# Highly Accurate Target Motion Compensation Using Entropy Function Minimization

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*Abstract*—One of the defects of stepped frequency radar systems is their sensitivity to target motion. In such systems, target motion causes range cell shift, false peaks, Signal to Noise Ratio (SNR) reduction and range profile spreading because of power spectrum interference of each range cell in adjacent range cells which induces distortion in High Resolution Range Profile (HRRP) and disrupt target recognition process. Thus Target Motion Parameters (TMPs) effects compensation should be employed. In this paper, such a method for estimating TMPs (velocity and acceleration) and consequently eliminating or suppressing the unwanted effects on HRRP based on entropy minimization has been proposed. This method is carried out in two major steps: in the first step, a discrete search method has been utilized over the whole acceleration-velocity lattice network, in a specific interval seeking to find a less-accurate minimum point of the entropy function. Then in the second step, a 1-D search over velocity is done in locus of the minimum for several constant acceleration lines, in order to enhance the accuracy of the minimum point found in the first step. The provided simulation results demonstrate the effectiveness of the proposed method.

*Keywords*—ATR, HRRP, motion compensation, SFW, TMP.

## I. Introduction

**A**UTOMATIC Target Recognition (ATR) generally refers to the use of computer processing to detect and recognize target signatures. High Resolution Range Profile (HRRP) method is one of the most useful ATR methods in radar field researches that projects target scatterers centers onto Radar Line of Sight (RLOS) and reveals some of the target structure characteristics in detail such as length, scatterers distribution, etc. As an important concept in radar, resolution is being used as a criterion for distinguishing two closely point targets [1]. the range resolution is inversely proportional to its bandwidth, thus resolution is improved by widening bandwidth. But it makes the transmitter and receiver infrastructure more complicated and impractical, increases interference at receiver's input and sampling rate of A/D [2].

One of the most convenient HRRP synthesis methods is Stepped Frequency Waveform technique (SFW) that yields High Range Resolution without requiring wide instantaneous bandwidth. Unfortunately SFW is highly sensitive to target radial motion that causes range cell shift, different Doppler shifts because of their distinct carrier frequencies for a given range rate from pulse to pulse, power divergence (power spreading from one range cell to adjacent range cells), SNR and range resolution reduction and consequently range profile distortion [3]. In Non-Cooperative Target Recognition (NCTR), an unknown targets HRR profile is compared with a set of HRR profiles stored in the template library. In fact, the

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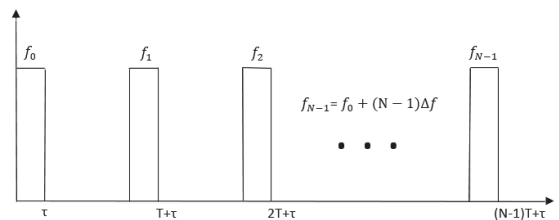


Fig. 1 Stepped Frequency Waveform

NCTR library consists of many HRR profiles obtained from stationary targets. Therefore any distortion of an unknown targets HRR profile will cause a significant deterioration in the identification performance. thus motion compensation should be taken place.

In [4]-[6] some methods are included by the bibliography. in [4] a method for velocity and acceleration estimation has been proposed by maximizing contrast function and minimizing correlation-like function. but these methods require a SNR as of 25 dB, which is too large for many practical implementations.

Also [5] has proposed a method in which motion effect compensation using entropy function minimization. Whereas sufficient accuracy for target motion parameters estimation isn't fulfilled.

An estimation-free method in [6] declares velocity effect elimination thereby exploiting two different PRIs, deriving the mathematical relation between them in the system model, even though has deficiencies like more implementation complexity since of using two PRIs instead of one and only effect of velocity eliminated and acceleration is not considered.

In this paper, we attempt to represent a method for target motion parameters estimation by minimizing entropy function using subarray averaging concept [5], [7]. This method improves accuracy for reasonable SNRs compared to the work due to [5].

## II. Generation of HRRP Using SFW

One of the most conventional approaches for HRRP generation is SFW technique which can achieve high range resolution and doesn't require wide instantaneous bandwidth [8]. The SFWs consist of a series of narrow-band pulses, and the frequency from pulse to pulse is stepped by a fixed frequency step. Fig. 1 illustrates a SFW pulse train with $PRI = T$ and pulse width of $\tau$.

The received signal is then sampled at the time t=nT whose mathematical equation is:

$$s(n) = \sum_{k=1}^{K} A_k exp\left( -j\frac{4\pi}{c}\left[f_0 + n\Delta f\right] r_{kn} \right)$$
$$n = 0, 1, 2, ..., N-1 \qquad (1)$$

where

$$r_{kn} = r_{k0} + v_r nT + \frac{1}{2} a_r n^2 T^2 \qquad (2)$$

In above equation $K$ denotes the number of target scatterer centers, $A_k$ is strength of the $kth$ target scatterer center, c is the speed of light, $nT$ is the corresponding delay time of the $n$th received pulse, $r_{k0}$ and $r_{kn}$ are the radial distance of RLOS from the $k$th target scatterer center at time $t = 0$ and $t = nT$ respectively, $N$ is the total number of the transmitted pulses, $f_0$ is the initial carrier frequency and $\Delta f$ is the frequency step for which carrier frequencies are different. $v_r$ and $a_r$ are the radial velocity and acceleration of target respectively which produce additional frequency components and cause Doppler effect which is:

$$f_d = \frac{2}{c}(f_0 + (n-1)\Delta f)(v_r + a_r nT) \qquad (3)$$

where $f_d$ is Doppler frequency.

This equation shows that transmitted pulses with different carrier frequencies commit different Doppler shifts and cause distortion in HRRP generated by this technique. Therefore TMP effects should be eliminated. If the TMPs could be estimated somehow, the phase error term emanated from Doppler effect in received signal would vanish as well. Since the received signal ($s(n)$) contains different frequency components caused by TMPs, the $s(n)$ is then multiplied in an exponential term including estimated TMP as below:

$$\hat{s}(n) = s(n)$$
$$\times exp\left( j\frac{4\pi}{c}\left[f_0 + n\Delta f\right].\left[\hat{v}_r nT + \frac{1}{2}\hat{a}_r n^2 T^2\right] \right)$$
$$= \sum_{n=0}^{N-1} A_k exp\left( -j\frac{4\pi}{c}\left[f_0 + n\Delta f\right].\left[(v_r - \hat{v}_r)nT \right.\right.$$
$$\left.\left. +\frac{1}{2}(a_r - \hat{a}_r)n^2 T^2\right] \right) \qquad (4)$$

where $\hat{v}_r$ and $\hat{a}_r$ are the estimations of $v_r$ and $a_r$ respectively. This multiplication is necessary so as to estimate TMP effects and $\hat{s}(n)$ is compensated at the end. Also it's supposed that the estimated TMP is sufficiently close to their true values.

### III. USING ENTROPY AS A COST FUNCTION FOR TMP ESTIMATION

As the target keeps moving, the total response of HRRP using SFW technique gets shifted, the peak decreases and causes distortion such as Side Lobe Levels (SLLs), blurring effects, etc along with increasing target velocity and acceleration which limits the target recognition accuracy [9]. One of the most conventional function for TMP estimation is entropy function. From image processing background, it represents the measure of image randomness and is minimized for focused images [5], [7]. entropy function is as follows:

$$E = -\sum_{n=0}^{N-1} \hat{g}(n) \ln\left( \hat{g}(n) \right) \qquad (5)$$

where

$$\hat{g}(n) = \frac{g(n)}{\sum_{n=0}^{N-1} g(n)} \qquad (6)$$
$$g(n) = IFFT\left( \hat{s}(n) \right) \qquad (7)$$

Then $\hat{v}_r$ and $\hat{a}_r$ could be estimated by minimizing entropy function. But with entropy function due to eq. 5 leads to many local minimums in entropy surface at velocity-acceleration space [9].

In [7] a method for decreasing this local minimums called "Subarray Averaging (SA)" has been proposed. In SA transmitting M overlapping subarrays of length L pulses for which the distance between consecutive of them is d, has been considered instead of transmitting an array of N pulses. The main idea is inspired by concept of Spectral Estimation, when overlapping in windowing is allowed in order to increase in correlation of covariance function and average of different windows to each other and avoid the very bad effect of deviated range profile data which is finally limited by resolution considerations (In conventional method, resolution is equal to $\frac{c}{2N\Delta f}$, but in SAEM method it's achieved by the equation $\frac{c}{2L\Delta f}$, therefore the resolution decreases by the ratio $\frac{L}{N}$ using SAEM method). The relation of entropy function due to [5] is as following:

$$\hat{E} = -\sum_{l=0}^{L-1} \hat{g}_m(l) \ln\left( \hat{g}_m(l) \right) \qquad (8)$$

where

$$\hat{g}_m(l) = \frac{\sum_{m=1}^{M} |g_m(l)|^2}{\sum_{m=1}^{M} \sum_{l=0}^{l=L-1} |g_m(l)|^2} \qquad (9)$$

$$g_m(l) = IFFT\left( \hat{s}(n + (m-1)d) \right)$$
$$= \frac{1}{L} \sum_{k=1}^{K} \sum_{n=0}^{L-1} A_k exp\left( j\phi(m, n, k, \hat{v}_r, \hat{a}_r) \right)$$
$$\times exp\left( \frac{j2\pi l\left[n + (m-1)d\right]}{L} \right)$$
$$m = 1, 2, ..., M \qquad l = 0, 1, ..., L-1 \qquad (10)$$

where

$$\phi(m, n, k, \hat{v}_r, \hat{a}_r) = -\frac{j4\pi}{c}\left\{ \left(f_0 + \left[n + (m-1)d\right]\Delta f\right) \right.$$
$$\times \left(r_{k0} + (v_r - \hat{v}_r)\left[n + (m-1)d\right]T \right.$$
$$\left.\left. +\frac{1}{2}(a_r - \hat{a}_r)\left[n + (m-1)d\right]^2 T^2\right) \right\}$$
$$(11)$$

If the effects of $v_r$ and $a_r$ are still persistent, $\hat{g}_m(l)$ would be absolutely blurred and distorted. 2-D gradient methods in
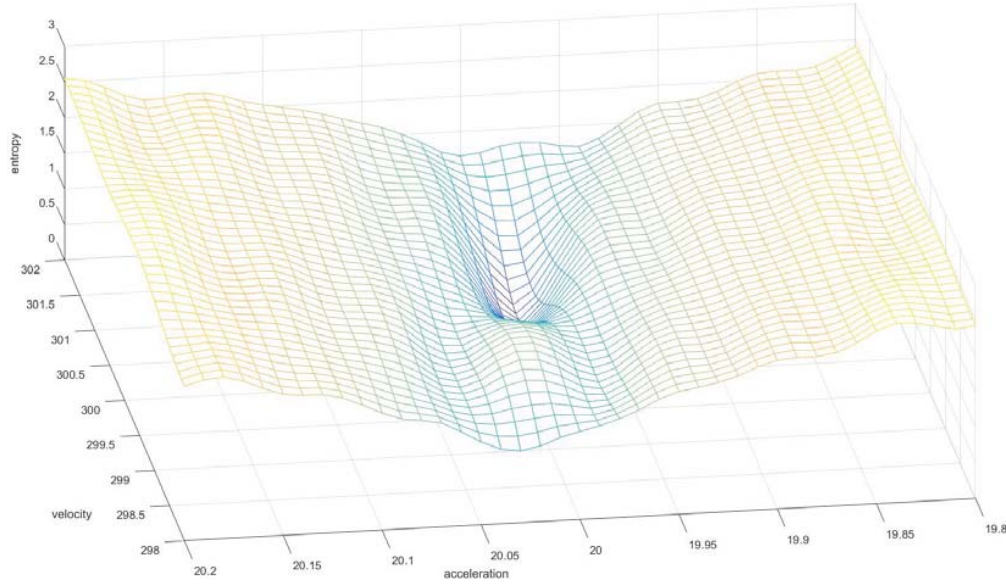
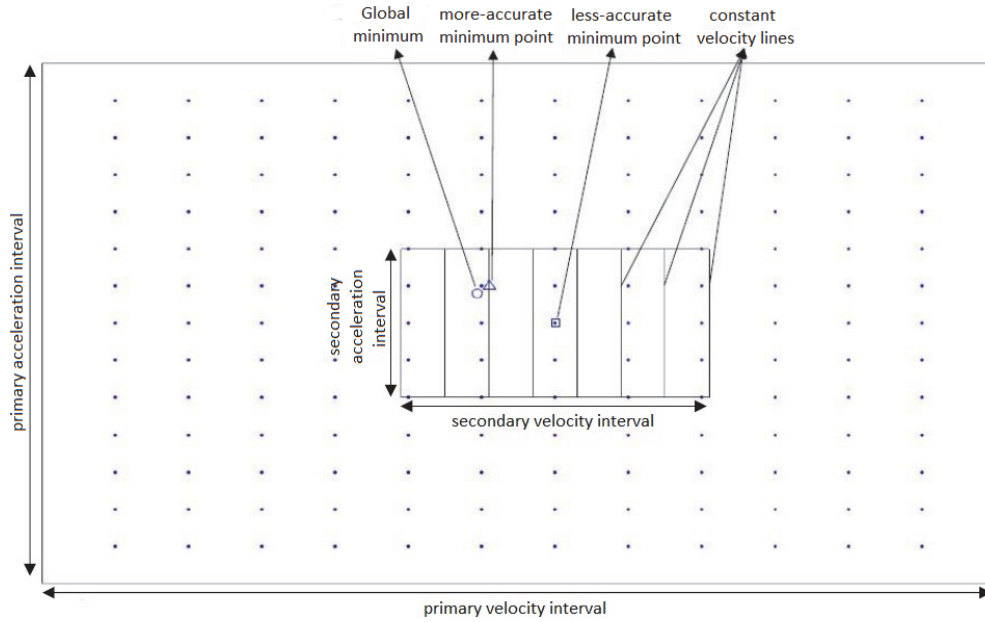Fig. 2 Entropy function with SA concept for $[\hat{v}_r, \hat{a}_r] = [300, 20]$



Fig. 3 The proposed method

velocity-acceleration space or Newton method using 1st and 2nd order derivatives of entropy function respect to $\hat{v}_r$ and $\hat{a}_r$ also with rather complicated calculations can be employed for optimizing entropy function.

In [5], [7], a proposed method consists of optimizing cost function using a series of linear 1-D searches. In this paper, a revised method for such an optimization has been introduced.

## IV. OPTIMIZATION OF THE COST FUNCTION FOR TMPS ESTIMATION

Fig. 2 illustrates entropy function of SA method for $[v_r, a_r] = [300, 20]$. It could be seen that the global minimum

of the entropy function is very close to true TMP values. Here a method has been introduced for TMP estimation whose steps are:

1) Initial search space allocation for velocity $[\hat{v}_{min}, \hat{v}_{max}]$ and acceleration $[\hat{a}_{min}, \hat{a}_{max}]$.
2) Velocity and acceleration space division to $n_{vel}$ and $n_{acc}$ lines and obtaining $n_{acc} \times n_{vel}$ sporadic points for producing lattice network (as of Fig. 3).
3) Entropy function calculation at this points and finding a point with approximate minimum (less-accurate minimum point) entropy value (shown in Fig. 3).
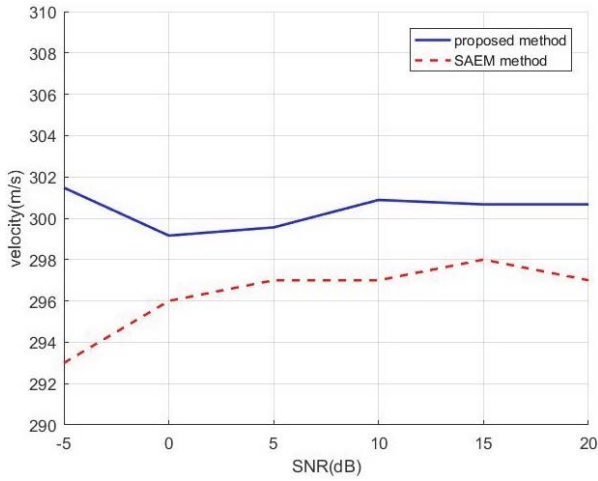4) 1-D search (like Golden Section Search (GSS) method)

Fig. 4 Averages of the the estimated velocity against SNR



Fig. 5 Standard deviations of the estimated velocity against SNR



Fig. 6 Averages of the estimated acceleration against SNR

employment in secondary velocity and acceleration interval for finding more-accurate minimum point around the point derived from step 3 over different constant-acceleration lines in velocity space (shown in Fig. 3).

We have concluded from many experimental results that proper values for $n_{vel}$ and $n_{acc}$ is about $200-300$ and $40-60$ respectively. Also the number of constant-acceleration lines used in step 4 is as of $20-40$. All this three values suffice for true TMP estimation. The secondary velocity-acceleration interval (The neighborhood of the less-accurate minimum point from step3) has been chosen as of $5-10$ percent of primary search interval. Increasing $n_{vel}$ , $n_{acc}$ and number of constant acceleration lines increases the simulation time without any remarkable improvement in TMP estimation. In [5] SAEM (Subarray Averaging and Entropy Minimization) method has been proposed to optimized cost function, while our proposed method guarantees a better accuracy (more accurate minimum point is very close to true minimum point) for TMP estimation compare to SAEM.

## V. SIMULATION RESULTS

In this section, the results of the proposed method have been compared to those of SAEM method in [5]. We assume the Target Motion and Signal Parameters to be as following:

Initial distance from point target to RLOS $r_0 = 10km$, initial carrier frequency $f_0 = 9GHz$, frequency step $\Delta f = 1MHz$, $PRF = 1KHz$, number of SFW pulses $N = 512$, bandwidth $BW_{total} = N\Delta f = 512MHz$, the length of each subarray $L = \frac{N}{4} = 128$, the interval of consecutive subarrays $d = 2$, total number of subarrays $M = \frac{N-L}{d} + 1 = 193$, radial velocity of target $v_r = 300m/s$, radial acceleration of target $a_r = 20m/s^2$, the initial search space for velocity and acceleration is: $[\hat{v}_{min}, \hat{v}_{max}] = [0, 600]m/s$ and $[\hat{a}_{min}, \hat{a}_{max}] = [-10, 30]m/s^2$ respectively.

The simulation has been performed 100 times for a data sequence mixed up to AWGN. Figs. 4 and 6 show mean values of the estimated velocity and acceleration for SNRs {-5, 0, 5,
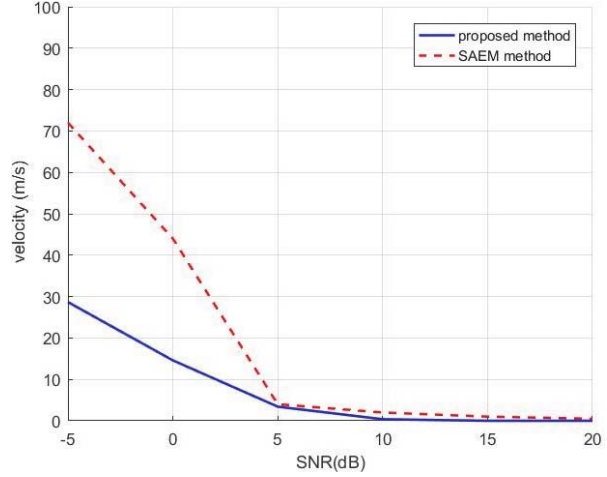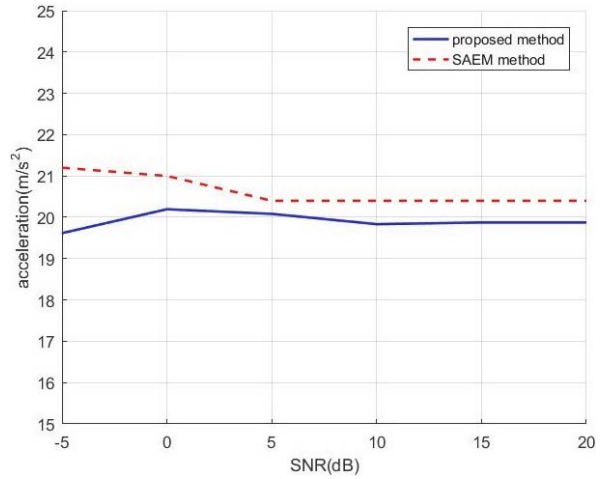
10, 15, 20} and Figs. 5 and 7 demonstrate their associated standard deviations for SAEM and proposed method. We can see in the figures that the proposed method is better than SAEM method in TMP estimation accuracy for all SNRs and the estimated standard deviations of velocity and acceleration in our method are low for SNRs 5-20 dB and their mean values are very close to the actual velocity and acceleration, thus we have reliable results.

## VI. CONCLUSION

In this paper a method for estimation of target motion parameters is proposed. The goal behind this method is to compensate phase errors caused by target motion in HRRP using SFW and thus achieve a focused HRRP. The TMPs were estimated with a very high accuracy. Our approach is to apply the concept of SA and use the entropy as a cost function and optimize it through many simulations, by 1-D searches in velocity-acceleration space. Thus, the motion effects of the target, is extremely decreased and focused HRRP is obtained.
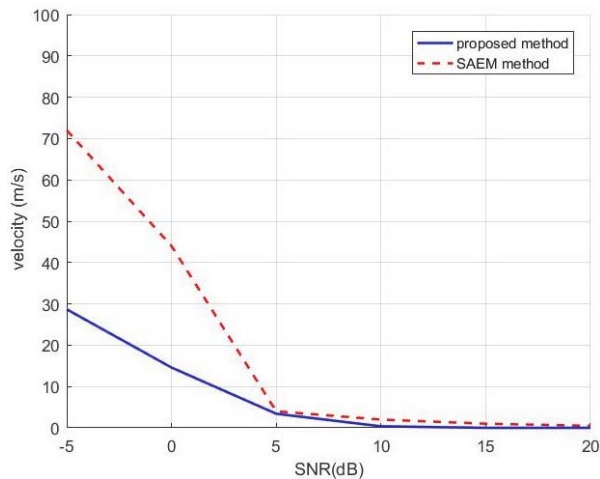
Fig. 7 Standard deviations of the estimated acceleration against SNR

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