} \sigma_o.R_o.h.d\theta = 2\sigma_o\beta.R_o.h = \int_{0}^{2\pi} \sigma_i.R_i.h.d\theta = 2\sigma_i\pi.R_i.h \quad (3)$$

$$M = \int_{-\beta}^{\beta} \left(\tau_o.R_o.h.d\theta\right).R_o = 2\tau_o\beta.R_o^2.h = \int_{0}^{2\pi}(\tau_i.R_i.h.d\theta).R_i = 2\tau_i\pi.R_i^2.h \quad (4)$$

A. FEM Model of a Pulley without Crack

Initially, the pulley has been designed as a solid circular disk with symmetric loading which is rotating at a certain angular velocity. As there is no asymmetric loading, the stress contour found to be symmetric about both the axes, and from FEM it is obvious that the center is the stress concentration zone. Later it has been designed for application of normal as well as shear loading on one side of its outer periphery which is the effect of the tension of the belt and torque due to the rotation. From the Von-Mises stress contour, the asymmetric contour can be shown. Later the shaft whole was considered, i.e. an annulus circular disk. The previous FEM modelling describes the highly stressed region adjacent to the inner periphery which is in touch with the motor shaft. As the center is the highly stressed region for both the cases, it is quite obvious that any crack will develop from the inner periphery.

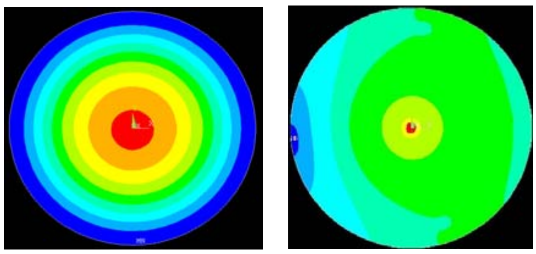

Fig. 2 Von-Mises contour plot for rotating circular disk with (a) symmetric loading, (b) asymmetric loading condition

B. FEM Model of a Pulley with a Single Crack

Further, the pulley has been designed with a crack on its inner periphery, i.e. emanating from the shaft hole. A parametric study was performed on the pulley in order to check the maximum stress concentration zone over the pulley surfaces with varying crack length and its orientation at a constant loading condition. A total of 289 cases of the problem are solved and plotted in MATLAB for each of the variations in crack length and crack orientation using finite element models of the disc under compression.

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:13, No:5, 2019

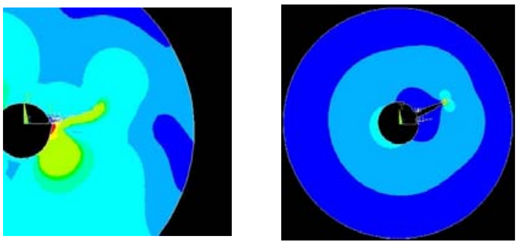

Fig. 3 Pre-existing crack with length and orientation varies

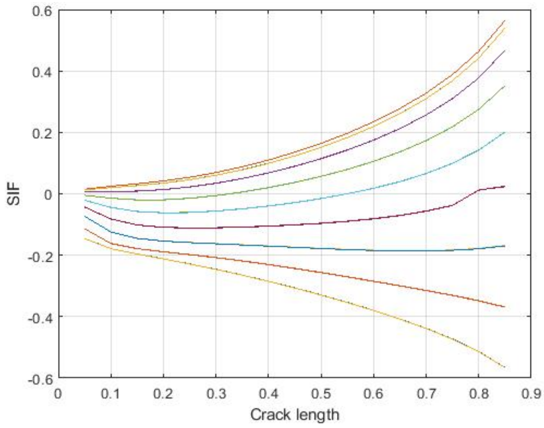

Fig. 4 Variation of K_I w.r.t crack length for different α

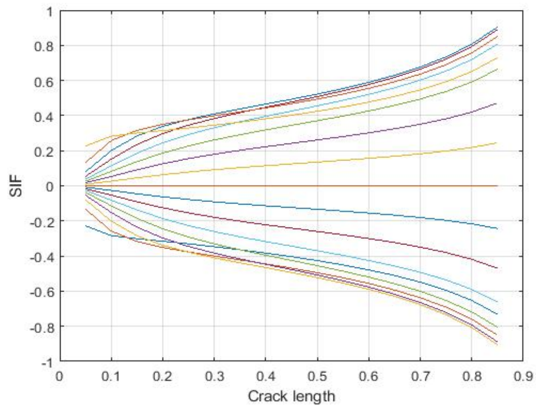

Fig. 5 Variation of K_{II} w.r.t crack length for different α

C. MATLAB Simulation

The results achieved from ANSYS (for both SIFs KI and KII) for a pulley during the parametric study with crack length vary from 0.05 to 0.85 and crack orientation vary from 100 to 1700 whereas the load on the outer periphery is kept constant are imported to MATLAB. Using MATLAB coding, we calculated the normalized KI and KII and using curve fitting toolbox we generated a 3D surface using those normalized SIFs' values. From the plotting, we can represent the surface (SIF vs a/w vs α) as a polynomial for both KI and KII where the accuracy is about 98% which can be directly read from MATLAB itself.

For K_I
$$f(x,y) = P_{00} + P_{10}x + P_{01}y + P_{20}x^2 + P_{11}xy + P_{02}y^2 + P_{30}x^3 + P_{21}x^2y + P_{12}xy^2 + P_{03}y^3 \quad (5)$$

P_{00} = -0.06346 , P_{10} = 0.1257, P_{01} = -0.006005, P_{20} = -0.1438, P_{11} = 0.03119, P_{02} = $6.945e^{-6}$, P_{30} = 0.004874, P_{21} = -0.03466, P_{12} = $1.542e^{-8}$, P_{03} = $2.18e^{-8}$

For K_{II}
$$f(x,y) = P_{00} + P_{10}x + P_{01}y + P_{20}x^2 + P_{11}xy + P_{02}y^2 + P_{30}x^3 + P_{21}x^2y + P_{12}xy^2 + P_{03}y^3 \quad (6)$$

$P_{00} = 0.1538$, $P_{10} = -3.844$, $P_{01} = -0.004594$, $P_{20} = 11.68$, $P_{11} = 0.01025$, $P_{02} = -5.56e^{-6}$, $P_{30} = -8.652$, $P_{21} = -6.42e^{-5}$, $P_{12} = 1.23e^{-5}$, $P_{03} = -5.279e^{-11}$

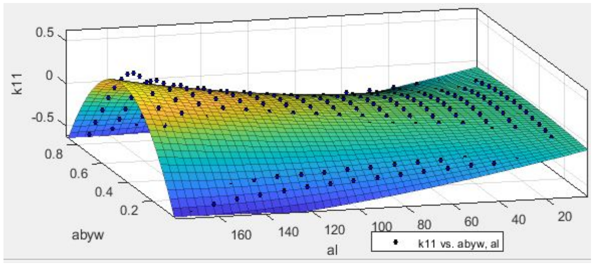

Fig. 6 Surface plot of K_I Vs a/w Vs α using MATLAB

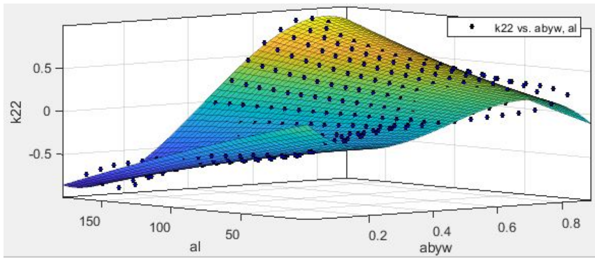

Fig. 7 Surface plot of K_{II} Vs a/w Vs α using MATLAB

D. FEM Model of a Pulley with Double Crack

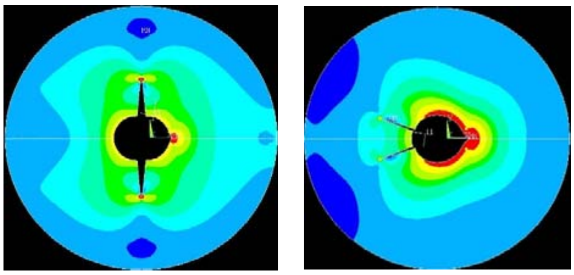

Fig. 8 Two identical cracks emerging from the shaft hole (a) vertical crack, (b) inclined orientation of both cracks

Here, a designed was done on a pulley with two identical edge cracks emerging from shaft hole using ANSYS and the value obtained for SIF at the crack tip for both cracks are plotted as a variation with respect to crack orientation. Total of 17 curves is shown (from 10^0 to 170^0) in each plot where each curve represent a variation of crack length. From the curves, one can observe that K_I is maximum at an orientation of 90^0 and slowly decreases. In the case of K_{II}, it is negative from 0 to 90^0. Exactly 0 at 90^0 and positive after 90^0 to 180^0. As K_{II} value is 0 when the cracks orientated vertically, The crack tip is under a pure Mode I fracture zone. For both crack tip, the value of SIF will be the same without consideration of body forces, i.e. gravity. If gravity will consider, we can relate it to a centre cracked Brazilan disk where the SIF value will be more for the lower crack

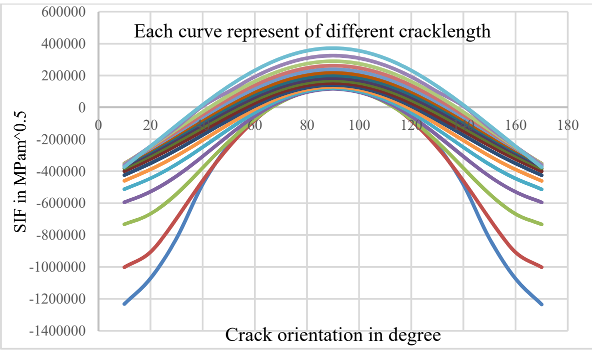

Fig. 9 Variation of K_I w.r.t crack orientation (α)

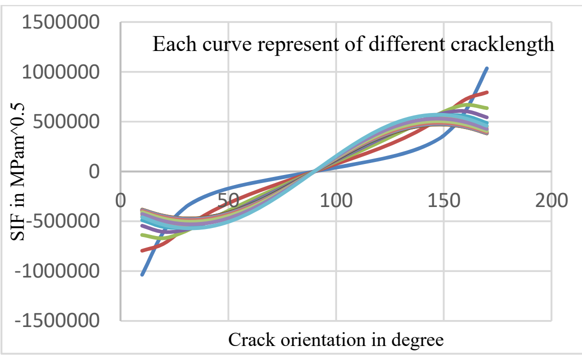

Fig. 10 Variation of K_{II} w.r.t crack orientation (α)

IV. Crack Kinking

As the growth of the crack takes place in the non-uniform stress field, the path of fracture is found to be curved. While studying brittle homogenous isotropic material, the path traced by the crack can be assumed as one which the local stress field at the tip is of pure Mode-I type (i.e. KII=0 at the crack tip).

In the historical chronology, one can find that research of fracture mechanics seeks the determination of stability condition of straight cracks extending in their own plane [10]. For a mixed mode, this is often not the mechanically most favourable path. In fact, a variety of methodologies have been employed to determine the direction of crack growth under mixed mode (tensile and shear) loading in homogeneous material.

The first calculations were those of Erdogan and Sih (1963) who developed the criterion that the crack will grow in the direction along the angle θ_c of maximum $\sigma_{\theta\theta}$ (hoop stress) linear the crack tip of the pre-existing (unkinked) crack in a linear elastic material. Using Maximum Tangential actress (MTS) criterion the optimal in-plane kink angle θ_c can be calculated from.

$$\sigma_{\theta\theta} = \frac{K_I}{4\sqrt{2\pi r}}\left(3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) - \frac{3K_{II}}{4\sqrt{2\pi r}}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) \quad (7)$$

Using, $\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \; and \; \frac{\partial}{\partial \theta}\left(\frac{\partial \sigma_{\theta\theta}}{\partial \theta}\right) < 0$ for maximum $\sigma_{\theta\theta}$

$$K_I \sin\theta_c + K_{II}\left(3\cos\theta_c - 1\right) = 0 \quad (8)$$

where K_I and K_{II} are the SIF at the crack tip for each case.

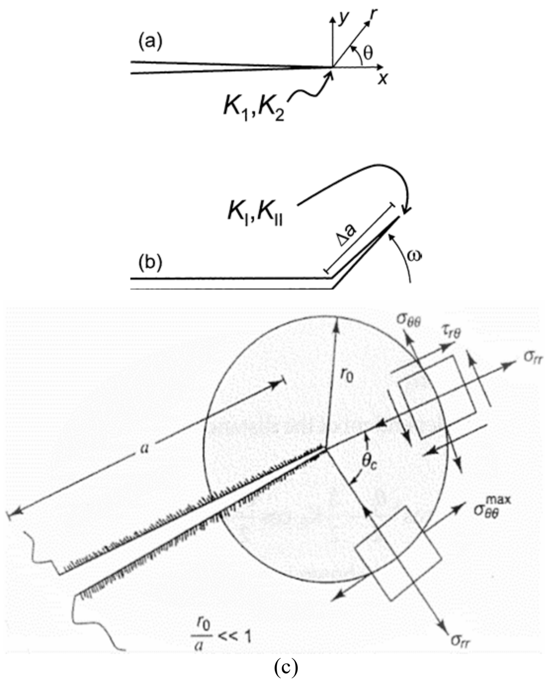

Fig. 11 Schematic of (a) straight crack, (b) kinked crack [11] and(c) crack kinking based on MTS criterion [12]

The design was considered on the pulley to predict the crack propagation path for a particular case of loading arc of $2\beta=140^0$ and crack orientated at 10^0 to the positive X-axis. Using Maximum Tangential Stress Criterion, the crack extension direction was established with the kink angle concept. Fig. 12 shows a crack subjected to Mode I and Mode II loadings. FE models showing the crack extension path for one existing crack is shown below.

Later, we have designed the pully for a loading arc of $2\beta=160^0$ where the initial crack was oriented at 70^0 with a crack length ratio of 0.1. Using ANSYS, we have the value for SIF K_I and K_{II} at the pre-existing crack tip. Using the same MTS criterion, we proceed for the kink angle and redesigned. In Fig. 13, we have the crack path which was initially mixed mode crack propagation and finally, the tip of the crack path converges to pure Mode-I.

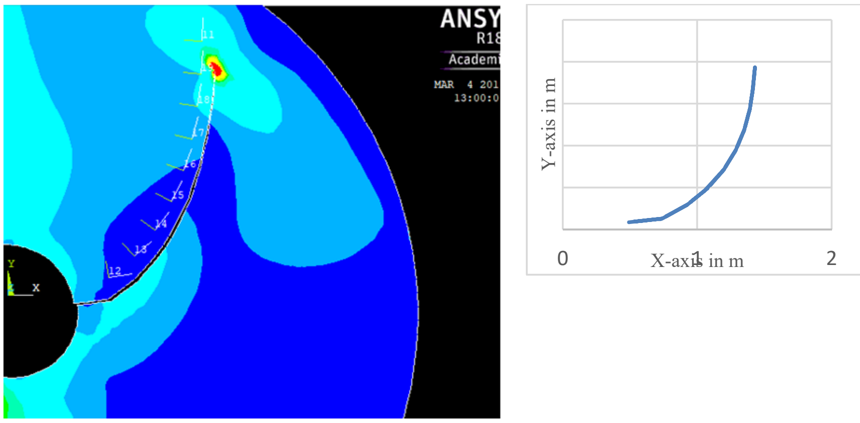

Fig. 12 Crack propagation path (a) using ANSYS, (b) using Excel

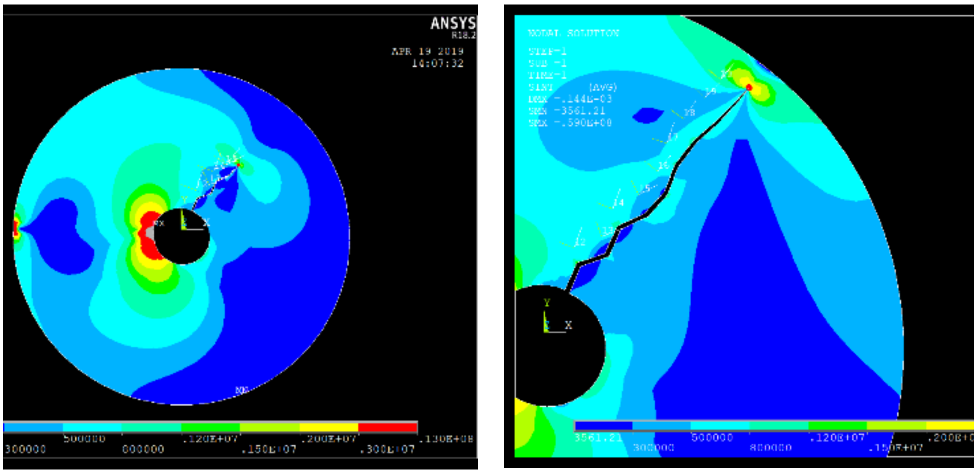

Fig. 13 Crack propagation using crack-kinking concept (a) at early stage, (b) just before physical separation

Further, our study is extended to the estimation of crack propagation path for different loading condition and with different parameters involves.

V. Conclusion

This paper mainly studied the fracture onset of an angled crack under far-field compressive loading. Based on our analysis, theoretical predictions of the kink angle and fracture stress agree well with our analysis using the numerical method. The asymmetric loading on the pulley surface due to belt tension and due to centrifugal force has been replaced by consistence nodal forces using FEM concept. From the design of the pulley as a solid disk, it is obvious that the center is the most stressed and critical region. Whereas considering the shaft whole i.e. pulley, the region near to inner radius is the stress concentration zone. From FE models of the solid disk shown here, it can be concluded that as normal tractions are symmetric about x-axis stresses generated due to it also maintains the symmetry, but it is not the case for application of normal along with shear traction. Considering a more realistic situation pulley with shaft hole, FE model and corresponding graphs shows the SIFs variation with the crack orientation and crack length at constant arc loading condition. From the resulting state of stress, stress concentration is observed near the constrained edge, which decays towards the outer edge on which normal and shear components of traction are distributed over a long arc of the boundary. As the surface is supporting the normal and tangential load (friction load) simultaneously at the same regions, stress state in the pulley is more intensified.

References

[1] Xu, Y. L. Stress intensity factors of a radial crack in a rotating compound disk. Engineering Fracture Mechanics, 44, (1993): 409–423.
[2] Bowie, O. L. and Neal, D. M. A modified mapping-collocation technique for accurate calculation of stress intensity factors. International Journal of Fracture Mechanics, 6, (1970): 199–206.
[3] Isida, M. Arbitrary loading problems of doubly symmetric regions containing a central crack, Engineering Fracture Mechanics, 7, (1975) 505–514.
[4] Pook, L. P., Berto, F., Campagnolo, A., and Lazzarin, P. Coupled fracture mode of a cracked disc under anti-plane loading. Engineering Fracture Mechanics, 128, (2014) 22–36.
[5] Dowson, D. "History of Tribology, 1979." 378-381.
[6] Beomkeun, Kim. "Finite element modeling and parametric study of an automotive V-belt pulley for durability improvement." *International Journal of Precision Engineering and Manufacturing* 16.7: 1517-24.
[7] Arnold, S. M., A. F. Saleeb, and N. R. Al-Zoubi. "Deformation and life

analysis of composite flywheel disk systems." Composites part B: engineering 33.6 (2002): 433-459.
[8] G. J. Nie, R. C. Batra, Stress analysis and material tailoring in isotropic linear thermoelastic incompressible functionally graded rotating discs of variable thickness, Composite Structures92(3)(2010)720–729.
[9] Blauel, J. G., J. Beinert, and M. Wenk. "Fracture-mechanics investigations of cracks in rotating disks." Experimental Mechanics 17.3 (1977): 106-112.
[10] Li, Xian-Fang, Kang Yong Lee, and Guo-Jin Tang. "Kink angle and fracture load for an angled crack subjected to far-field compressive loading." Engineering Fracture Mechanics 82 (2012): 172-184.
[11] T. L. Becker Jr., R. M. Cannon and R. O. Ritchie. “Finite crack kinking and T-stresses in functionally graded materials.” International Journal of Solid and Structures 38 (2001): 5545-5563.
[12] Elements of Fracture Mechanics by Prashant Kumar, Publisher McGraw Hill Education (India) Private Limited.