FEM Study of Different Methods of Fiber Reinforcement Polymer Strengthening of a High Strength Concrete Beam-Column Connection

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Abstract—In reinforced concrete (RC) structures, beam-column connection region has a considerable effect on the behavior of structures. Using fiber reinforcement polymer (FRP) for the strengthening of connections in RC structures can be one of the solutions to retrofitting this zone which result in the enhanced behavior of structure. In this paper, these changes in behavior by using FRP for high strength concrete beam-column connection have been studied by finite element modeling. The concrete damage plasticity (CDP) model has been used to analyze the RC. The results illustrated a considerable development in load-bearing capacity but also a noticeable reduction in ductility. The study also assesses these qualities for several modes of strengthening and suggests the most effective mode of strengthening. Using FRP in flexural zone and FRP with 45-degree oriented fibers in shear zone of joint showed the most significant change in behavior.

Keywords—High strength concrete, beam-column connection, FRP, FEM.

I. INTRODUCTION

BEAM-COLUMN connections are the most important region of RC structures which has a tremendous effect on the behavior of RC Frames. When the structure is under severe earthquake load, it is possible that the column at the top and the bottom of the joint is under reverse moments. This will cause a high shear force which has a higher amount than the shear in the adjacent beam and column. This causes failure in the connection zone. Because of the complex behavior of the connection zone, there is no analytical model available to study beam-column connection, directly. Therefore, codes do not imply connections and there are just some recommendations for confinement, prevention of shear failure and suitable anchorage length to prevent failure due to bars slip.

An appropriate ductile behavior of connections in earthquake influences energy absorption of the structure. The utilization of high strength concrete (HSC), which has more brittle behavior than normal strength concrete, has been increased in high-rise buildings. This brittleness of HSC can easily affect the ductility of the structure. Utilization of FRP can also have effects on energy absorption and ductility of joints. In this research, the behavior of HSC connections strengthened with FRP has been studied, and comparisons in various cases have been presented.

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The past researches illustrate an enhancement in the behavior of beam-column connections by strengthening them. In 2003, Antonopoulos et al. [1] studied experimentally on the behavior of strengthened joint under seismic loading and reports showed a rise in load carrying capacity and energy absorption. Kim and LaFave [2] studied 139 experimental models to assess some important parameters as like confinement and column axial pressure in 2007. Parvin and Granata [3] in 2000 investigated the effect of FRP on concrete joints especially in exterior joints. Their studies show increasing flexural strength and also decreasing in ductility. In 2010, Parvin et al. [4] studied rehabilitation of concrete frame joints with inadequate shear and anchorage details. The shear failure was the mechanism of failure for the specimens before strengthening, but after using carbon fiber reinforced polymer (CFRP), delamination of FRP layer happened first. There was a considerable rise in load-carrying capacity.

In previous researches, mostly the behavior of joints with normal strength concrete has been studied, and there are no many works on effects of strengthening of high strength concrete beam-column connections by FRP. This study is based on finite element modeling of high strength concrete and FRP. For analyzing of RC, CDP model has been used which is the stronger method than Concrete Smeared Cracking model and Concrete Brittle Cracking model.

II. SPECIMENS DETAILS

In the present study, in order to have a comparison with an experimental study, the initial model is adapted from the study of Parvin et al. [4], which is an exterior beam-column connection with inadequate shear reinforcement. This model is displayed in Fig. 1.

In all models, the joint is the same that we consider as the basic model and name it as JP. The name of different models represents concrete compressive strength and type of FRP that is used for strengthening of each model. Letters C and G represent CFRP and Glass Fiber Reinforced Polymer (GFRP), respectively.

III. MATERIAL PROPERTIES

A. High Strength Concrete

In this study, various amounts of compressive strength for concrete have been used for comparing the effects of strengthening in different compressive strengths. In most concrete codes, the strengths which are more than 50 MPa, are

categorized as HSC. For this class of concrete, there are some stress-strain equations which are needed for modeling of the non-linear behavior of concrete. The offered relation for stress-strain by CEB-FIP2010 [5] is:

$$f = -f_c'(\frac{k\eta - \eta^2}{1+(k-2)\eta}) \qquad |\varepsilon|' < |\varepsilon_{c\,\lim}|$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{cl}} \qquad k = \frac{E_{ci}}{E_{cl}} \qquad E_{cl} = \frac{f_c'}{\varepsilon_{cl}} \qquad (1)$$

where ε_{cl} is the strain related to the maximum stress, and f_c' is the compressive strength. The concrete in tension has linear behavior before reaching the tensile strength and the fracture happens. In this paper, the CEB-FIP2010 relation [5] for tension strength has been used which, for HSC, is:

$$f_{ctm} = 2.12 \ln(1 + 0.1(f_{ck} + 8)) \qquad (2)$$

And finally, the elasticity module formula according to CEB-FIP equation is:

$$E_{ci} = E_{co}\alpha_E (\frac{f_{ck} + 8}{10})^{\frac{1}{3}} \qquad (MPa) \qquad (3)$$

It should be considered that the mentioned equations are related to HSC or concrete with compressive strength higher than 50 MPa.
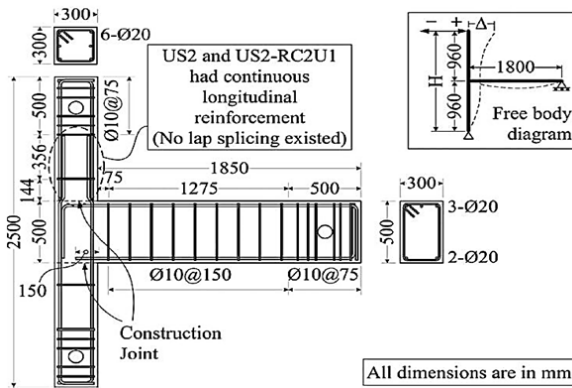

Fig. 1 Detail of base model joint

B. FRP

The two types of FRP used for the analyzed models here are CFRP and GFRP. High tensile strength and also linear stress-strain behavior before failure are the most noticeable properties of carbon and glass fibers. The last-mentioned behavior results in a brittle fracture of FRP which can be considered as a weakness compared to an elastoplastic material as steel.

The failure mechanisms that can happen in a strengthened member are: crushing of concrete before yielding in steel, FRP rupture after yielding of steel bars, crushing of concrete after

yielding of steel bars, debonding of FRP and concrete splitting under FRP layer [6]. The mode of failure for each model will be determined after analyzing.

The strengthening of a member by FRP can be performed by various approaches. Flexural strengthening and shear strengthening are the most useful methods for strengthening of a RC joint. All the relations and formulas used in this research for FRP such as delamination criteria are according to ACI 440.2R-08 [7]. Mechanical properties for GFRP and CFRP used for modeling in this study are presented in Table I [6].

TABLE I
MECHANICAL PROPERTIES OF CFRP AND GFRP

Properties	direction	GFRP	CFRP
	E_x	21000	62000
Elastic Modulus (MPa)	E_y	7000	4800
	E_z	7000	4800
	ν_{xy}	0.26	0.22
Poisson's ratio	ν_{xz}	0.26	0.22
	ν_{yz}	0.3	0.3
Tensile Strength (MPa)	T_{xx}	600	958
	G_{xy}	1520	3270
Shear Modulus (MPa)	G_{xz}	1520	3270
	G_{yz}	2650	1860
Thickness (mm)		1	1

C. Reinforcement Steel

The steel used for reinforcing of joints for longitudinal and transverse bars is class A615 according to ASTM. For this class, the yielding stress is 420 MPa, and ultimate stress is 620 MPa. The elasticity module and Poisson's ratio are 200 GPa and 0.3, respectively [8].

IV. MODELING METHOD

For FE modeling of RC, there are three main methods in a finite element software such as ABAQUS which are: concrete smeared cracking, concrete brittle cracking and CDP. From the three methods available in ABAQUS for analyzing RC, the CDP has more accurate results compared to other methods because of considering more parameters to obtain closer results to real models. In this study, the CDP method has been used [9] and [10].

The behavior of steel is considered as biaxial, and for FRP, it is linear until the fracture happens, and for concrete, as mentioned, it is nonlinear for compression and linear for tension, using CEB-FIP2010 equations. FRP behavior is introduced as an orthotropic material to the software. The solid element for concrete, truss element for steel rebar and also shell element for FRP has been employed [11].

The rotation-moment diagram is the result that can be analyzed for studying the behavior of these models and compare the load-carrying capacity. To approach that result, calculating the moment and rotation is needed. In the software, it is available to highlight which output is needed to avoid unnecessary results and to reduce analysis time. To approach this diagram, the rotation and related moment step by step are needed. To find the rotation of joint in every step, we need displacement of three points. We have one point on beam (A),

one on column (B) and the point (O) is exactly center of joint. Using these three points, we have two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. These two vectors are defined for beam and column, respectively. After loading, from the first step, beam and column start to rotate and it leads to change in defined vectors. So, for each step, two different vectors are generated that we call these vectors $\overrightarrow{OA'}$ and $\overrightarrow{OB'}$. Firstly $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular, but when loading or controlled displacement at the end point of beam starts, the angle between these two vectors is not 90 degree anymore and it shows the rotation of join. Using the inner product, we can find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OA'}$ and also $\overrightarrow{OB}$ and $\overrightarrow{OB'}$. And the difference between these two angles can present the rotation of the joint for every step. There will be a related support reaction for each step that can show the moment for each step (Fig. 2).
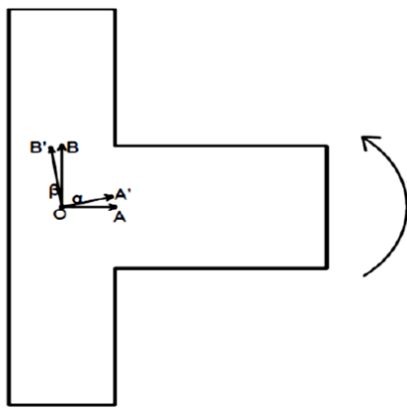

Fig. 2 Using inner product to find rotation of joint

$$\cos \alpha = \frac{\overrightarrow{OA} . \overrightarrow{OA'}}{|OA|.|OA'|}$$

$$\cos \beta = \frac{\overrightarrow{OB} . \overrightarrow{OB'}}{|OB|.|OB'|} \qquad (4)$$

$$\theta = \beta - \alpha$$

In this study to verify FE modeling, an experimental flexural strengthened beam studied by Barros and Fortes [12] has been used. A comparison between FEM and experimental results is displayed in Fig. 3. This figure presents a reasonable result for FE modeling of strengthened RC, and the observed differences are less than 10 percent for moment.

V. STRENGTHENING OF CONNECTION BY *GFRP* AND *CFRP*

A comparison between non-strengthened joint and strengthened joint by CFRP and GFRP has been aimed for the first modeling in this study. The JP80-O is a beam-column connection which has not any FRP for strengthening it. JP80-CA and JP80-GA are the joints which are strengthened by CFRP and GFRP, respectively.

In this study, for all models, there are several crucial points that have been specified in the moment-rotation diagram, the moment that first cracking in concrete appears, yielding point

and point that delamination of FRP happens. These points help us to assess the mechanism of failure.
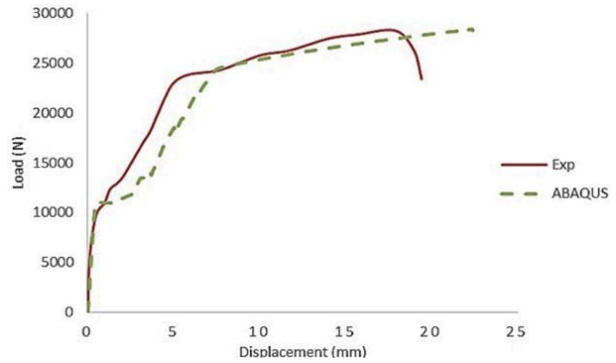

Fig. 3 FEM analyze and experimental results

To control the delamination, the equation by ACI 440.2R-08 has been used which refers to the strain of FRP where if FRP in flexural member reached to that limit in strengthened zone, then delamination will occur. This strain is according to ACI440.2R-08 [7]:

$$\varepsilon_{pd} = 0.41 \sqrt{\frac{f'_c}{nE_t t_f}} \le 0.9 \varepsilon_{f_u} \qquad (5)$$

By considering $f_c' = 80\,MPa$ and $t_f = 1mm$ for glass and carbon fiber layer and considering the maximum limits, the delamination strain can be calculated where, for carbon and glass, sequentially it is 0.0139 and 0.0257.

The strain in the fracture point for concrete obtained from FEM analyzing in the CFRP strengthened model is 0.0138 and for GFRP strengthened model is 0.014. It means that the delamination did not happen before fracture bin concrete. The diagrams for referred models have been displayed in Fig. 4.
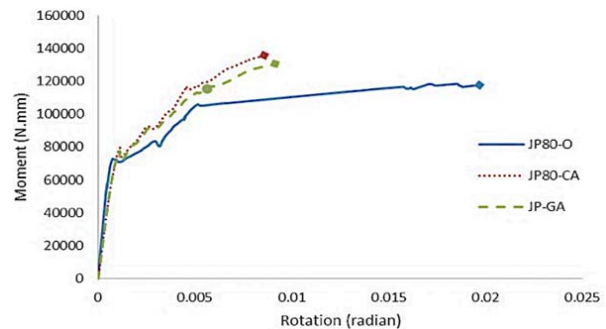

Fig. 4 Moment-rotation diagram for CFRP and GFRP strengthened joints

All criteria quantities are presented in Table II. These results show an increase of 28 and 32% in cracking moment of *GFRP* and *CFRP* respectively and also the rotation of joint in the cracking moment is two times more than non-strengthened models.

TABLE II
RESULT OF ANALYZE FOR *CFRP* AND GFRP AND NON-STRENGTHENED JOINT

specimen		JP80-0	JP80-CA	JP80-GA
Rotation (radian)	Crack initiation	0.000481	0.000999	0.000993
	Steel yielding	0.00522	0.00602	0.00591
	Fracture	0.01967	0.00875	0.00916
Moment (N.m)	Crack initiation	57830	76294	74314
	Steel yielding	105422	122371	116803
	Fracture	117597	136736	130436
Ductility ($\frac{\theta_u}{\theta_y}$)		3.77	1.45	1.55

The first model illustrated that using FRP leads to a considerable rising in load bearing capacity but a decrease in ductility of connection zone which is effective in seismic response of joint.

VI. SHEAR STRENGTHENING OF CONNECTION

In this part, the purpose is to study the behavior of several models where FRP is used only in the shear zones. These zones influence the shear performance of joint. The used names for models, considering in Fig. 5, are JP80-CB, JP80-CC and JP80-CE.

To consider the delamination, according to the equation from ACI440.2R-08 for shear strengthening:

$$\varepsilon_{fe} = k_v \varepsilon_{fu} = \frac{k_1 k_2 L_e}{11900} \quad L_e = \frac{23300}{(n_f t_f E_f)^{0.58}} = 38.7$$

$$k_1 = (\frac{f_c'}{27})^{2/3} = 2.06 \quad k_2 = \frac{d_{fv} - 2L_e}{d_{fv}} = 0.828 \tag{6}$$

$$\varepsilon_{fe} = 0.0055$$

where the result is more than ACI recommended strain of delamination in shear strengthening, so the strain of $\varepsilon_{fe} = 0.004$ will be counted as the delamination strain.

The rotation-moment curve for B, C and E which is resulted by FEM analyzing of these models has been displayed below in Fig. 6.

As the results confirm, when the FRP only used on column (C), it does not affect a lot in specified moments and rotations, so it does not seem to be an effective way to strengthening. On the other hand, strengthening beam and column simultaneously (E), has effects on all criteria moments. The moment capacity increased nearly 7 percent but the ductility factor as defined before has been decreased about 30 percent.
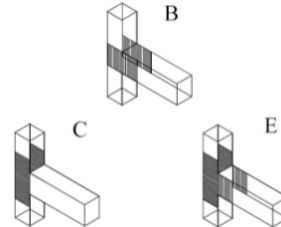

Fig. 5 Studied modes for shear strengthening

Controlling delamination by the strain of 0.004, we can observe that delamination occurs after cracking and before yielding of steel bars which is not a desirable performance for strengthened connection. Therefore, in the next strengthening mode, the opposite side of the column has been strengthened too. According to Fig. 7, this mode is specified by letter R.

There are three models for this mode in this study. The first joint has strengthened by one layer of FRP and other two are with 45-degree oriented fibers, one model by one layer in one orientation and the other model with two-layer having perpendicular oriented fibers by 45 degrees to column and beam orientations.

The results of analysis for these models are shown in Fig. 8.
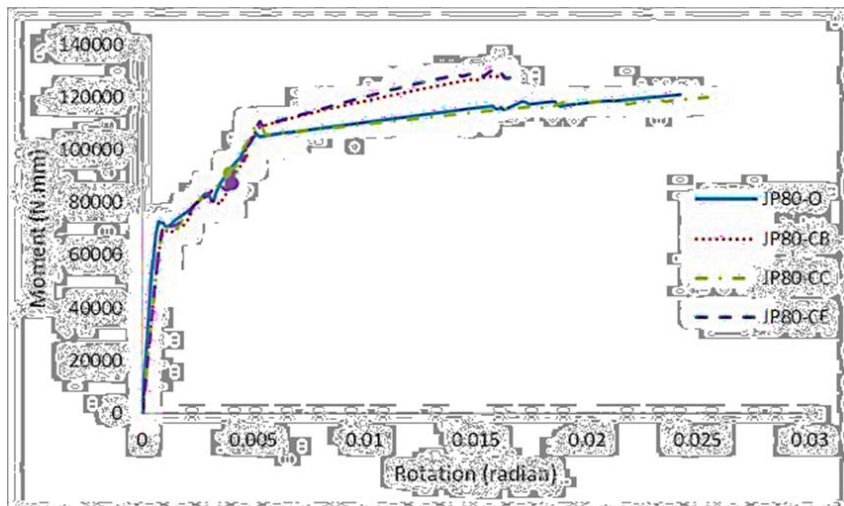

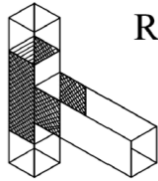Fig. 6 Moment-rotation diagram for B, C and E modes of strengthening

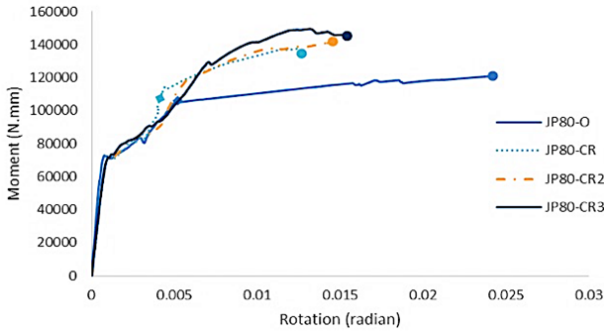Fig. 7 Shear strengthening using 1- and 2- layer of FRP

Fig. 8 Shear strengthening using 1- and 2-layer of FRP

Noticing the graph, it is obvious that there are considerable changes in all properties that have been discussed which presented in Table III.

TABLE III
RESULT OF ANALYZE FOR R MODE STRENGTHENING

	specimen	JP80-0	JP80-CR	JP80-CR3
Rotation (radian)	Crack initiation	0.000481	0.00069	0.0007
	Steel yielding	0.00522	0.00682	0.00710
	Fracture	0.01967	0.01407	0.01544
Moment (N.m)	Crack initiation	57830	57620	59872
	Steel yielding	105422	116803	122371
	Fracture	117597	141875	145148
Ductility($\frac{\theta_u}{\theta_y}$)		3.77	2.06	2.17

VII. STRENGTHENING OF FLEXURAL ZONE

Next model considers the mode of strengthening that FRP is used only for flexural zones. To realize what zones mainly are under flexural tensile, the none-strengthened joint model is used, so it will be easily recognizable which elements bearing tensile stress. So, the shape of strengthening will be according to Fig. 9.
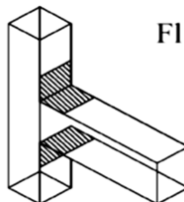

Fig. 9 Strengthening of element that are under flexural tensile stress

The result for analyzing of this model which is named Fl has been displayed as the moment-rotation graph in Fig. 10.

This result illustrates a considerable reduction in ductility and this mode of strengthening shows a completely brittle behavior, however, the amount of rising in the concerned moments is noticeable.
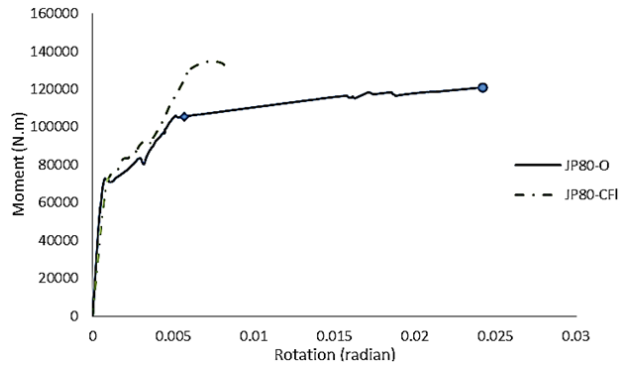

Fig. 10 Moment-rotation diagram for flexural strengthened joint

The ductility parameter as defined previously, reduced to 1.39 which shows that this mode has more brittleness compared to other modes that were discussed before.

For assessing delamination, looking through all elements of FRP, the maximum strain is 0.0049 when concrete reaches to crashing level. The ACI440 suggests controlling strain of 0.0139. So, delamination does not happen before fracture of joint. The result for this analysis is presented in Table IV.

This model indicates that using FRP in order to retrofit of joint, at tensional zones has a dramatic effect on the behavior of beam-column connection. But, it should be referred that in this modeling, method of loading which is displacement at the end of beam determines this behavior and flexural strengthening effect, but also it is considerable that, in all joints, loading is similar to this model.

TABLE IV
RESULT FOR ANALYZING OF FL MODE

	specimen	JP80-0	JP80-CFl
Rotation (radian)	Crack initiation	0.000481	0.000745
	Steel yielding	0.00522	0.00627
	Fracture	0.01967	0.00875
Moment (N.m)	Crack initiation	57830	63506
	Steel yielding	105422	132001
	Fracture	117597	130715
Ductility($\frac{\theta_u}{\theta_y}$)		3.77	1.39

VIII. SHEAR AND FLEXURAL STRENGTHENING

Using simultaneously of FRP in both flexural and shear zones is modeled as displayed in Fig. 11. In this mode, there are two layers of 45 degrees oriented of fibers in order to shear strengthening and one layer of fiber in beam and column direction to strengthening of zones that carry flexural tension. Fig. 12 is the moment-rotation diagram for the JP80-CA2 model which has two layers of 45-degree fibers comparing to non-strengthened joint result.
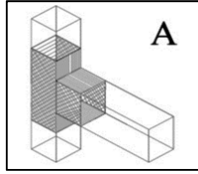
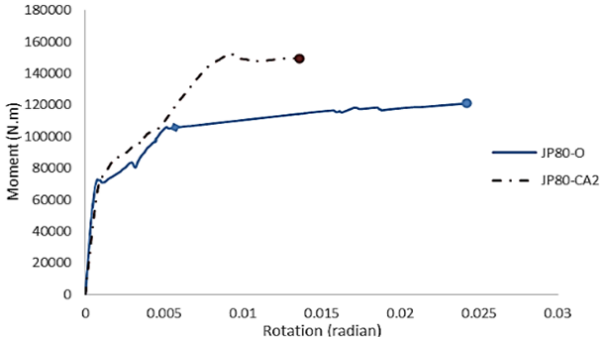Fig. 11 FRP used to strengthening all around joint

Fig. 12 Moment-rotation diagram for CA2 mode

Maximum strain at failure point of concrete in this mode reduced to 0.0069, meanwhile, the debonding strain for CFRP is 0.0139. So, debonding in this mode of strengthening did not happen. The result of analysis asserts a noticeable reduction in ductility. On the other hand, the dramatic rises of crucial moments and rotations are visible. Table V presents all quantities and differences of this mode of strengthening in compare to joint with no FRP.

TABLE V
RESULT FOR ANALYZING OF A-MODE OF STRENGTHENING

	specimen	JP80-0	JP80-CFl
Rotation (radian)	Crack initiation	0.000481	0.000822
	Steel yielding	0.00522	0.00852
	Fracture	0.01967	0.0139
Moment (N.m)	Crack initiation	57830	67915
	Steel yielding	105422	148736
	Fracture	117597	149510
Ductility($\frac{\theta_u}{\theta_y}$)		3.77	1.63

Fig. 13 is a comparison between all mentioned modes of strengthening due to their moment-rotation performances.
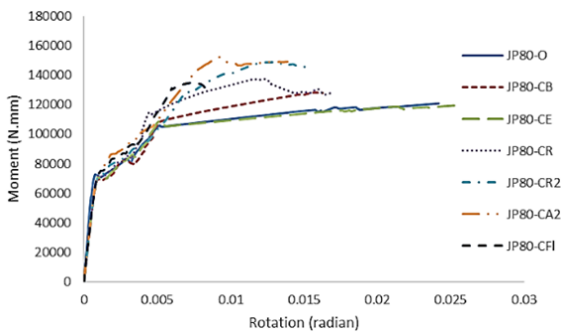

Fig. 13 Moment-rotation diagram for CA2 mode

Looking through Fig. 13, it is obvious that JP80-CA2 has the maximum load-bearing capacity and JP80-CE shows the same performance that the non-strengthened one does. In this mode, FRP layers only used to cover two sides of the column that does not carry much flexural tension and only there is shear stress. So, it is obvious that the E-mode of strengthening nearly has no effect.

IX. EFFECT OF FRP LAYER NUMBERS

To raise the load-carrying capacity of joints, instead of using only one layer of FRP, some additional layers are used for the next models. To understand the effect of number of FRP layers on a concrete beam-column connection, there is a comparison between base joint strengthened with various layer numbers. One FRP layer up to four FRP layers is used in these models.

The strengthening of the joint with different numbers of layers has been done using A mode which contains FRP layers in all sides of the column and beam. The number used in naming of each model presents the number of layers for that. So, after analyzing, the results displayed as a moment-rotation curve in Fig. 14.
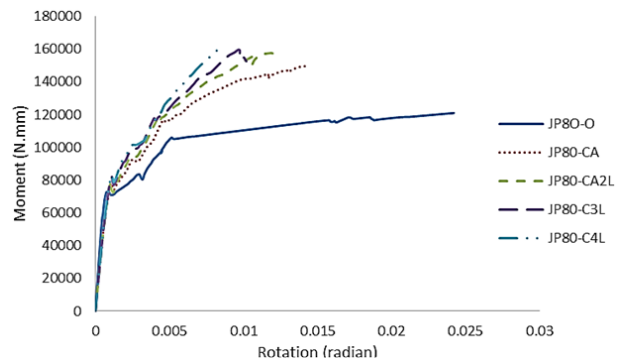

Fig. 14 Moment-rotation diagram for CA2 mode

As the graph asserts, strengthening the joint by the first layer of FRP show a dramatic change in capacity and also in ductility, but adding other layers up to 4th layer did not make too much difference in performance. The maximum moment for one layer strengthened increased to 25 percent rather than non-strengthen, whereas for 2 layers, 3 and 4 layers, this number is 27, 29 and 32 percent, respectively.

X. EFFECT OF LENGTH OF STRENGTHENING

Using FRP for greater length can delay debonding of FRP layer, but to understand the effect of length on moment capacity of strengthened joint and its rotation, there are three more models with an additional length of FRP based on A form of strengthening, which is displayed in Fig. 9. In basic A model, the length of FRP on the beam is 50 cm from the column and in the column direction has been continued 25 cm from top and bottom of the beam. The names are based on length on column (C) or beam (B). JP80-CA100B is strengthened exactly like JP80-CA but the length of the FRP

layer on the beam is 100 cm. and also for JP80-CA100B75C, length of FRP layer for the column is 75 cm. The result of the analysis has been displayed in Fig. 15.
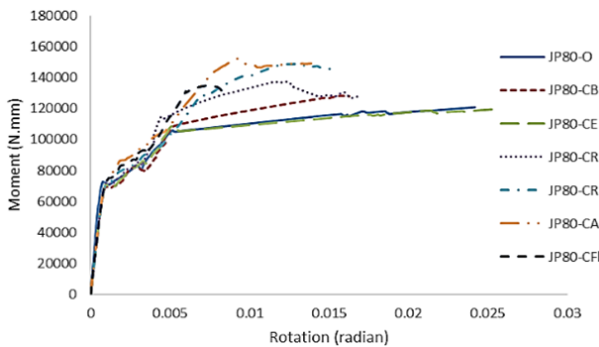

Fig. 15 Moment-rotation diagram for CA2 mode

Looking through the moment-rotation graph, it can be observed that length of strengthening does not have too much effect on the behavior of joint. Actually, it asserts that using FRP in joint in limited length around the zone has the most effect. Increasing length of FRP on beam from 25 cm up to 100 cm has led to rising of the ultimate moment about only 5 percent and decreasing of ultimate rotation about 7 percent is also realizable.

XI. ENERGY ABSORPTION IN STRENGTHENED HSC JOINTS

One of the most important parameters in structures is energy absorption before the fracture happens. There are many methods to figure out the amount of energy dissipation. Calculating the area under the load-displacement or moment-rotation curve to the failure point can present this parameter and show how much energy has been absorbed before the joint collapse. There is a comparison between non-strengthened model and JP80-CA2 which covered with FRP used fiber in two directions and according to the results had the most increase of load-bearing capacity in Fig. 16.
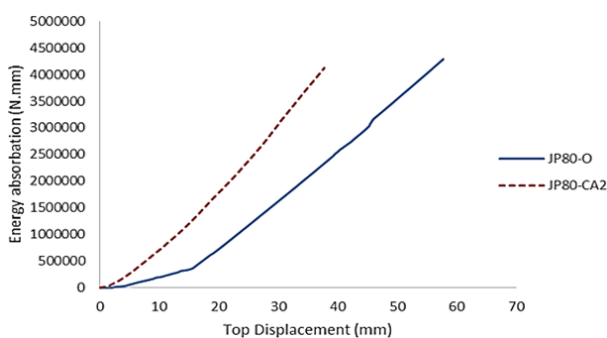

Fig. 16 Energy dissipation of a strengthened joint and joint with no FRP

This graph shows that the ultimate absorbed energy in two models, approximately is equal, however, this absorption has occurred in lower displacement in strengthened mode.

However, using FRP reduce the ductility parameter based on the ratio of fracture rotation to yielding rotation, but ductility actually defines as the amount of energy absorbed before failure in structure occur. So, it is obvious that using FRP did not lead to the reduction in energy dissipation in the joint but only in a lower rotation of join structure collapsed.

XII. INITIAL CRACKING AND YIELDING

The effect of FRP on the location of initial cracking in concrete and yielding should be considered because it is determining parameter in the joint performance. Cracking in concrete determines by SDEG parameter in ABAQUS. When SDEG is equal to 1 in an element, the cracking in that element initiates. The result of analysis indicates that in join without FRP, the first elements that show initial cracks in concrete are around the beam and column junction which is not desirable performance. But using FRP made this initiation of crack happens far from the column on the beam (Fig. 17), and this can be one of the aims in retrofitting of connection. Also, the location of yielding of steel bars is important in the behavior of connection which using result it shows the same changes as the crack initiation showed. Transferring of the location of steel yielding to a location far from the column can be a positive change that actually happened after strengthening.
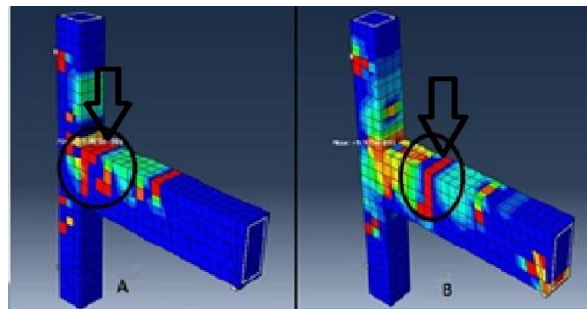

Fig. 17 Location of initiate crack (A) in a non-strengthened joint. (B) in A-mode strengthened joint

XIII. CONCLUSION

To summarize, because the beam-column connection is the most important zone in determining the behavior of RC frames, it is necessary to study methods of design, analyze and retrofitting them, and one of the impressive ways to strengthening joints is using FRP. This study which is based on finite element modeling illustrates firstly a noticeable increase in load bearing capacity after using FRP is some modes of strengthening. Using fiber reinforced polymers in shear and flexural stress zones (mode A), showed the most effect on capacity. Also transferring of the location of first cracks and steel yielding from the location of the intersection to a location out of connection zone on the beam is a desirable performance in a joint which using FRP made this happen. However, using FRP leads to a brittle behavior, but it does not result in less energy absorption.

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