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Performance Analysis of M-Ary Pulse Position Modulation in Multihop Multiple Input Multiple Output-Free Space Optical System over Uncorrelated Gamma-Gamma Atmospheric Turbulence Channels

Hechmi Saidi, Noureddine Hamdi

***Abstract*—The performance of Decode and Forward (DF) multihop Free Space Optical (FSO) scheme deploying Multiple Input Multiple Output (MIMO) configuration under Gamma-Gamma (GG) statistical distribution, that adopts M-ary Pulse Position Modulation (MPPM) coding, is investigated. We have extracted exact and estimated values of Symbol-Error Rates (SERs) respectively. A closed form formula related to the Probability Density Function (PDF) is expressed for our designed system. Thanks to the use of DF multihop MIMO FSO configuration and MPPM signaling, atmospheric turbulence is combatted; hence the transmitted signal quality is improved.

***Keywords*—FSO, MIMO, MIMO, multihop, DF, SER, GG channel.

I. Introduction

THE ever increasing bandwidth requirement of present and emerging communication systems is the driving force behind research in optical communications (both fibre optics and optical wireless). Optical communication guarantees abundant bandwidth, which translates into high data rate capabilities. The huge bandwidth available on the fibre ring networks that form the backbone of modern communication technology is still not available to end users within the access network. This is mainly due to the bandwidth limitations of the copper wire based technologies that, in most places, connect the end users to the backbone. This places a restrictive limit on the data rate/download rate available to end users. This problem is termed "ccess network bottleneck" and a number of approaches, including fibre to the home (FTTH), Digital subscriber loop (DSL) or cable modems, power-line communication, local multipoint distribution service (LMDS), ultra-wide band technologies (UWB) and terrestrial FSO, have been proposed to tackle this bottleneck. Free space optical communication or better still, laser communication is an agelong technology that entails the transmission of information laden optical radiation through the atmosphere from one point to the other. Optical signal traversing the atmospheric channel suffers attenuation due to scattering and absorption of the signal by aerosols, fog, atmospheric gases and precipitation. In the event of thick fog, the atmospheric attenuation coefficient exceeds 100 dB/km, this potentially limits the achievable FSO link length to less than 1 kilometre. But even in clear atmospheric conditions when signal absorption and scattering are less severe with a combined attenuation coefficient of less than 1 dB/km, the atmospheric turbulence significantly impairs the achievable error rate, the outage probability and the available link margin of a terrestrial FSO communication system. Compared to RF communication path, FSO offers high capacity and free bandwith [1]. It has lately been used as first-mile solution for access background [2]. Refractive index's fluctuations were investigated in [3]. GG turbulence distribution matchs well weak and strong turbulence circumstances as in $\{$[4]-[5]$\}$. Other impairments like photons' absorption and scattering were taken into consideration by [6].

A. Related Works

MIMO-FSO transmission network with MPPM signaling following not only log-normal but also Rayleigh law for fading problem has been studied in [7], the BER performance of the above network using OOK under log-normal distribution for fading turbulence problem in independent and dependent channels has been investigated in [8]. Outage probability analysis of a similar network deploying Selection Combining(SC) method has been studied in [9], whereas this network's closed-form bounds using PPM coding for many other fading models has been derived in [10] . Moreover, exponentiated Weibull fading model has been specially investigated in [11]. By $\{$[12],[13]$\}$, GG distribution had well described channels turbulence even if the MIMO channels' GG computation has been approximated through both gamma distribution and GG one. The outage probability's closed-form formula for above system assuming BPPM has been derived in [14]. The GG collected signals' result combined summation was nearly modeled by the $\alpha - \mu$ distribution [15]. DF protocol

Hechmi Saidi is with the University of Tunis El Manar, National Engineering School of Tunis, Communication System laboratory, Tunisia (e-mail: hechmi.saidi@rns.tn).
Noureddine Hamdi is with the University of Tunis El Manar, National Engineering School of Tunis, Communication System laboratory, Tunisia (e-mail: noureddine.hamdi@ept.rnu.tn).

and MPPM signaling are considered in [16] without taking into account misalignment fading. In Quantize and Forward(QF) protocol, the transmitted symbol's log-likelihood is roughly approximated and quantized [17]. FSO is transparent to traffic type and protocol, making its integration into the access network almost hitch free. To further enhance its seamless integration, an understanding of the capabilities, limitations and performance analysis of an optical wireless system that is based on some of the existing access network traffic types, modulation techniques, is therefore highly desirable.

B. Contributions

In this work, we are interested in dealing with a MIMO FSO DF multihop system over GG tubulence uncorrelated channels and using MPPM modulation technique. We have extracted exact and estimated values of SERs respectively. The PDF's closed form formula is expressed for our designed system. This manuscript is arranged as follow: The scheme model is introduced in Section II. We also present in this section its statistical model. In Section III, we express the end to end BER. Section IV reveals simulations results and their discussion. Finally, V concludes the paper.

II. System Model

A model of DF multihop MIMO FSO communication system is adopted.

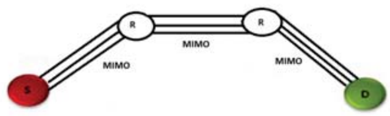

Fig. 1 DF multihop MIMO FSO network with N optical sources and N optical detectors.

A. GG Channel Distribution Overview

The GG distribution's marginal PDF is introduced like in [4]:

$$g(K_s)=\frac{2(xy)^{\frac{x+y}{2}}}{\Omega\Gamma(x)\Gamma(y)}\left(\frac{K_s}{\Omega}\right)^{\frac{x+y}{2}-1}B_{x-y}\left(2\sqrt{\frac{xyK_s}{\Omega}}\right), \quad where K_s\geq 0, \tag{1}$$

and knowing that $\Gamma(.)$ represents gamma function, $B_c(.)$ describes c_{th} order case of modified Bessel function related to second kind, K_s illustrates average received information photon count for each signal slot and $\Omega = E\{K_s\}$ is K_s's mean. Here $E\{.\}$ constitutes the expected value. The parameters x and y are GG distribution's shape parameters, associated to refractive as well as diffractive turbulence effects which are related to large-scale as well as small-scale eddies effective number. $\chi^2_{SC} := var\{K_s\}/E\{K_s^2\}$ denotes scintillation index related to x and y as:

$$\chi^2_{SC}=\frac{1}{x}+\frac{1}{y}+\frac{1}{xy} \tag{2}$$

Concerning weak towards moderate turbulence, χ^2_{SC} belongs to [0, 0.75], whereas $\chi^2_{SC}>0.75$ case of strong turbulence [18].

B. MIMO-FSO Model

The MIMO-FSO link includes N transmit and N receive apertures as shown in Fig. 1. The MPPM symbol period is composed of Q intervals (time slots). Besides, an optical power seems solely transmitted during w times slots, where w belongs to {1,2,..., $\frac{Q}{2}$}. We transmit each MPPM frame through all N transmitters. Using equal gain combining, the average received photon count for every signal slot is described as

$$K_{S_{on}}=\sum_{m=1}^{N}\sum_{n=1}^{N}K_{S_{mn}}+K_b \tag{3}$$

where $K_{s_{mn}}$ represents average received signal photon count concerning mth source as well as nth receiver, K_b denotes average received photon count for every slot because of background noise. Each Q summation of N^2 GG random variables has a PDF as:

$$Z^j=\sum_{m=1}^{N}\sum_{n=1}^{N}Y^j_{mn},\ j\in\{1,2,...,Q\} \tag{4}$$

where Y^j_{mn} denotes the detected photon count in slot$j\epsilon\{1,2,...,Q\}$ having both n and $m\epsilon\{1,2,...,N\}$.

C. MIMO Configuration Case of Independent Channels

If the distance from m laser source to n photo detector, where {m,n} belong to {1,2,...,N}, $d_{mn}>\rho_c$ where ρ_c denotes correlation distance, every channel between transmitter and receiver is judged independent. ρ_c is nearly equal to $\sqrt{\lambda L}$ case of weak turbulence, having a wavelength as λ and a link distance as L like in [3]. Therefore, the PDF of Z defined as N^2 independent GG random channels variables' summation might be expressed using $\alpha-\mu$ distribution: $\{$[15],[3],[19]$\}$:

$$f_Z(z)=\frac{\alpha\mu^{\mu}Z^{\alpha\mu-1}}{\hat{Z}^{\alpha\mu}\Gamma(\mu)}exp\left(-\mu\frac{Z^{\alpha}}{\hat{Z}\alpha}\right) \tag{5}$$

with $\alpha,\mu>0$ as distribution parameters and $\hat{Z}\ =^{\alpha}\ \sqrt{E\{Z^{\alpha}\}}$ denotes α-basic mean value. Some moment-based factor estimators are employed to evaluate α, μ and $\hat{z}$, in$\{$[15],[3],[19]$\}$, in solving following nonlinear equations:

$$\frac{\Gamma^2(\mu+\frac{1}{\alpha})}{\Gamma(\mu)\Gamma(\mu+\frac{2}{\alpha})-\Gamma^2(\mu+\frac{1}{\alpha})}=\frac{E^2\{Z\}}{E\{Z^2\}-E^2\{Z\}}$$

$$\frac{\Gamma^2(\mu+\frac{2}{\alpha})}{\Gamma(\mu)\Gamma(\mu+\frac{4}{\alpha})-\Gamma^2(\mu+\frac{2}{\alpha})}=\frac{E^2\{Z^2\}}{E\{Z^4\}-E^2\{Z^2\}} \tag{6}$$

$$\hat{Z}=\frac{\mu^{\frac{1}{\alpha}}\Gamma(\mu)E\{Z\}}{\Gamma(\mu+\frac{1}{\alpha})}$$

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By multinomial expansion technique's exploitation, the expected moments $E\{Z\}, E\{Z^2\}$, and $E\{Z^4\}$ are calculated like:

$$E(Z^\nu) = \sum_{j_1=0}^{\nu} \sum_{j_2=0}^{j_1} \dots \sum_{j_{N^2-1}=0}^{j_{N^2-2}} \binom{\nu}{j_1} \binom{j_1}{j_2} \dots \binom{j_{N^2-2}}{j_{N^2-1}} \times E\{L_1^{\nu-j_1}\} E\{L_2^{j_1-j_2}\} \dots E\{L_{N^2}^{j_{N^2-1}}\} \quad (7)$$

where $L_i = K_{s_{mn}}$ for $i = ((m-1)N)+n$ while the integer $\nu > 0$. The vth moments of L_i are given by:

$$E\{L_i^\nu\} = \left(\frac{xy}{\Omega}\right)^{-\nu} \frac{\Gamma(x+\nu) + \Gamma(y+\nu)}{\Gamma(x)\Gamma(y)} \quad (8)$$

III. Performance Analysis

A. MIMO-FSO Link

1) SER: Let us define the received vector

$$Z = (Z^1, Z^2, ..., Z^Q) \quad (9)$$

, where the jth entry in this vector Z^j represents a summation over slot j as introduced in (4). In addition to these Q slots, we have w ON slots for carrying signal while $Q-w$ OFF slots for carrying no signal. Let

$$P_{MIN} = p_1(z_{min})^m (1 - P_1(z_{min}))^{w-m} \quad (10)$$

knowing that z_{min} represents lowest photon amount per symbol signal time slots. Whereas, $p_1(.)$ represents probability of photon count for signal time slots. $P_1(.)$ denotes its cumulative statistical distribution. If a number of one or else more among OFF slots has count >= to that case of ON slots , we have a transmission error . The SER case of MPPM coding in absence of turbulent atmosphere path is introduced as in [20]:

$$SER = \sum_{z_{min}=0}^{\infty} \sum_{l=1}^{Q-w} \sum_{m=1}^{w} \binom{w}{m} \binom{Q-w}{l} \times P_{MIN} P_0(z_{min}-1)^{Q-w-l} [(1-P_0(z_{min}))^l + p_0(z_{min})^l \left(1 - \frac{1}{\binom{l+m}{m}}\right)] \quad (11)$$

Where $p_0(.)$ represents probability of photon count for non-signal time slots. Whereas, $P_0(.)$ represents its cumulative statistical distribution. Adopting Poisson distribution, the above probabilities are described as:

$$p_0(k) = \frac{K_b^k}{k!} e^{-K_b}, p_1(k) = \frac{(z+K_b)^k}{k!} e^{-(z+K_b)}$$
$$P_0(k) = \sum_{j=0}^{k} \frac{K_b^j}{j!} e^{-K_b}, P_1(k) = \sum_{j=0}^{k} \frac{(z+K_b)^j}{j!} e^{-(z+K_b)} \quad (12)$$

having $k\epsilon\{0, 1, 2, ...\}$, z representing the average received signal photon amount per ON time slot. Regular SER might be determined as a result of averaging (11) considering z value. Assuming that solely P_{MIN} in (11) depends on our channel statistical distribution, we can obtain average SER from replacing P_{MIN} by its average $P_2(z_{min})$ in (11):

$$P_2(z_{min}) = \int_0^\infty p_1(z_{min})^m (1 - P_1(z_{min}))^{w-m} f_Z(z) dz \quad (13)$$

which after algebraic manipulations can be expressed as:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \binom{j+mz_{min}}{B} r(j) \times \frac{e^{-wk_b} k_b^{j+mz_{min}-B}}{z_{min}!^m} \int_0^\infty z^B e^{-wz} f_Z(z) dz \quad (14)$$

where r(j), having each integer $j \geq (w-m)(z_{min}+1)$, is described like
$r(j) = \sum_{(S_1,S_2,...,S_{w-m}) \in \chi(j)} \frac{1}{S_1!S_2!...S_{w-m}!}$
over the set of vectors $\chi(j)$, where
$\chi(j) = (S_1, S_2, ..., S_{w-m}) \in N^{w-m} : \sum_{i=1}^{w-m} S_i = j$ and $\forall \ell \in \{1, 2, ..., w-m\}$, $z_{min}+1 \leq S_\ell \leq j-(w-m-1)(z_{min}+1)$

2) Exact SER Case of MIMO Independent Channels: Having (5) as well as (14), we can obtain:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \binom{j+mz_{min}}{B} \times r(j) \frac{\alpha\mu^\mu e^{-wk_b} k_b^{j+mz_{min}-B}}{z_{min}!^m \Gamma(\mu) \hat{z}^{\alpha\mu}} \int_0^\infty z^{\alpha\mu+B-1} e^{-wz} e^{-\frac{\mu z^\alpha}{\hat{z}^\alpha}} dz \quad (15)$$

Adopting Meijer-G mathematical function [21], having $\alpha = t/v\epsilon\ Q$ and using basic integrations formulae as in [21] , we can get:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \left(\frac{\binom{j+mz_{min}}{B} r(j)\alpha\mu^\mu \nu^{0.5} t^{\alpha\mu+B-\frac{1}{2}}}{e^{wk_b} z_{min}!^m \Gamma(\mu) \hat{z}^{\alpha\mu} (2\Pi)^{\frac{t+\nu-2}{2}}} k_b^{j+mz_{min}-B} \times w^{-(\alpha\mu+B)} G_{t,\nu}^{\nu,t} \left[\frac{\mu}{\hat{z}^\alpha \nu}{}^\nu \frac{t}{w}{}^t \,\middle|\, \begin{pmatrix} \frac{1-\alpha\mu-B}{t}, ..., \frac{t-\alpha\mu-B}{t} \\ 0, \frac{1}{\nu}, ..., \frac{\nu-1}{\nu} \end{pmatrix} \right] \right) \quad (16)$$

3) Approximate SER: Through Gauss-Laguerre method for quadrature law [18]. We can rewrite (13) like:

$$P_2(z_{min}) = \frac{e^{(-wk_b)}}{z_{min}!^m} \int_0^\infty e^{(}-wz)(z+k_b)^{mz_{min}} \times [e^{(}z+k_b) - \sum_{B=0}^{j+mz_{min}} \frac{(z+k_b)^j}{j!}]^{w-m} f_z(z) dz \quad (17)$$

Based on Gauss-Laguerre method for quadrature rule, this integration might be estimated:

$$P_2(z_{min}) \approx \sum_{i=1}^{c} \wedge_i \frac{e^{(-wk_b)}}{z_{min}!^m} (\vee_i + k_b)^{mz_{min}} \times [e^{(}\vee_i + k_b) - \sum_{j=0}^{z_{min}} \frac{(\vee_i+k_b)^j}{j!}]^{w-m} f_z(\vee_i) dz \quad (18)$$

where $\vee_i$ represents i-th basis for the Laguerre polynomial $L_c(x)$ where $c > 1$. $\wedge_i$ denotes the corresponding weighting coefficient.

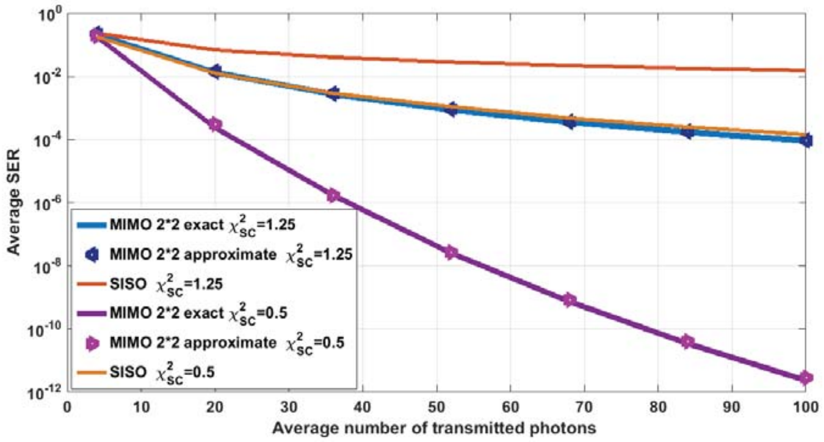

Fig. 2 Comparison of MIMO schemes FSO link by changing the number of transmitters and receivers under strong and moderate turbulence

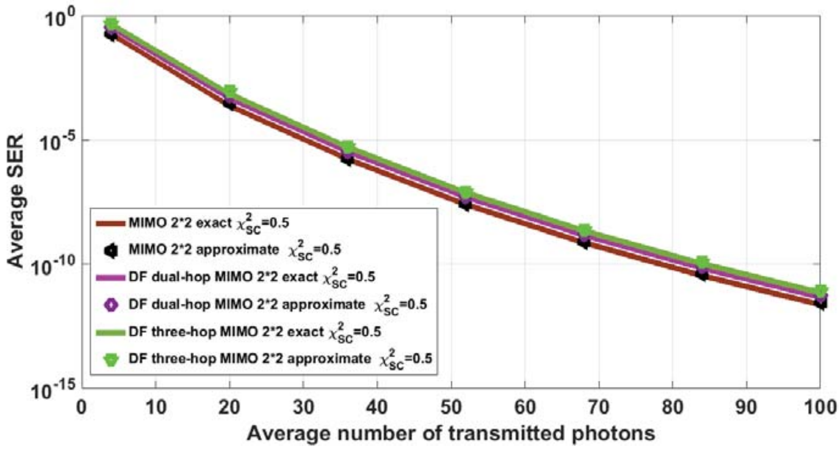

Fig. 3 Comparison between N hops DF relaying MIMO-FSO system under moderate turbulence

B. BER for Serial Relaying DF-DF Scheme

The SEP for the three-hop DF relaying system is given by [16]:

$$P_t^e = 1-(1-P^e_{SR_1})*(1-P^e_{R_1R_2})*(1-P^e_{R_2D}) = 1-(1-P_2(z_{min}))^3$$

Once the MPPM symbol is detected, it is mapped to a string of $log_2(M)$ bits through the inverse of the encoding mapping. There are $M/2$ symbol errors that will produce an error in a given bit in the string, and there are $(M-1)$ unique symbol errors. Thus, assuming all symbol errors are equally, the resulting BER can be expressed as

$$BER = \frac{M}{2\times(M-1)}P_e \tag{19}$$

IV. Simulations

In this section, we numerically investigate the SER of MIMO-FSO system under GG turbulence. Fig. 2 showed the average SERs versus average number of transmitted photons of MIMO-FSO(with $N=2$) and SISO-FSO links, both adopting MPPM techniques with $(Q,w) = (4,2)$. Two levels of turbulence intensity were assumed, strong turbulent channels with parameters($x=8, y=1, \chi^2_{SC}=1.25$)and moderate turbulent channels with parameters($x=5, y=4, \chi^2_{SC}=0.5$). Same background noise of $K_b=1$ photon per channel was assumed as well. In fact, when SNR(the number of transmitted photons) was growing, SER was decreasing. Both exact and approximate expressions were depicted in the Fig. 2, they were approximately the same. It was also clear from the Fig. 2 that there was a significant improvement in the SER when using MIMO systems when compare to that of the SISO systems. For example for 2I2O for moderate turbulence, we had a numerical $SER=10^{-11}$ when the average number of transmitted photons $SNR=100$ while for SISO we had $SER=10^{-4}$ under same turbulence and fo the same SNR.

As mentioned above, when the average number of transmitted photons(SNR) was growing, the SER was deacreasing. Fig. 3 showed also that when the number of hops increased which means that the distance between resource and destination was growing, the average SER seemed to be little worse but didn't affect so much the whole system performance. Thanks to multihop configuration, we could send data for far distances. For example for single hop for moderate turbulence, we had $SER = 10^{-11}$ when the average number of transmitted photons $SNR = 100$ while for three-hop we had $10^{-11} < SER < 10^{-10}$ which meaned that $SER_{3hops} \approx \mathrm{SER}_{1hop}$ under same turbulence and for the same SNR. Indeed we gained 2Km (the distance between two nodes = 1Km) more having almost the same quality of service i.e. the same data rate.

V. Conclusion

As a conclusion, we have considered a multihop MIMO-FSO communication, investigated DF relaying and compared between obtained schemes networks performances. For performance analysis, we have calculated the average symbol error probability function of signal-to-noise ratio.The biggest challenge for us was the choice of GG channel for FSO link, multiple transmitters and multiple receivers (MIMO) and MPPM to improve the whole system's performance which made our calculation harder. Our expressions were used to investigate the DF multihop MIMO system performance which were much better than simple FSO network.

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