Dynamic Stability Assessment of Different Wheel Sized Bicycles Based on Current Frame Design Practice with ISO Requirement for Bicycle Safety

Milan Paudel, Fook Fah Yap, Anil K. Bastola

***Abstract*—**The difficulties in riding small wheel bicycles and their lesser stability have been perceived for a long time. Although small wheel bicycles are designed using the similar approach and guidelines that have worked well for big wheel bicycles, the performance of the big wheelers and the smaller wheelers are markedly different. Since both the big wheelers and small wheelers have same fundamental geometry, most blame the small wheel for this discrepancy in the performance. This paper reviews existing guidelines for bicycle design, especially the front steering geometry for the bicycle, and provides a systematic and quantitative analysis of different wheel sized bicycles. A validated mathematical model has been used as a tool to assess the dynamic performance of the bicycles in term of their self-stability. The results obtained were found to corroborate the subjective perception of cyclists for small wheel bicycles. The current approach for small wheel bicycle design requires higher speed to be self-stable. However, it was found that increasing the headtube angle and selecting a proper trail could improve the dynamic performance of small wheel bicycles. A range of parameters for front steering geometry has been identified for small wheel bicycles that have comparable stability as big wheel bicycles. Interestingly, most of the identified geometries are found to be beyond the ISO recommended range and seem to counter the current approach of small wheel bicycle design. Therefore, it was successfully shown that the guidelines for big wheelers do not translate directly to small wheelers, but careful selection of the front geometry could make small wheel bicycles as stable as big wheel bicycles.

***Keywords*—**Big wheel bicycle, design approach, ISO requirements, small wheel bicycle, stability and performance.

I. Introduction

BICYCLES share a long history of more than 200 years with us. Since the invention of the bicycle in 1817 by Karl Von Drais, it endured various design modifications before evolving as a relatively safe vehicle "safety bicycle" in 1890 [1]. The safety bicycle had two equal sized pneumatic wheels, drive chain, brake callipers, and closely resembles the modern bicycles. Ever since the evolution of the safety bicycle, a number of frame builders or bicycle manufacturers have been established, and a plethora of bicycle designs have emerged in the market. The other descendants of the bicycle are scooters and motorcycles.

The motorcycle design procedures have matured over the time. Various simulation software is radially available to evaluate the performance of the motorcycle during the design procedure. However, bicycle design is still a matter of craftsmanship rather the engineering. Most of the frame builders use trial and error methods or some empirical guidelines derived based on the experiences [2]-[6] to design the bicycle frames. Those guidelines are basically used to maintain the cyclist's ergonomic in the bicycle. Cyclist's body parameters such as overall height, torso length, leg inseam length, arm length etc. are taken as the design input. Then, the frame design parameters such as saddle height, handlebar height, saddle to handlebar reach, bottom bracket height reach, stack are selected based on these rider's anthropomorphic parameters and the derived guidelines. On the other hand, headtube angle and trail are chosen based on the experiences. This bicycle design methodology has remained stagnant for a long time. This is because the guidelines are formulated based on years of experimentations on the bicycle design and their performances. Therefore, these guidelines are considered to be well optimised to ensure the stability and rideability of the bicycle. Nevertheless, these guidelines were mainly proposed for designing big wheel bicycles (wheel size diameter greater than 20-inches).

The bicycle frame design is the crucial aspect of their riding performance. Especially, the position of the rider and the geometry of the front steering assemblies determines the performance of a bicycle. According to the current bicycle design practice, most of the design parameters related with the ergonomic are selected using cyclist's anthropomorphic measurements and steering parameters such as headtube angle, trail and fork offset are selected within a certain range. For example, most of the bicycle is designed with headtube angle ranging from 65^0 to 75^0 and trail ranging from 50-70 mm, and fork offset is generally maintained around 45 mm. Often, the trail values are regarded as a stability indicator. A longer trail is often recommended for better stability. However, there are very limited information in published documents regarding detail guidelines for selecting appropriate head tube angle and trail. As the bicycle manufacturing has become a global industry, the International Organization for Standardization (ISO) also has developed some guidelines for bicycle design for safety and stability. The ISO standard also uses headtube angle, trail or fork offset as the governing parameters for the

Milan Paudel is with the Nanyang Technological University, Institute for Sports Research, 50 Nanyang Avenue, 639798 Singapore (corresponding author, phone: +65-81486808, e-mail: milan002@e.ntu.edu.sg).

Fook Fah Yap is with the School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798 Singapore (e-mail: mffyap@ntu.edu,sg).

Anil K. Bastola is with the Nanyang Technological University, Institute for Sports Research, 50 Nanyang Avenue, 639798 Singapore (e-mail: anilkuma001@e.ntu.edu.sg).

stability and the safety of the bicycle. It provides an acceptable range for the headtube angle, the fork offset, and trail. All bicycle manufacturing industries should consider the ISO recommended range for the bicycles design. In the other words, bicycles have been designed using the provided guidelines by the ISO standards. This procedure of designing bicycles has worked acceptably for big wheel bicycles, and their performances are perceived to be stable and safe. However, the performance of small wheel bicycles (wheel size diameter 20-inches or less) is not as well as the big wheel bicycle.

Fig. 1 Comparison between typical big wheel bicycle and Moulton small wheel bicycle

In recent time, there is a rapidly growing popularity of small wheel bicycles in many modern cities worldwide. People are migrating towards the cities because of the urbanisation and places are becoming more crowded. Their compactness and portability make them suitable for riding in crowded places and for easy storage in small places. Although these bicycles have relatively small wheels, the overall frame design is very much similar to big wheel bicycles. According to Hon [7], the founder of Dahon bicycle, the traditional guidelines evolve over a century of optimization and should be adhered to. A comparison between a typical big wheel bicycle and small wheel Moulton bicycle [8] is shown in Fig. 1. As can be seen, the position of the saddle, handlebar, bottom bracket is almost the same. To increase the trail, the headtube angle has been decreased and wheel base in greater than the typical big wheel bicycle. This is because, the guidelines for big wheel bicycle have been working well, and there are not adequate guidelines for small wheel bicycles. Therefore, small wheel bicycle design relies on traditional guidelines proposed for big wheel bicycles. In the other words, most of the bicycles are designed with similar guidelines and methodology which consider cyclist ergonomics and use trail as an indicator of stability. As a result, bicycles seem to have similar design configurations irrespective of the wheel size. However, the performance of different wheel size bicycles is markedly different. One way to evaluate the performance is to ride these bicycles hands-free [9]. The small wheel bicycles are not as stable as the big wheel bicycles. The ride of the small wheel is often described as the "twitchy", or "wobbly". The ride becomes more difficult as the wheel size decreases. As the number of small wheelers is increasing in recent time, it is of interest that we should consider the aspect to improve the stability and the rideability of these small wheel bicycles. In this way, we could also decrease the number of accidents caused by the instability of small wheel bicycles.

In this paper, we are comparing the performance of the different wheel sized bicycles. The paper presents the performance of typical 26-inch, 20-inch, and 16-inch wheel size bicycles designed with the traditional methodology and within the ISO recommended criteria. The validated mathematical model has been used as the tool to evaluate and compare the performance of the different wheel sized bicycle. The major contribution of this paper is to investigate whether it is possible to improve the small wheel bicycle designs within ISO recommended criteria such that these bicycles will have comparable stability and riding performance as a typical big wheel bicycle.

II. ISO Standard for Bicycle Stability and Safety

The ISO is a worldwide federation for preparing the international standards through the technical committees. The standardization of the bicycle "Cycles — Safety requirements for bicycles" has been prepared by Technical Committee ISO/TC 149 [10]. The aim is to make bicycles as safe as possible and ensure stability of the bicycles. Part2: Requirements for city and trekking, young adult, mountain and racing bicycles (BS EN ISO 4210-2:2014), recommended the following design criteria for bicycles front steering geometries:

a) The steering head angle is not more than 75° and not less than 65° in relation to the ground line;
b) The steering axis intersects a line perpendicular to the ground line, drawn through the wheel centre, at a point not lower than 15 % and not higher than 60 % of the wheel radius when measured from the ground line.

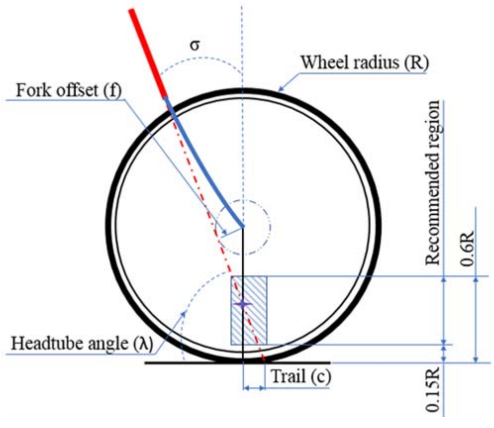


Fig. 2 Bicycles steering geometry criteria recommended by ISO

Fig. 2 depicts the above-mentioned criteria. A relation can be derived for fork offset (f) and trail (c) in terms of wheel radius (R) and headtube angle (λ) satisfying the ISO recommended criteria for safety requirements.

$$Fork\ offset\ (f) = R * (1 - K) * \cos(\lambda) \quad (1)$$

where K is defined in terms of the wheel radius and represents the point of intersection of steering axis with a line drawn vertically from the wheel axle centre. The limiting values for K are $0.15R$ and $0.6\,R$. Equation (1) could also be modified as:

$$Fork\ offset\ (f) = R * (1 - K) * \sin(\sigma) \tag{2}$$

Similarly, the trail for bicycle front system is defined as:

$$Trail\ (c) = \frac{R * \cos\lambda - f}{\sin\lambda} \tag{3}$$

Substituting either (1) or (2), in (3), range of ISO recommended trail value could be obtained as:

$$Trail\ (c) = R * K * \tan(\sigma) \tag{4}$$

Thus, the trail is maximum for a particular headtube angle when the point of intersection is at its highest point, i.e. $0.6R$ and minimum at $0.15R$. Table I presents the range of ISO recommended trail range for different wheel size bicycle.

TABLE I
ISO RECOMMENDATION FOR FRONT GEOMETRY

Headtube angle	Trail (mm) for 26-inch (674 mm) wheel bicycle		Trail (mm) for 20-inch (486 mm) wheel bicycle		Trail (mm) for 16-inch (414 mm) wheel bicycle	
	Min	Max	Min	Max	Min	Max
65	23.57	94.29	17.00	67.99	14.48	57.92
66	22.51	90.03	16.23	64.91	13.82	55.30
67	21.46	85.83	15.47	61.89	13.18	52.72
68	20.42	81.69	14.73	58.91	12.55	50.18
69	19.40	77.62	13.99	55.97	11.92	47.68
70	18.40	73.59	13.27	53.07	11.30	45.21
71	17.41	69.62	12.55	50.20	10.69	42.77
72	16.42	65.70	11.84	47.37	10.09	40.36
73	15.45	61.82	11.14	44.58	9.49	37.97
74	14.49	57.98	10.45	41.81	8.90	35.61
75	13.54	54.18	9.77	39.07	8.32	33.28

To understand how the bicycles are being designed, headtube and trail for 91 different big wheel bicycles (including 30 bicycles from Tour De France 2013) were analysed. The headtube and trail data were taken from PhD thesis of Prince [11]. The front geometry of these 91 bicycles was plotted against the ISO recommended values as shown in Fig. 3.

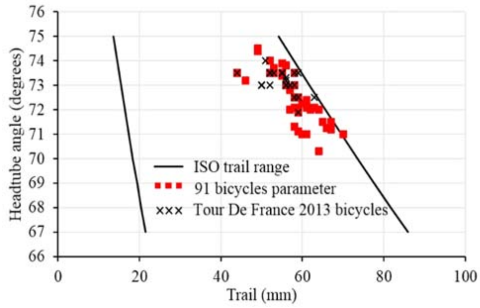


Fig. 3 Bicycle front geometry plotted against ISO recommended criteria

Almost all bicycles seem to meet the ISO recommended standards. These 91 different bicycles include both the high-performance bicycles used in the Tour de France 2013 and the normal bicycle designs from different manufacturers. These bicycles also represent a range of bicycle designs which are well accepted as good design for stability and safety. This, in a way, validates the current ISO recommendation for big wheel bicycle designs. Though the ISO standard covers a wide range of headtube angle and trail values, it is interesting to know that the current bicycle designs are confined within a very narrow region. Most of the bicycles were found to be designed with headtube angle in between 71-74 degrees and trail in between 50-65 mm. Table II shows the detail breakdown for the selected data set.

TABLE II
FRONT GEOMETRY OF TOUR DE FRANCE 2013 BICYCLES

		Trail range (mm)							
		40-45	45-50	50-55	55-60	60-65	65-70	70-75	*Total*
Headtube angle	<70	0	0	0	0	0	0	0	**0**
	70-71	0	0	0	0	1	0	0	**1**
	71-72	0	0	**0**	**4**	**2**	5	1	**12**
	72-73	0	0	**0**	**9**	**10**	0	0	**19**
	73-74	2	1	**15**	**32**	**0**	0	0	**50**
	74-75	0	3	6	0	0	0	0	**9**
	>75	0	0	0	0	0	0	0	**0**
Total		**2**	**4**	**21**	**45**	**13**	**5**	**1**	**91**
Headtube angle (Mean): 72.8 degrees				***Trail (Mean): 56.5 mm***					
Headtube angle (Mode): 73.0 degrees				***Trail (Mode): 55.0 mm***					

It is very obvious that the shallower headtube angle normally produces longer trail and vice versa. However, as can be seen, only a few bicycles have very shallower headtube angle and very longer trail. This is because the cyclists prefer both the good handling and stability. Based on the experiences and guidelines, it is generally recommended that a headtube angle of 73 degrees and a trail of 56 mm are an ideal combination for both stability and handling. The argument seems to be well accepted as the headtube angle range 73 to 74 degrees included about 55% of the bicycles. Similarly, the trail range 55 to 60 mm included about 50% of the bicycles. Overall mean for headtube and trail is also about 73 degrees and 56 mm respectively. In addition, amidst the selected bicycles, 73 degrees headtube angle and 55 mm trail are the most repeated values. Therefore, 73 degrees headtube angle and trail range from 50-60 mm (51% of selected bicycles) were selected in this study. The dynamic performance within the selected range of front geometry was evaluated and set as a reference. The performance will be compared with small wheel bicycles within ISO recommended front geometry criteria.

III. Performance Comparison of Different Wheel Sized Bicycles

A bicycle is a complex engineering system with low speed lateral instability. In the other words, a bicycle is laterally unstable at low speed therefore requires cyclist's manual control on the handlebar to ride. However, when a bicycle forward speed crosses some threshold value, it does not necessarily require handlebar control, i.e. a bicycle can balance itself and move forward without falling over. This property is called the self-stability of the bicycle. This property enables the cyclist to perform hands-free riding and qualitatively measures the stability of the bicycles. This intriguing property of the bicycles had been attracting many researchers worldwide. A number of mathematical models and theories have been proposed to describe the self-stability of a bicycle [12]-[16]. The trail theory proposed by Jones [14] is still considered while designing the bicycles. According to this theory, longer trail helps to improve the stability of bicycles. An accurate mathematical model has recently been benchmarked [17] and validated [18] after careful review of the past literatures. The benchmarked model treats the bicycle as a system of four rigid bodies: - rear wheel, rear frame where a rigid rider could be added, front wheel, and front steering assembly consisting of handlebar and fork. The model includes 25 different bicycle related parameters. This includes mass related parameters such as mass, centre of mass(COM), moment of inertia (MOI), and geometrical parameters such as wheelbase, headtube angle, and trail. The model also includes kinematic constraints in the system. The equations for motion is defined for lean angle (ϕ) and steering (δ) with constant forward speed as

$$M\ddot{q} + vC_1\dot{q} + [gK_0 + v^2K_2]q = f \quad (5)$$

where $q = [\square, \square]^T$ and forcing terms $f = [T_\square, T_\square]^T$ are the lean torque and steer torque. For the self-stability in riding hands-free, $f = 0$, because rider theoretically does not need to provide any manual control in hands free riding. The coefficients $M, C1,$ and K are the mass matrix, damping like matrix and stiffness matrix. Assuming the solution of the form $e^{\lambda t}$, the characteristic equation of motion can be defined as:

$$det\,(M\lambda^2 + vC_1\lambda + [gK_0 + v^2K_2]) = 0 \quad (6)$$

The roots of above characteristics equations are called the eigenvalues. A bicycle will be self-stable if the eigenvalues have the negative real parts. The self-stable region could be predicted via analysing these eigenvalues. Castering mode always has negative real parts, therefore this mode is considered always stable. The remaining modes, namely the weave mode and the capsize mode, determine the stability of a bicycle. The capsize mode represents the motion of the bicycle dominated by leaning of the bicycle, whereas the weave mode represents the oscillation of the bicycle in headed direction.

An adult rider was modelled on the bicycle rear frame as explained by Moore [19]. A typical solution of (6) is shown in Fig. 4. The solution represents the calculated eigenvalues with respect to the forward speed for a typical big wheel bicycle having 73^0 headtube angle and 56 mm trail as shown in Fig. 5. A bicycle will be self-stable if all the eigenvalues have negative real parts.

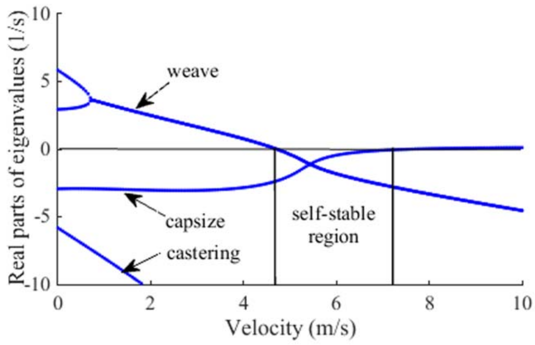
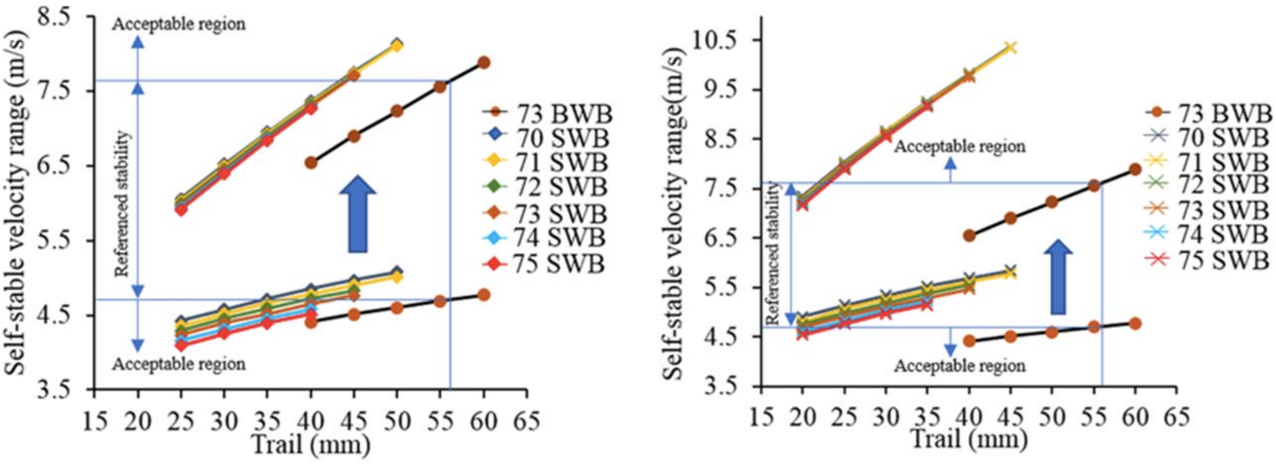


Fig. 4 Eigenvalues analysis for predicting bicycle self-stability

The weave eigenvalues initially have positive real parts. However, the positivity of the real parts decreases with increase in velocity and crosses the zero-reference line to become negative. The velocity at which the weave eigenvalues cross the zero-reference line is called the weave critical velocity. Conversely, the capsize mode is initially negative but with the increasing velocity, eigenvalues become less negative and cross the zero-reference line. Again, the velocity at which capsize eigenvalues cross the zero-reference line is called the capsize critical velocity. On the other hand, the castering mode always has the negative real parts of eigenvalues, thus considered as a stable mode. Therefore, the self-stability of a bicycle is bound by the weave critical velocity as the lower boundary and the capsize critical velocity as the upper boundary. In this way, the mathematical model is used as a tool to study the self-stability characteristic of bicycles, and compare the performance of the different wheel sized bicycles within ISO recommended standard. Here, it should be noted that the position of the cyclist on the rear frame, ergonomics of an adult cyclist, was kept almost similar for all bicycles, and the front steering geometry was varied to analyse the dynamic performance.

Fig. 5 A typical big wheel bicycle

The range of self-stable velocity of big wheel bicycle (26-inches) with 73^0 headtube angle and trail in a range of 40-60 mm was calculated and compared with the self-stability of the small wheel bicycles (20-inches and 16-inches). The headtube angle and corresponding range of trail was limited by the ISO design criteria. Fig. 6 represents the self-stability comparison between the big wheel bicycle (73^0 headtube angle and 40-60 mm trail) and small wheel bicycles. Fig. 6(a) shows the comparison between selected big wheel bicycle with 20-inches wheel bicycle self-stable region. Similarly, Fig. 6(b) depicts the comparison between big wheel bicycle and 16-inches wheel bicycle. The figures show the capsize critical velocities (upper two curves) and the weave critical velocities (lower two curves) for each headtube angle and trail.



(a)

(b)

Fig. 6 Self-stability range comparison over a range of front geometry: (a) 26-inches big wheel bicycle (BWB) vs 20-inches small wheel bicycle (SWB), (b) 26-inches big wheel bicycle (BWB) vs 16-inches small wheel bicycle (SWB)

It could be easily noticed that the region of the self-stable velocity is not similar. The self-stable region has shifted upward towards higher speed region for small wheel bicycles as shown by the arrows in Fig. 6. This means that the small wheel bicycles must attend higher speed to become self-stable as compared to the big wheel bicycle. The requirement of higher speed to become self-stable increases as the wheel size decreases. That is, the smaller the wheel size, the higher is the speed required for bicycle to become self-stable. Therefore, a cyclist needs to pedal hard to maintain the higher velocity. Thus, 16-inches wheel bicycles are more difficult to control than 20-inches bicycle.

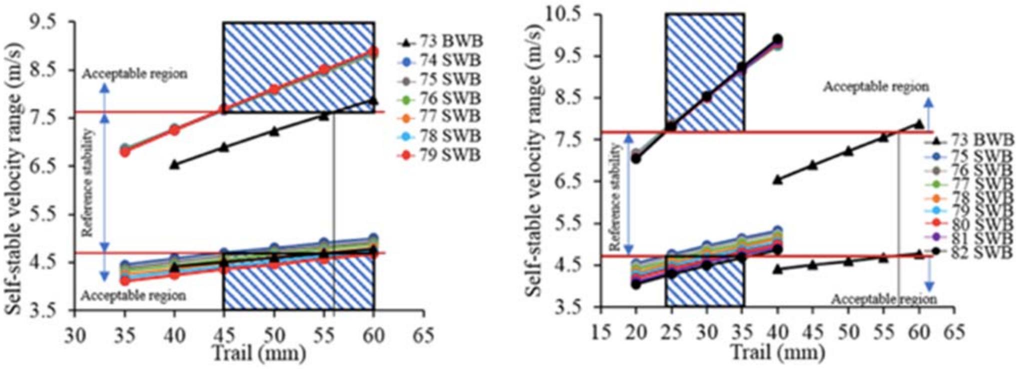
As mentioned in Section I, the trail is often used as an indicator for the bicycle stability. The trail reduces drastically as the wheel size becomes smaller. Since 56 mm trail is generally recommended for big wheel bicycles, small wheel bicycles also seek to maintain similar trail by reducing the headtube angle and fork offset. Therefore, most small wheel bicycles have shallower headtube angle about 71-72 degrees. Therefore, we first searched for the combination of front geometry parameters for small wheel bicycles, which would exhibit a similar self-stable region as the big wheel bicycle within 71^0 to 72^0.

The self-stable region of the big wheel bicycle having 73 degrees headtube angle and 56 mm trail was calculated to be 4.71 m/s to 7.63 m/s as shown by the horizontal lines in Fig. 6 (a). The main objective was to maintain this self-stable range in small wheel bicycles by tinkering around with the front geometry parameters. This means that any combination of front geometry whose self-stable region starts at 4.71 m/s or less and ends at 7.63 m/s or more is accepted. Both the lower limit and the upper limit of the self-stability should be satisfied at the same time. The acceptable regions are marked with the arrowhead in the figures. For example, consider a 20-inches wheel bicycle with 71^0 headtube angle and 50 mm trail. The upper limit of the self-stable velocity for the 20-inch wheel bicycle with 71^0 headtube angle and 50 mm trail is above than the referenced big wheel bicycle upper velocity limit, which is acceptable. On the other hand, the lower limit for the same geometrical configuration is above the referenced velocity. In fact, the self-stable velocity range for 20-inch wheel bicycle with 71^0 headtube angle and 50 mm trail was

International Journal of Medical, Medicine and Health Sciences
ISSN: 2517-9969
Vol:12, No:9, 2018

calculated to be 5.01 m/s to 8.11 m/s. Therefore, 71^0 headtube angle and 50 mm trail did not provide comparable self-stability as the big wheel bicycle, and thus, the combination was neglected. A similar analysis was carried out for both 20-inches wheel bicycle and 16-inches wheel bicycle as presented in Fig. 6. The headtube angles were varied from 70^0 to 75^0, and trail range was selected on the basis of the ISO recommended trail range for each wheel size.

It was also found that even if small wheel bicycles had equal trail as big wheel bicycles by reducing the headtube angle and fork offset, the self-stable region was not still comparable. For example, the same big wheel bicycle with 73^0 headtube angle and 56 mm trail have a self-stable region from 4.70 m/s to 7.63 m/s. A 20-inch wheel bicycle with 71.5^0 headtube angle and the same trail exhibits self-stable region from 5.41 m/s to 9.36 m/s and, a 16-inch wheel bicycle with 71^0 headtube angle has self-stable region from 6.12 m/s to 11.15 m/s. We could not find any combination of front geometry parameters for small wheel bicycles exhibiting a similar self-stable range the big wheel bicycle if we reduce the headtube angle and fork-offset to match a similar trail value. Therefore, this analysis has clearly showed that the use of the trail values as an indicator for the stability of bicycle is not adequate. The approached might have worked well for the big wheel bicycles, but it does not translate directly to the small wheelers. As this traditional approach did not work for the small wheel bicycles, it was believed that small wheels are inherently less stable, and there is nothing that could be done to improve the performance of the small wheelers. However, if we carefully analyse Fig. 6 (a), stability of the big wheel bicycle with 73^0 headtube angle and 50 mm trail approximately matches with 20-inches wheel bicycle with 74^0-75^0 headtube angle and 40 mm trail. The self-stable region of 20-inches bicycle with 74^0 and 40 mm trail is 4.58 m/s to 7.28 m/s which is very close to self-stability (4.6 m/s to 7.23 m/s) for big wheel bicycle with 73^0 headtube angle and 50 mm trail. The observation is indeed very interesting as it was counter to the current design approach reducing the headtube angle. A comparable self-stability was found when we increased the headtube angle making it steeper and reduced the trail.



(a) (b)

Fig. 7 Identification of front geometry for small wheel bicycles showing comparable self-stability as big wheel bicycle: (a) 26-inch wheel bicycle (BWB) vs 20-inch wheel bicycle (SWB), (b) 26-inch wheel bicycle(BWB) vs 20-inch wheel bicycle(SWB)

IV. Performance Comparison of Different Wheel Sized Bicycle Beyond ISO Recommended Criteria

In Section III, we found that increasing the headtube angle could help small wheel bicycles to have a similar self-stability as compared to big wheel bicycles. Therefore, in this section, we are analysing the performance of small wheel bicycles outside the ISO recommended criteria as shown in Table I. The main objective of this analysis was to find if it is possible to obtain some combinations of front steering geometry that will enable small wheel bicycles to have similar self-stability as big wheel bicycles. Since the 73-degree headtube angle and 56 mm trail are very common for good performance bicycle, these combinations of headtube angle and trail were selected as a reference. Therefore, we varied the headtube angle from 74 degrees to 79 degrees for 20-inch wheel bicycle and 75 degrees to 83 degrees for 16-inch wheel bicycles. The trail range was extended further beyond the ISO recommended value. We searched for the combination of the headtube angle and trail which will at least exhibit the self-stable range from 4.71 m/s to 7.63 m/s. Again, the selected front geometry should have self-stable region started from 4.71 m/s or less to 7.63 m/s or more. The results are presented in Fig. 7.

As we increased the headtube angle, the self-stable range shifted towards the low velocity region. This allowed us to identify some combination of the headtube angle and trail which exhibited a comparable self-stability as big wheel bicycles. It was interesting to know that the 45-mm trail and 74 degrees headtube angle are the nominal values for 20-inch wheel bicycle to exhibits referenced big wheel bicycle self-stability. Similarly, the nominal values for the 16-inch wheel bicycles were 24 mm trail and 76 degrees headtube angle. This analysis for small wheel bicycles was based on the horizontal location of the cyclist about 29 cm to 30 cm from the rear wheel axle. The nominal values might slightly change depending upon the chain stay length and seat tube angle for the cyclists that is the position of the cyclist on the bicycle.

However, the analysis provides a strong evidence that increasing the headtube angle helps to improve the stability of the small wheel bicycles. Furthermore, the allowable range of the trail was found to be increased with the increase in head tube angles. This is counter to the general trend obtained as per the ISO recommended criteria, Fig. 1. For example: - 20-inch wheel bicycle with 75 headtube angle and trail ranging from 45-48 mm contributed to match referenced self-stability value for big wheel bicycle. The trail range increased to 45-56 mm for 77 degrees. The observation is also true for the 16-inch wheel bicycle. In the other words, the steeper is the headtube angle, the greater is the allowable trail range. Table III lists the detail of trail range for headtube angles which are beyond the ISO recommended limit.

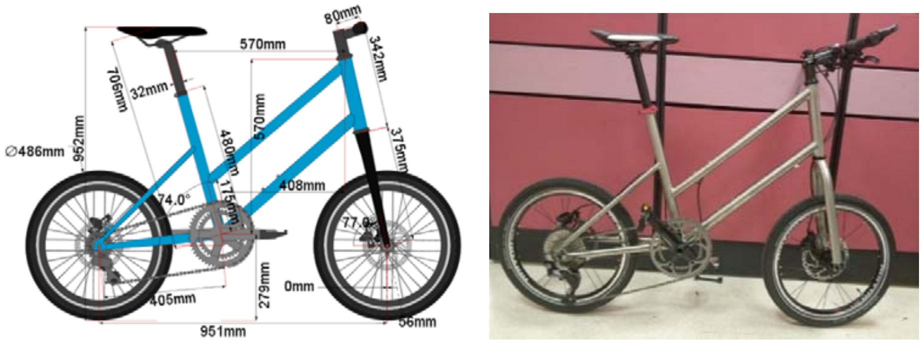


Fig. 8 Prototype of 20-inch wheel bicycle

TABLE III
FRONT GEOMETRY OF TOUR DE FRANCE 2013 BICYCLES

Headtube angle (degrees)	Trail for 20-inch wheel bicycle (mm)		Headtube angle (degrees)	Trail for 16-inch wheel bicycle (mm)	
	Min	Max		Min	Max
74	45	45	76	24	25
75	45	48	77	24	27
76	45	51	78	24	28
77	45	56	79	24	30
78	45	58	80	24	32
79	45	60	81	24	33
80	NA	NA	82	24	35

A. Prototype Design and Preliminary Riding Test

A 20-inch wheel bicycle was built to experimentally verify the results from mathematical analysis. The prototype bicycle has a very steep headtube angle. In addition, the trail is as the same as the big wheel bicycle, which is considerably longer than the ISO recommended values. The schematic bicycle diagram and the prototype are shown in Fig. 8. The bicycle was designed considering the ergonomics of an adult cyclist. Since the hands-free riding is an aspect to acknowledge the stability, the prototype bicycle was tested for hands-free riding and compared with the typical big wheel bicycles and well known small wheel bicycles such as typical mountain bicycle, Airnimals racing bicycle, Moulton spaceframe bicycle, Brompton bicycle. The prototype was tested by four different cyclists who were not very skilled on riding hands-free. The prototype was found to corroborate the mathematical analysis as the bicycle could easily be ridden hands-free as compared to the used big wheel bicycles and small wheel bicycles. The bicycle had calculated self-stability from 4.64 m/s to 8.52 m/s. The self-stability of the bicycle started at relatively low forward speed than the referenced value, therefore it showed a good low speed stability. Thus, the preliminary riding tests in a way verify that small wheel bicycles should be designed with the steeper headtube angle compared to the big wheel bicycles. A proper combination of steeper headtube angle and trail improves the stability and the rideability of the small wheel bicycles.

This analysis provided a strong evidence for improving the small wheel bicycle performance with proper selection of the front steering geometry. Furthermore, we have scientifically proved that the small wheels are not to be blamed, as per most of the cyclist does, for lesser stability of the small wheelers. In fact, the lesser stability was found to be the result of an inappropriate bicycle design practice for small wheel bicycle based guidelines formulated for big wheel bicycles.

V. CONCLUSION

This paper analyses dynamic stability performances of different wheel sized bicycles in relation with the self-stability. The stability performances are studied for typical big wheel and small wheel bicycles designed based on traditional design guidelines within the frame of ISO recommendation. A total of 91 big wheel bicycles, including 30 bicycles from the Tour De France 2013, are considered to understand the current approach for the front steering parameter selection. These bicycles collectively represent both high performance bicycles and normal bicycles. Although ISO recommendation covers a wide range of front steering parameters (headtube angle ranging from 65 degrees to 75 degrees and trail ranging from 13 mm to 95 mm), most of the bicycles are found to be designed within a narrow bandwidth of headtube angle from 70-degrees to 75- degrees and trail ranging from 50 mm to 65 mm. Amongst this narrow range, headtube angle of around 73 degrees and trail around 55-60 mm is found to cover most of the bicycle designs. Therefore, a big wheel bicycle having 73 degrees headtube angle and 56 mm trail is selected as reference steering geometry. The stability performance the big wheel bicycle is calculated using the validated mathematical model and compared with the range of small wheel bicycle steering geometry as recommended by the ISO criteria.

The stability range for 26-inch, 20-inch, and 16-inch wheel bicycles are found noticeably different. Big wheel bicycles are stable around moderate speed, whereas small wheel bicycles require higher forward speed to become self-stable. This could be correlated with the general experience of difficulties in riding small wheel bicycles. It is found that the difficulties are not only because of the reduced wheel size, lesser moment of inertia or gyroscopic effects but also the design approach used for small wheelers. We have illustrated the possible improvement of small wheelers by increasing the headtube angle and properly selecting the trail. However, most combinations of the front geometry, i.e. headtube angle and trail, are found outside the ISO recommended criteria. Therefore, the current guidelines and ISO recommendations work well for big wheel bicycles, it does not directly translate to the small wheelers and the information that we have about the small wheel bicycle designs is not adequate to ensure the stability and performance.

The study has recommended that the bicycle design should be modified properly according to the wheel size used. In fact, the direct measurement of the stability should be used as the design objective to improve the bicycle design for better stability and performance. The negative effect of the small wheels could be compensated with proper steering geometry and the performance of the small wheelers could be improved to make as stable as big wheel bicycles.

APPENDIX

TABLE IV
BENCHMARK BICYCLE PARAMETERS FOR PROTOTYPE BICYCLE

Parameters	Values
Wheelbase	950.8 (mm)
Trail	56.1 (mm)
Headtube angle	77 (degrees)
Rear wheel	
Radius	243 (mm)
Mass	1.752 (kg)
MOI	0.022, 0.0436 (kg-m^2)
Rear frame (including rider)	
Mass	75.985 (kg)
COM	0.291,0,1.024 (m)
MOI	$\begin{bmatrix} 9.48 & 0 & -2.02 \\ 0 & X & 0 \\ -2.02 & 0 & 2.55 \end{bmatrix}$(kg-m^2)
Front frame	
Mass	1.7 (Kg)
COM (u, v)	(0.23, 0.68) m
MOI	$\begin{bmatrix} 0.135 & 0 & -0.01 \\ 0 & X & 0 \\ -0.01 & 0 & 0.013 \end{bmatrix}$(kg-m^2)
Front wheel	
Radius	243 (mm)
mass	1.181 (kg)
MOI	0.027, 0.051 (kg-m^2)

REFERENCES

[1] Wilson D. G., Papadopoulos J. Bicycling science. MIT press; 2004.
[2] Proteus P. The Proteus framebuilding book: A guide for the novice bicycle framebuilder Proteus Design Inc. ; 1975.
[3] Kolin M. J., Denise M., la Rosa D. The Custom Bicycle. Rodale Press; 1979.
[4] Talbot R. P. Designing and Building Your Own Frameset: An Illustrated Guide for the Amateur Bicycle Builder. Manet Guild; 1984.
[5] Paterek T. The Paterek Manual for bicycle framebuilders. Framebuilders' Guild; 1985.
[6] BikeCad. BikeCad: Bicycle design software 2017. Available from: https://www.bikecad.ca/
[7] Hon D. T. Folding bicycles: A treatise. Dahon bicycle; 2016.
[8] Company M. B. Features (cited 2016 25 Jan). Available from: http://www.moultonbicycles.co.uk/features.html
[9] Forester J. Report On Stability Of The Da Hon Bicycle1989 cited. http://www.johnforester.com/index.html
[10] Cycles -- Safety requirements for bicycles -- Part 2: Requirements for city and trekking, young adult, mountain and racing bicycles, ISO 4210-2:2015
[11] Prince J. An investigation into bicycle performance and design: Auckland University of Technology; 2014.
[12] Whipple F. J. The stability of the motion of a bicycle. Quarterly Journal of Pure and Applied Mathematics. 1899;30(120):312-321.
[13] Timoshenko S. P., Young D. H. Advanced dynamics. McGraw-Hill Book Company, Inc.; 1948.
[14] Jones D. E. The stability of the bicycle. Physics today. 1970;23(4):34-40.
[15] Hand R. S. Comparisons and stability analysis of linearized equations of motion for a basic bicycle model. Cornell University; 1988.
[16] Sharp R. S. On the stability and control of the bicycle. Applied Mechanics Reviews. 2008;61(6):060803.
[17] Meijaard J. P., Papadopoulos J. M., Ruina A, Schwab A. L. Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences2007 p. 1955-1982. s
[18] Kooijman J., Schwab A., Meijaard J. Experimental validation of a model of an uncontrolled bicycle. Multibody System Dynamics. 2008;19(1-2):115-132.
[19] Moore J. K., Hubbard M., Kooijman J., Schwab A. A method for estimating physical properties of a combined bicycle and rider. ASME 2009 international design engineering technical conferences and computers and information in engineering conference: American Society of Mechanical Engineers; 2009. p. 2011-2020.