

Hybrid Methods for Optimisation of Weights in Spatial Multi-Criteria Evaluation Decision for Fire Risk and Hazard

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Abstract—The challenge for everyone involved in preserving the ecosystem is to find creative ways to protect and restore the remaining ecosystems while accommodating and enhancing the country social and economic well-being. Frequent fires of anthropogenic origin have been affecting the ecosystems in many countries adversely. Hence adopting ways of decision making such as Multicriteria Decision Making (MCDM) is appropriate since it will enhance the evaluation and analysis of fire risk and hazard of the ecosystem. In this paper, fire risk and hazard data from the West Gonja area of Ghana were used in some of the methods (Analytical Hierarchy Process, Compromise Programming, and Grey Relational Analysis (GRA) for MCDM evaluation and analysis to determine the optimal weight method for fire risk and hazard. Ranking of the land cover types was carried out using; Fire Hazard, Fire Fighting Capacity and Response Risk Criteria. Pairwise comparison under Analytic Hierarchy Process (AHP) was used to determine the weight of the various criteria. Weights for sub-criteria were also obtained by the pairwise comparison method. The results were optimised using GRA and Compromise Programming (CP). The results from each method, hybrid GRA and CP, were compared and it was established that all methods were satisfactory in terms of optimisation of weight. The most optimal method for spatial multicriteria evaluation was the hybrid GRA method. Thus, a hybrid AHP and GRA method is more effective method for ranking alternatives in MCDM than the hybrid AHP and CP method.

Keywords—Compromise programming, grey relational analysis, spatial multi-criteria, weight optimisation

I. INTRODUCTION

MCDM can generally be described as a tool for deriving priorities from a set of available alternatives; based on a set of criteria with different significance. It is used for making choices in the presence of multi-conflicting criteria. Many researchers have proposed different methods based on quantitative measurement for the selection of most optimal alternative from a set of alternatives [1]. The most frequently used MCDM methods include: Methods based on quantitative initial measurements i.e. AHP and Fuzzy Theory Set [1], methods based on quantitative measurement i.e. Technique for

Ordering Preference by Similarity to Identical Solution (TOPSIS) [2], Linear Programming Technique for Multidimensional Analysis of Preference [3], Complex Proportional Assessment (COPRAS) [4], Additive Ratio Assessment methods, GRA and CP [5]; Comparative preference methods based on pair-wise comparison alternatives i.e. Preference Ranking Organization method for Enrichment Evaluations (PROMETHEE) [3].

In MCDM, usually the evaluation criteria are associated with different weights, with the weights of the criteria having large impact on the selected alternative. Technique and decision-making methods in MCDM are dynamic [6]-[9], [3]. There are two ways of determining the weight associated with each criterion based on the importance attached to it: direct explication and indirect explication [10]. The direct explication is where weights are assigned through questionnaire surveys, conventional rules and expert interviews before the data of each alternative are collected. On the contrary, indirect explication represents the importance of the alternatives being evaluated. The weights are a reflection of the data [11]. However, it must be noted that optimality is complicated whenever multiple objectives are considered in the evaluation of a solution [12]. The most widely used concept for obtaining the optimal solution of a problem which involves multiple objectives is the concept of Pareto optimality. The concept is such that the improvement in one objective leads to the detriment of the other. This concept of Pareto optimality usually serves as a processing stage for a MCDM. In this case the information necessary to support the selection of the most optimal solution from the set of possible solutions are provided [13].

This paper is aimed at making alternative decision rules in spatial multicriteria evaluation and analysis of fire risk and hazard data from the West Gonja Area of Ghana (WGA). The AHP, GRA and the CP are the MCDM methods considered in this paper. The AHP is used to determine the weight of the various criteria based on expert's relative preferences for the various criteria. The optimal alternative will be selected based on the result obtained by the hybrid AHP-GRA or the AHP-CP. Research has been conducted in the use of GRA and/or CP for the selection of the best alternative. Reference [14] used CP for multi-objective route planning; adaptation of CP approach for multicriteria decision analysis by [15]; [16] assesses the fire safety of underground building based on GRA; [3] used grey additive ratio assessment method for multiple criteria analysis; and [17] used GRA for criteria

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TABLE I
THE FUNDAMENTAL EVALUATION SCALE FOR RELATIVE IMPORTANCE OF THE VARIOUS CRITERIA

Relative importance of criteria i over Criteria j	Value
Equal	1
Small important	3
Average importance	5
Importance	7
Great importance	9
Intermediate	2, 4, 6, 8

$$A = a_{ij}(i, j = 1, 2, \dots, n)$$

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{bmatrix} = \begin{bmatrix} 1 & \alpha_{12} & \dots & \alpha_{1n} \\ 1/\alpha_{21} & 1 & \dots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/\alpha_{n1} & 1/\alpha_{n2} & \dots & 1 \end{bmatrix}$$

where a_{ij} is an integer and a lies in the interval $0 < a < 10$, then $a_{ij} = \frac{1}{a}$, $a_{ij} = 1$ if $i=j$ [23].

The degree of association of the importance of each criterion is compared to others using a scale ranging from 1 to 9.

Step2. Normalisation using pairwise comparisons: The element in the matrix is normalised by dividing each element by the sum of its column.

$$A_{ij} = \frac{R_i}{R_j} \tag{1}$$

$$\frac{\frac{R_i}{R_j}}{\frac{\sum_{i=1}^n R_i}{R_j}} = \frac{R_i}{R_j} \times \frac{R_j}{\sum_{i=1}^n R_i} = \frac{R_i}{\sum_{i=1}^n R_i} \tag{2}$$

The sum of each row is divided by the number of elements in the row.

$$\left(\frac{R_i}{\sum_{i=1}^n R_i} + \dots + \frac{R_i}{\sum_{i=1}^n R_i} \right) \frac{1}{n} = \frac{n \cdot R_i}{\sum_{i=1}^n R_i} \cdot \frac{1}{n} = \frac{R_i}{\sum_{i=1}^n R_i} \tag{3}$$

B. CP

CP is a multicriteria method used to determine a subset of possible solutions (compromise set) with the best alternative having the shortest distance from an ideal point for which all criteria are optimised. The highest possible value is the optimal solution found from the compromise set based on the decision maker's preference, whereas the corresponding distance functions are obtained by a family of p-metrics [24].

CP has been successfully applied in various areas for the determination of an optimal alternative based on a set of conflicting criteria. Some areas where CP has been successfully applied include: water resources, transportation, sustainability, environmental issues (fire risk management and flood management) etc. Reference [24] used CP model to select the suitable site for borrow pits, [25] used CP for site selection, [26] used CP to evaluate the alternative options in the context of long-term water resources planning, [27] used CP for man power planning, [28] extended CP to introduce spatial CP, A modified CP was used by [29] to deal with

problems of hierarchical nature, [30] used CP for making sustainability rankings (an application to European paper industry) and [31] used CP method based on multi-bounds formulation and dual approach for multicriteria structural optimisation to enhance the reliability and efficiency of multicriteria optimisation procedure.

A MCDM problem with discrete number of alternatives can be generally described as follows: If X is a finite set of n alternatives, $a, b \in X$ and m is the set of different evaluation criteria $l_i, i = 1, 2, \dots, m$, then alternative 'a' is considered better than alternative 'b' based on the *ith* criterion, if $l_i(a) > l_i(b)$. The decision problem will be represented by a set of $n \times m$. The element (i, j) of the matrix, where $j = 1, 2, \dots, m$ and $i = 1, 2, \dots, n$, is the evaluation of the *jth* alternatives with respect to the *ith* criteria.

CP is based on Minkowski L_p metric and it can be shown by:

$$\min L_{p,i} = \left\{ \sum_{j=i}^n W_j^p \left(\frac{y_i^* - y_{ij}}{y_j^* - y_j^{**}} \right)^p \right\}^{\frac{1}{p}} \tag{4}$$

$$y_i^* = \max (y_{ij}), i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, m \tag{5}$$

$$y_j^{**} = \min (y_{ij}), i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, m \tag{6}$$

where $L_{p,i}$ is the distance metric of the *ith* alternative for a given parameter p , y_i^* and y_j^{**} are the most preferable and the worst performance rating of *jth* criterion, and p is metric, $p \in [1, \infty)$ [31]. W_i is the standardized form of the criterion weight, w_i , and it represents the relative preference of the decision maker among the *i*-criteria, with the sum of the weights of the criteria equals one; y_i^* is the ideal value for the criterion *i* as shown in (5). y_j^{**} is the worst value for the criterion *i* as described in (6).

The problem of MCDM can be solved using CP approach by computing the ideal value y^* , and the worst value, y^{**} using (5) and (6) respectively. The obtained values are then put in (1) to obtain L_p distance values from the ideal points. The optimum alternative has the shortest distance value for each p. This means that an alternative with the lowest value for L_p metric will be the best compromise solution. The parameter p acts as a weight attached to the deviation according to their magnitudes [24]. It is assumed in this research that $p=1$ thus serving as a balance factor, where all weighted deviations perfectly balance each other. Thus, W_i becomes the weight for a deviation which shows the relative significance attached to that criterion [24].

C. GRA

Due to the advancement in science, technology and the progress of mankind, there is a gradual improvement of human understanding in matters concerning systems' uncertainties. The Grey System Theory is an interdisciplinary theory [32] extending across the fields of both natural science and social science. Today, research on system uncertainties has been taken to a higher height. GRA is one of the effective

techniques that can be used to solve uncertainty problem under a discrete data and information incompleteness [17]. It uses the concept of “grey” to describe features. Grey is a term used to describe data incompleteness. On the contrary, an insufficient data is called “black” [33]. Grey System Theory is concerned with intrinsic structure of the system given such limited data [34].

In GRA, the most optimal alternative can be obtained by calculating the Grey Relational Grade (GRG) using the process described in the following sub-section:

Normalisation

In order to compare data with different measurement units, there is the need to normalize the data to take values ranging between the interval from 0 - 1. Normalisation of data can be achieved in four ways [35] based on the expectation of the decision maker: For data with;

Larger-the-better Characteristic

$$r_i^*(k) = \frac{r_i^{(O)}(k) - \min r_i^{(O)}(k)}{\max r_i^{(O)}(k) - \min r_i^{(O)}(k)} \quad (7)$$

Smaller-the-better Characteristic

$$r_i^*(k) = \frac{\max r_i^{(O)}(k) - r_i^{(O)}(k)}{\max r_i^{(O)}(k) - \min r_i^{(O)}(k)} \quad (8)$$

There exists a target value to be reached for the original data.

$$r_i^*(k) = 1 - \frac{|r_i^{(O)}(k) - TG|}{\max(\max r_i^{(O)}(k) - TG, TG - \min r_i^{(O)}(k))} \quad (9)$$

where *TG* =target value

Dividing sequence value by the first value of sequence:

$$r_i^*(k) = \frac{r_i^{(O)}(k)}{r_i^{(O)}(1)} \quad (10)$$

where $r_0^{(O)}(k)$ and $r_i^{(O)}(k)$, $i = 1, 2, \dots, m; k = 1, 2, \dots, n$ (m is the number of alternatives and n the number of criteria) represent the original reference sequence and comparable sequence respectively [33].

Grey Relational Coefficient (γ)

The grey relational coefficient can be calculated as follows:

$$\gamma[(r_0^*(k), r_j^*(k))] = \frac{\max_{j \in I} \max_k |r_0^*(k) - r_j^*(k)| + \xi \min_{j \in I} \max_k |r_0^*(k) - r_j^*(k)|}{\min_{j \in I} \min_k |r_0^*(k) - r_j^*(k)| + \xi \max_{j \in I} \min_k |r_0^*(k) - r_j^*(k)|} \quad (11)$$

for $0 < \gamma \leq 1$

In literature [12], the distinguishing coefficient (ξ) in (8) is usually given the value 0.5.

Grey Relational Grade (GRA)

The GRG is the weighted sum of the Grey Relational Coefficient as shown in (12):

$$\gamma(r_0^*, r_j^*) = \sum_{k=1}^n \beta_k \gamma[(r_0^*(k), r_j^*(k))] \quad (12)$$

where

$$\sum_{k=1}^n \beta_k = 1 \quad (13)$$

GRA is used to assess the degree of the influence of each factor. After the overall ranking index has been determined for the respective alternative, the alternative with the smallest overall ranking index has the highest priority [5].

III. RESULTS AND DISCUSSION

Data from the West Gonja Area of Ghana for fire risk and hazard were used to rank the land cover types that contributed to fire severity. The data were used to provide a more objective conclusion in terms of the application of GRA and CP for optimal weight determination. The ranking of the alternatives was based on fire hazard factors of different significance. The values of the various land cover alternatives were assigned according to expert’s advice. This case study presents the ranking of the various land cover types based on the fire hazard factors. These criteria are shown in Table II. The weights of the criteria and the sub-criteria are obtained as a result of pairwise comparison (Table III).

TABLE II
CRITERIA FOR RANKING THE LAND COVER TYPES

Criteria	Sub-criteria	Definitions
Fire Hazard	F_1	Land use
	F_2	Elevation
	F_3	Slope
	F_4	Aspect
	F_5	Temperature
	F_6	Relative humidity
	F_7	Wind force
Fighting Capacities	C_1	Fire-brigade
	C_2	Watch-tower
	C_3	Helicopter water source
Response Risk	R_1	Land cover friction
	R_2	Elevation friction
	R_3	Slope friction

TABLE III
WEIGHT OF CRITERIA AND SUB-CRITERIA OBTAINED BY PAIRWISE COMPARISON

Criteria	Weight	Sub-criteria	Weight
Fire Hazard	0.657	F1	0.4184
		F2	0.0427
		F3	0.0969
		F4	0.1442
		F5	0.2041
		F6	0.0643
		F7	0.0294
Fire Fighting Capacities	0.068	C1	0.1932
		C2	0.7235
		C3	0.0833
Response Risk	0.279	R1	0.7482
		R2	0.0714
		R3	0.1804

TABLE IV
RANKING RESULTS OBTAINED USING HYBRID AHP-GRA AND AHP-CP

Land cover types	Road L1	Water L2	Agriculture L3	Shrub L4	Plantation L5	Natural Forest L6	Settlement L7	Highest Vulnerability to fire	Ranking Order
Methods	I	II	III	IV	V	VI	VII	VIII	
GRA	0.7070	0.7866	0.5181	0.4303	0.3840	0.3709	0.5817	L6	$L_6 > L_5 > L_4 > L_3 > L_7 > L_1 > L_2$
CP	0.3011	0.2190	0.1887	0.1076	0.0382	0.0058	0.2285	L6	$L_6 > L_5 > L_4 > L_3 > L_2 > L_7 > L_1$
	7th	5th	4th	3rd	2nd	1st	6th		

IV. CONCLUSIONS

The GRA and the CP methods are based on the idea that the best alternative has the smallest distance from the ideal point and can therefore be said to be distance-based approach methods. From Table IV, both the hybrid AHP-CP and AHP-GRA gave the highest priority to the natural forest (L6) which agrees well with the experts' judgment, and therefore prove to be effective methods for fire risk assessment. In Table IV, the AHP-GRA method rank the various land cover types in accordance with their vulnerability to fire in the following ascending order: natural forest, plantation, shrub, agriculture, settlement, road and water which also conform to experts' knowledge. The ranking by the AHP-CP is in the ascending order: natural forest, plantation, shrub, agriculture, water, settlement and road. This does not agree well with the experts' knowledge to some extent. It can therefore be concluded that hybrid AHP-GRA method is a more effective method for ranking alternatives than the hybrid AHP-CP method (Table IV). It is recommended that optimisation of weights estimated by methods and/or hybrid methods used in spatial multicriteria decision analysis should be considered.

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