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Proton Radius Calculation for Muonic Hydrogen 2S-2P Transition Experiment

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Abstract**—**Scientists are making attempts to solve proton radius puzzle. In this paper, the calculated value matches the experiment observation within 0.1%, compared to those obtained from CODATA, and muonic hydrogen scattering experiments of 4%. The calculation is made based on the assumption that the muonic hydrogen system has (Ep – Eμ) energy state (or frequency mix state of $\nu_p - \nu_\mu$), which interacts resonantly with the incoming photon of energy 206.2949(32) meV. A similar calculation is also made for muonic deuterium 2S-2P transition experiment with an accuracy of 1% from the experimental observation. The paper has also explored the theoretical as well as experimentation advancements that have led towards the development of results with lesser deviations.

Keywords**—**2s-2p transition, muonic hydrogen, proton radius, scattering experiment, photon, quantum, Lamb shift.

I. Introduction

The proton radius puzzle has startled a number of scientists and has remained a puzzle for many years. This has led towards challenging theories of structural Physics through different experimentation [1]. The recent advancement in the domain of muonic hydrogen Lamb shift has opened doorways to further research while enhancing the debate of muonic hydrogen and electronic hydrogen [2]. Recent experiments based on laser spectroscopy of muonic hydrogen [3], and muonic deuterium [2], [3] has generated a puzzle for the proton radius.

In this paper, an approach is presented that will start from some basic physics laws and make the calculation based on the new physics arguments. The calculated results match the experimental measurement for muonic hydrogen with closer values for the experimental measurement of muonic hydrogen and deuterium.

II. Literature Review

Hydrogen is a unique atom as most of the theories incorporated with it are applied without any approximations. Hence, the discrepancies arising from the potential differences in theoretical and experimental domains have unveiled enhancement of theoretical as well as experimental accuracy. It also holds the potential for development of new insights.

Pertaining to the muonic hydrogen radius puzzle [4], a muon is 200 times heavier as compared to an electron [5]. As per particle physics' reigning theory, the proton must interact with muon and/or electron in the same manner. The proton shrinking in the vicinity of muon would lead towards the existence of a fundamental force that acts between muons and protons; however, it is not between the electrons and protons. Reference [5] has pointed out that the radius of proton is quite difficult to measure thereby resulting in making the measurement quite error-prone. Nevertheless, it becomes quite difficult during the circumstances when the proton is surrounded by the electron as witnessed in a hydrogen atom.

Hydrogen, being the smallest element in the periodic table, possesses a simplified structure having proton at the center. In 1933, Stern highlighted that the deviation of proton's magnetic moment's deviation from the Dirac relativistic theory [6]. It has also indicated that the proton instead of electron represents the structure. During 1947, 2S-2P Lamb shift measurements along with 1S-hyperfine splitting within hydrogen have shown deviation from the Dirac equation [7]-[9]. This development has lead towards the inception of QED (quantum electrodynamics). The deviation has also shifted focus towards the measurement of energy levels in hydrogen with greater accuracy and it has peaked with Hanch's frequency comb laser experimentations during the 1990s [7]-[9].

The QED advancements have allowed the prediction of proton radius and Rydberg constant along with a slow time-dependent variation of different fundamental constants. However, the energy levels of hydrogen are slightly modified due to the fact that the proton determines the size instead of an electron. Hence, for the precise prediction of these energy levels, it is quite necessary to have root-mean-square radius of the proton. As part of the historical advancements, the predictions of proton radius are quite commonly based on the scattering of electrons on the protons. During the experiment, a beam of electrons is scattered on the liquid hydrogen target. The hydrogen energy level prediction accuracy had become quite limited by the extracted results of electron-proton scattering experiment. Moreover, it has also limited the comparison between the measurements and theoretical framework. For an enhanced checking mechanism between the predicted values and observed measurements of hydrogen energy levels QED, it has become quite necessary for the precise determination of the proton radius of hydrogen [4].

Different groups of researchers have made attempts to predict the proton radius around 0.88 femtometers. However, Pohl's group made the advancements in 1998 for measurement of proton radius through muonic hydrogen; mainly because of the reason of muon's heft allows significantly fruitful measurement of proton size. After twelve years, another study has found out that the value obtained from the regular hydrogen has been found even more precise around 0.84 femtometers thereby falling quite short of the stated average

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from other researchers. Based on that, a better estimation has become possible using the measurement of Lamb shift in muonic hydrogen [1]. The puzzle is in the proton quite commonly known as the positively charged particle present in the atomic nuclei that is inherently the glucon and quark fuzzy ball. It is measured to be somewhat larger if it is orbited by the electron as compared to the proton orbited by the muon. A muon is similar to electron; however, it is around 207 times heavier than electron [5]. By the study, it has also been observed that the proton's finite size and its effect have been enhanced in muonic S states. A shift in S states has been observed mainly because of the reason that at the proton's location, the muonic wave function is non-zero.

A measure of Lamb shift in the muonic hydrogen has been first considered through effects of electron vacuum polarization [6], [7] together with Lamb shift in electronic, regular hydrogen [8] and subsequently dominated by self-energy of an electron [9]. Afterwards, the first successful muonic hydrogen observation through x-rays has been discovered [10]. Laser spectroscopy of muonic proton's 2S-2P transition requires the muonic proton to be in the metastable 2S state. For achieving that, a number of researchers and scientists have failed to observe the long-lived 2S muonic proton atoms whenever the muons are stopped in the molecular hydrogen gas condition [11]-[13]. The very first observation of the long-lived muonic proton atoms under 2S state has been the inception [14], [15] of the recently developed Lamb shift measurements [3].

From the laser spectroscopy experiments [16], where the transition frequencies for $2S^{F=01}{}_{1/2} - 2P^{F=1}{}_{3/2}$ and $2S^{F=1}{}_{1/2} - 2P^{F=2}{}_{3/2}$ in muonic hydrogen system were measured, the charge radius of the proton may be extracted to be:

$$R_e = 0.84087(39)\,fm$$

The value obtained is around 4% smaller values as compared to the CODATA radius [17] bearing 7σ discrepancy. The experiments have been performed at Switzerland in Paul Scherrer Institute. The experiment has begun with slowing muons and subsequently, sending them towards hydrogen target. The muons are then stopped and captured on the hydrogen atoms in a number of orbits. The muons cascaded down quite quickly and almost all the muons have cascaded to 1S state with few being stuck at 2S metastable state. The muons of interest are the ones that got stuck in the 2S state. With a short delay to allow the cascades to finish, the hydrogen atom is shined via tunable laser. With the certain frequency of tunable laser light, the 2P state transition is observed as shown in Fig. 1 (a).

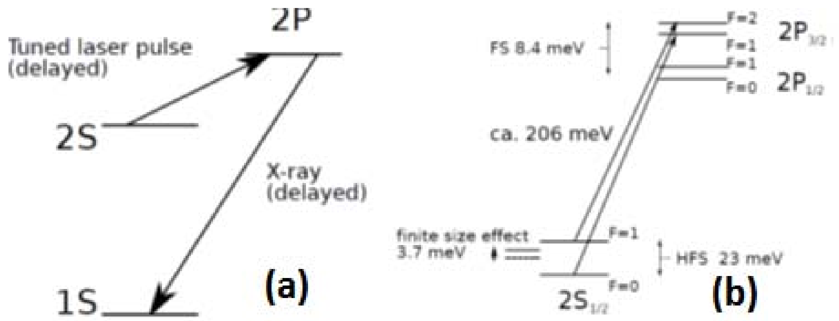

Fig. 1 Muonic Hydrogen Energy Levels [2]

2P state has decayed spontaneously to a ground state having x-ray emissions. The x-ray detector detects and defines the desired splitting energy. The measured two lines as shown in Fig. 1 (b) depict the 2P and 2S states quite elaborative. The subscripts besides 2S and 2P states indicate the angular momentum of electron spin and orbital motion. On the other hand, F shows orbital motion, electron and proton spin, and total angular momentum. Proton size does not have any impact on the P states; however, S states are affected by energy level on overall S state along with hyperfine splitting. The Lamb Shift can be defined as the removal of hyperfine splitting of $2S_{1/2}$-$SP_{1/2}$ [2].

Considering ordinary muonic hydrogen presents some discrepancies without any clear explanations; hence, studying other systems has become an essential part of the calculations. For that, deuterium comprising of a proton and a neutron has been selected. A muonic deuterium having muon orbiting the deuteron was used as a means of comparison between the deuteron radius and that of electron orbited regular deuterium. The deuteron radius puzzle has been explored to seek answers for proton's radius. During the experimentation, it has been found that the muons or electron orbiting the deuteron at a certain energy level has spent substantial time within the deuteron thereby reducing the attraction felt by the muon or electron. Hence, the more time muon or electron spends with the deuteron; it becomes less strongly bonded thereby making it easier to move to different energy levels. Muon, being a heavier particle than electron, orbits much closer to the deuteron. It makes the deuteron as the most precise probe for the radius. For estimating the proton radius, a laser is fired for the muonic deuterium gas thereby causing the muons to jump from the lower to a higher energy level that could not overlap with the nucleus. The energy needed by the muon for transitioning had revealed that the weak bonding force of muon when it was residing within the deuteron.

The measurement of the muonic proton's Lamb shift has been considered the backbone of the atomic spectroscopy but the recent advancements pertaining to laser technology and muon beams has made it possible. As part of the study [1], the energy difference between the states of $2S_{1/2}^{F=1}$ and $2S_{3/2}^{F=2}$ muonic proton atoms is determined via pulsed laser spectroscopy at wavelength around 6.01 μm. The transition has been chosen because it facilitates six allowable 2S-2P optical transitions. Apart from that, all the transitions observed are spectrally separated [3]. Besides that, spectroscopy of the exotic atoms can also lead towards the proton radius puzzle. Precise laser spectroscopy of positronium $(Ps \equiv e^{+}e^{-})$ [18], [19] or that of muonic $(Mu \equiv \mu^{+}e^{-})$ [20] nature can test the bound-state QED that are free from any effects of finite nuclear sizes. The improved spectroscopy pertaining to 1S-2S energy state transition along with the hyperfine splitting of the ground state is under development within the domains of positronium and muonic QED. Laser spectroscopy of the muonic deuterium also possesses tremendous potential to shed ample light on the proton radius puzzle [21]. Hence, a newly charged deuterium radius can be readily compared with the one having isotope shift considering 1S-2S energy state transition of electronic D and H [22].

III. Calculations for Muonic Hydrogen Proton Radius

From Einstein energy formula, the particle energy is

$$E = h\upsilon \tag{1}$$

For the muonic hydrogen system, there are two different frequencies associated with the two different kinds of particles. For muon,

$$\nu_{\mu} = E_{\mu} / h \tag{2}$$

and for proton,

$$\nu_{p} = E_{p} / h \tag{3}$$

The two frequencies will mix to generate different frequency or energy states locally for the system, E_p, E_μ, $(E_p - E_\mu)$, $(E_p + E_\mu)$. These energy states will coexist in the system. 2S-2P transition resonance will happen when the system energy at

$$E_{\mu p} = E_{p} - E_{\mu} \tag{4}$$

state interacting with the incoming photon of energy at 206.2949(32) meV. Hence,

$$\nu_{\mu p} = \nu_{p} - \nu_{\mu} \tag{5}$$

Assuming that the energy density (ρ) is considered to be the same for both Ep and Eμp, which can be expressed as:

$$E = \left(\frac{4}{3}\pi r^{3}\right)\rho C^{2} \tag{6}$$

Hence, the radius $r_{\mu p}$ can be calculated by:

$$r_{\mu p} = r_{p}\left[\frac{E_{p} - E_{\mu}}{E_{p}}\right]^{\frac{1}{3}} \tag{7}$$

where E_p=1.6726219×10^{-27} kg·C^2 [17], E_μ = 1.883532711×10^{-28} kg·C^2, r_p = 0.875877 fm from CODATA. By putting the values, following result is obtained:

$$r_{\mu p} \approx 0.84168\, fm$$

The accuracy of the calculated value $r_{\mu pcal}$ from the experiment measurement value $r_{\mu pmea}$ can be given by:

$$\eta_{\mu p} = \frac{(0.84087 - 0.84168)}{0.84087} = -9.63x10^{-4}$$

The calculated proton radius matches the experimental measurement value quite well, comparing to the accuracy obtained from CODATA value:

$$\eta_{\mu pCODATA} = \frac{(0.84087 - 0.875877)}{0.84087} \approx -4.16\%$$

Similarly, for the muonic deuterium system, 2S-2P transition resonance will happen when the system energy is in the state given by:

$$E_{\mu d} = E_{d} - E_{\mu} \tag{8}$$

Under these circumstances, there are two frequencies associated with the muon. Recall (3):

$$\nu_{\mu} = E_{\mu} / h$$

and for deuterium:

$$\nu_{d} = E_{d} / h \tag{9}$$

Hence, the radius $r_{\mu d}$ can be calculated using:

$$r_{\mu d} = r_{d}\left[\frac{E_{d} - E_{\mu}}{E_{d}}\right]^{\frac{1}{3}} \tag{10}$$

where E_d can be given by:

$$E_{d} = 2.014 \cdot 1.6726219x10^{-27} kg \cdot C^{2}$$

The radius $r_{\mu d}$ is given by:

$$r_{\mu d} = 2.10172\ fm$$

The accuracy of the calculated value $r_{\mu dcal}$ from the experiment measurement value $r_{dCODATA}$ is given by:

$$\eta_{\mu d} = \frac{(2.1256278 - 2.10172)}{2.1256278} = 0.011 \approx 1.1\%$$

The accuracy value obtained is close to that obtained from CODATA data:

$$\eta_{\mu dCODATA} = \frac{(2.1256278 - 2.142421)}{2.1256278} \approx 0.8\%$$

IV. Conclusion And Recommendations

Hydrogen can be used as a working platform for the development tool for even more strongly bonded systems. The advancements could lead towards enhancement in Rydberg's constant as the determination would lead towards linking fine structure constant, electron mass and the Planck constant. The hydrogen's proton radius could also serve to provide a benchmark in the domain of lattice quantum chromodynamics having an aim to model the proton along with its key constituents including the glucons and quarks.

Proton radius puzzle has fascinated the scientists and researchers for more than forty years. It has resulted in the development of various theoretical advancements through experimentations. The discovery of the anomalously large magnetic moment of the proton by Otto Stern [6] indicates that the proton is not an elementary particle; the journey of proton radius prediction has started. However, the muonic hydrogen has presented some discrepancies that have allowed researchers to focus on muonic deuterium thereby allowing better estimation because of relatively greater bonding force with the deuteron as compared to the electron. The study has presented proton radius calculation for the muonic hydrogen 2S-2P transition experiment and compared the calculated value with that of CODATA for analyzing the potential deviations. The calculated value significantly matches the experiment observation within 0.1%. The calculation is made based on the assumption that the muonic hydrogen system has (Ep – Eμ) energy state (or frequency mix state of $\nu_p - \nu_\mu$), also interacts resonantly with the incoming photon of energy 206.2949(32) meV.

Similar equation to (7) can be used for other calculations. For electron hydrogen system, since electron mass m_e or equivalent energy Ee is ~ 200 times smaller than that of muon, the proton radius calculation remains almost the same, or $r_{ep} \sim r_p$.

Positron and electron annihilation can be another interesting example. Since the masses or equivalent energies for positron and electron are the same, the calculated combined radius could be zero, or annihilation happened. Lastly, a calculation is made for muonic deuterium 2S-2P transition experiment with an accuracy of 1% from the experimental observation. For muonic hydrogen scattering experiments, the obtained scattering amplitude could have two extra peaks for matching proton radius of rμp- (Ep – Eμ), and rμp+ (Ep + Eμ). For muonic hydrogen laser spectroscopy experiments, it will be interesting to observe both transition resonances, those match the proton radius of rμp- (Ep – Eμ) and rμp+ (Ep + Eμ).

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