

Calculation of the Thermal Stresses in an Elastoplastic Plate Heated by Local Heat Source

M. Khaing, A. V. Tkacheva

Abstract—The work is devoted to solving the problem of temperature stresses, caused by the heating point of the round plate. The plate is made of elastoplastic material, so the Prandtl-Reis model is used. A piecewise-linear condition of the Ishlinsky-Ivlev flow is taken as the loading surface, in which the yield stress depends on the temperature. Piecewise-linear conditions (Tresca or Ishlinsky-Ivlev), in contrast to the Mises condition, make it possible to obtain solutions of the equilibrium equation in an analytical form. In the problem under consideration, using the conditions of Tresca, it is impossible to obtain a solution. This is due to the fact that the equation of equilibrium ceases to be satisfied when the two Tresca conditions are fulfilled at once. Using the conditions of plastic flow Ishlinsky-Ivlev allows one to solve the problem. At the same time, there are also no solutions on the edge of the Ishlinsky-Ivlev hexagon in the plane-stressed state. Therefore, the authors of the article propose to jump from the edge to the edge of the mine edge, which gives an opportunity to obtain an analytical solution. At the same time, there is also no solution on the edge of the Ishlinsky-Ivlev hexagon in a plane stressed state; therefore, in this paper, the authors of the article propose to jump from the side to the side of the mine edge, which gives an opportunity to receive an analytical solution. The paper compares solutions of the problem of plate thermal deformation. One of the solutions was obtained under the condition that the elastic moduli (Young's modulus, Poisson's ratio) which depend on temperature. The yield point is assumed to be parabolically temperature dependent. The main results of the comparisons are that the region of irreversible deformation is larger in the calculations obtained for solving the problem with constant elastic moduli. There is no repeated plastic flow in the solution of the problem with elastic moduli depending on temperature. The absolute value of the irreversible deformations is higher for the solution of the problem in which the elastic moduli are constant; there are also insignificant differences in the distribution of the residual stresses.

Keywords—Temperature stresses, elasticity, plasticity, Ishlinsky-Ivlev condition, plate, annular heating, elastic moduli.

I. INTRODUCTION

RESIDUAL stresses and deformations occurring in the metal structure after welding work play an important role in the further operation of this product. Therefore, studies aimed at modeling the welding processes in order to predict residual stresses and deformations in the zone of the point impact of welding are currently relevant.

One of the model problems of the welding process is the problem of the theory of temperature stresses about heating point of a plate. Similar problems were discussed repeatedly [1]-[6], but in them, the elastic moduli did not depend on temperature, although a linear or parabolic error was taken for

the yield limit. The dependence of the elastic modulus on temperature is fairly well known [7]-[9], but the effect of the same on the solution of problems, temperature stresses have not been studied to date. Therefore, this paper is based on the simplest problem of the temperature of the elastic moduli. Thanks to the use of a piece of small plastic potential, namely: the conditions of the plastic flow of Ishlinsky-Ivlev, it is possible to obtain an analytical solution of the problem with constant elastic moduli. Similar decision algorithms are described in [10], [11].

II. FORMULATION OF THE PROBLEM

Elastic deformation - Consider the round aluminum plate the initial temperature, which is equal to room temperature T_0 . In the center of the plate, we arrange a cylindrical coordinate system $r_i (r, \varphi, z)$, place a solid heat source with a radius R_0 in the center of coordinates, and heat the plate according to the law:

$$T|_{r \leq R_0} = qt, \quad (1)$$

where q is the heating rate, t is the time, the index after the comma indicates the derivative with respect to the corresponding coordinate. At some point in time t_* , conditional $T(R_0, t_*) = 0.85T_p$, and $0 < \tau \leq 1$, T_p — the melting point of the plate material, the heating is stopped and the plate is left at room temperature. Due to the heat removal from the surface of the plate to the environment, the plate will cool down. As with heating and cooling, the temperature distribution across the plate at each instant is found from the heat equation

$$T_{,t} = a(rT_{,r})_{,r} + \chi(T_0 - T), \quad (2)$$

where a is the coefficient of thermal diffusivity, χ is the coefficient, which depends on the heat transfer, thermal conductivity, the density of the material and the thickness of the plate. We shall not take into account the heat-conducting influence of deformation, that is, we further assume that the mechanical problem is disconnected. Now, according to the temperature field calculated according to (1), (2) at each moment of time, we calculate the mechanical parameters of strains and stresses.

We consider, deformation d_{ij} $i, j = (r, \varphi, z)$ in the material

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small and consisting of reversible e_{ij}^e and irreversible e_{ij}^p constituent

$$d_r = e_r^e + e_r^p, \quad d_\varphi = e_\varphi^e + e_\varphi^p, \quad d_z = e_z^e + e_z^p, \\ d_r = u_{r,r}, \quad d_\varphi = u_r r^{-1}.$$

To find the stresses, we use the Duhamel-Neumann law, which connects an irreversible deformation, e_{ij}^e stresses σ_{ij} and the current temperature T

$$\begin{aligned} \sigma_r &= (\lambda + 2\mu)e_r^e + \lambda(e_\varphi^e + e_z^e) - 3K\alpha(T - T_0), \\ \sigma_\varphi &= (\lambda + 2\mu)e_\varphi^e + \lambda(e_r^e + e_z^e) - 3K\alpha(T - T_0), \\ \sigma_z &= (\lambda + 2\mu)e_z^e + \lambda(e_r^e + e_\varphi^e) - 3K\alpha(T - T_0), \end{aligned} \quad (3)$$

where α is the coefficient of linear expansion, all-round compression module K and parameters of Lamé λ , μ are represented by the following dependences in terms of the Young's modulus E and Poisson's coefficient ν :

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}.$$

Under conditions of a significant change in temperature, it has been experimentally confirmed that the elastic moduli vary with the temperature change [7]-[9]. Therefore, we take as its simplest dependences the changes in the elastic moduli

$$\begin{aligned} E(t) &= E_0 + (E_p - E_0)f(t), \\ \nu(t) &= 0.5 - (0.5 - \nu_0)f(t), \\ \tau(T) &= \tau(t) = (T_p - T(t))(T_p - T_0)^{-1}. \end{aligned} \quad (4)$$

Here, the constants E_0, ν_0 are the Young's modulus and Poisson's coefficient at room temperature, T_p is the melting point of the metal. Irreversible deformations accumulate in the plate under the conditions of the stresses of the loading surface $f(\sigma_{ij})=0$ in the space of stresses. In this way, we assume that the loading surface is independent of kinematics and the history of deformation. As a condition of plasticity, we take the condition of the maximum reduced shear stress (Ishlinsky-Ivlev criterion), then the loading surface takes the form of the Ivlev prism in the space of principal stresses [12].

$$\max|\sigma_i - \sigma| = \frac{4}{3}k, \quad \sigma = \frac{1}{3}\sigma_{kk}.$$

For yield stress $k(T)$, we assume the following relationship

$$k(T) = k_0 \tau^2(T).$$

Assuming the conditions of the Mises maximum principle, we have the relations of the associated law of plastic flow

$$de_{ij}^p = d\phi \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}}, \quad d\phi > 0. \quad (5)$$

So, to find the stresses and strains in the material of a plate subject to annular heating, we will use the relationships described above and the laws to which we must add a single non-trivial equation of equilibrium

$$\sigma_{r,r} + r^{-1}(\sigma_r - \sigma_\varphi) = 0 \quad (6)$$

Assuming that, at the initial time $t=0$, there were no irreversible deformations in the material, for a plane stress state $\sigma_z = 0$ the Duhamel-Neumann law will be written in the form

$$\begin{aligned} \sigma_r &= (4\mu(\lambda + \mu)u_{r,r} + 2\lambda\mu r^{-1}u_r - 6K\mu\theta)(\lambda + 2\mu)^{-1}, \\ \sigma_\varphi &= (2\lambda\mu u_{r,r} + 4\mu(\lambda + \mu)r^{-1}u_r - 6K\mu\theta)(\lambda + 2\mu)^{-1}, \\ \theta &= \alpha(T - T_0) \end{aligned} \quad (7)$$

Substituting (7) into the equilibrium equation (6), we obtain a differential equation with respect to displacements u_r

$$\xi u_{r,rr} + \zeta u_{r,r} + \omega u_r + \beta = 0, \quad (8)$$

where $\xi, \zeta, \omega, \beta$ are some functions from temperature and radius. In the case of the dependence of the elastic moduli on the temperature (4), they will be written in the form

$$\begin{aligned} \xi &= 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}, \\ \zeta &= 4(\mu^2\lambda_r + \mu_r(\lambda^2 + 2\lambda\mu + 2\mu^2))(\lambda + 2\mu)^{-2} + 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}r^{-1}, \\ \omega &= 2(2\mu^2\lambda_r + \lambda^2\mu_r)(\lambda + 2\mu)^{-2}r^{-1} - 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}r^{-2}, \\ \beta &= -\theta(8\mu^2\lambda_r + 2(3\lambda^2 + 4\lambda\mu + 4\mu^2)\mu_r)(\lambda + 2\mu)^{-2} - 6K\mu\theta_r(\lambda + 2\mu)^{-1}. \end{aligned} \quad (9)$$

And then, the equation of equilibrium (8), (9) can be solved at each time step only numerically, for example, by a different method, using boundary conditions simulating the free boundary on $r=R$ and lack of movement in the center $r=0$

$$u_r|_{r=0} = 0, \quad \sigma_r|_{r=R} = 0 \quad (10)$$

If we neglect the dependence of the elastic moduli on the temperature, then the functions $\xi, \zeta, \omega, \beta$ take the form

$$\xi = 1, \zeta = r^{-1}, \omega = -r^{-2}, \beta = -1.5K\theta_r(\lambda + \mu)^{-1} \quad (11)$$

And from the equilibrium equation (8), (11), we find the displacements

$$u_r = 1.5K\alpha(\lambda + \mu)^{-1} r^{-1} \int_l^r \rho T(\rho) d\rho + 0.5rC_h + r^{-1}C_{h+1}, \quad (12)$$

$$l = 0, \quad h = 1,$$

where C_1 and C_2 are the integration constants (time functions).

$$\begin{aligned} \sigma_r &= -3K\mu(\lambda + \mu)^{-1} r^{-2} \alpha \int_l^r \rho T(\rho) d\rho + 3K\mu(\lambda + 2\mu)^{-1} C_1 - 2\mu r^{-2} C_2 \\ \sigma_\phi &= 3K\mu(\lambda + \mu)^{-1} r^{-2} \alpha \int_l^r \rho T(\rho) d\rho + (\mu^2(\lambda + \mu)^{-1} - 3\mu) \alpha T + \\ &\quad + 3\mu K(\lambda + 2\mu)^{-1} C_1 + 2\mu r^{-2} C_2 \end{aligned} \quad (13)$$

The displacements and stresses in the absence of the dependence of the elastic moduli on the temperature are found from (12) and (13) together with the boundary conditions (10). In the case of the dependence of elastic moduli on the temperature of displacement, as mentioned earlier, from the numerical solution of the equilibrium equation written in the displacements (8) and the boundary conditions (10), and then the stresses are calculated.

Plastic flow—In the center of the plate $r = 0$, the Ishlinsky-Ivlev condition is satisfied in both calculations $\sigma_r + \sigma_\phi = -4k$. This leads to the formation of an elastoplastic boundary $n(t)$, which divides the material of the plate into two regions: elastically deformable $n(t) \leq r \leq R$ and the area prone to plastic flow $0 \leq r \leq n(t)$. In an elastically deformable region, the stresses and displacements are calculated from (12) and (13) with allowance for $l = n(t)$. In the plastic flow region $0 \leq r \leq n(t)$, the Duhamel-Neumann law (3) can be rewritten with the presence of developing irreversible deformations e_r^p, e_ϕ^p, e_z^p ,

$$\begin{aligned} \sigma_r &= (4\mu(\lambda + \mu)(u_{r,r} - e_r^p) + 2\lambda\mu(r^{-1}u_r - e_\phi^p) - 6K\mu\theta)(\lambda + 2\mu)^{-1}, \\ \sigma_\phi &= (2\lambda\mu(u_{r,r} - e_r^p) + 4\mu(\lambda + \mu)(r^{-1}u_r - e_\phi^p) - 6K\mu\theta)(\lambda + 2\mu)^{-1}. \end{aligned} \quad (14)$$

Irreversible deformations in the region of plastic flow are related to each other by a consequence of the associated law of plastic flow (5) $e_r^p = e_\phi^p$. From the plastic flow condition $\sigma_r + \sigma_\phi = -4k$ and (14), we express the irreversible deformation

$$e_r^p = 0.5(r^{-1}u_r + u_{r,r}) - (\lambda + 2\mu)(3\mu K)^{-1}k - \theta \quad (15)$$

Taking into account the above, the equilibrium equation assumes the form

$$\xi = 1, \zeta = r^{-1}, \omega = -r^{-2}, \beta = -2\mu^{-1}k_r \quad (16)$$

Integrating the equation of equilibrium (8), (16) and substituting (15) and (14), we obtain expressions determining the displacements, stresses and irreversible deformation in the region of plastic flow

$$\begin{aligned} u_r &= 2(\mu r)^{-1} \int_0^r \rho k(\rho) d\rho - C_3(2\mu r)^{-1} - rC_4, \\ e_r^p &= -4((3\mu)^{-1} + (9K)^{-1})k - \theta + C_4, \\ \sigma_r &= r^{-2} \int_0^r \rho k(\rho) d\rho + r^{-2}C_3. \end{aligned}$$

Integration factors C_3 and C_4 (time functions), together with the coefficients C_1 and C_2 from the relation (13) and (14), are found from the boundary conditions simulating the continuity of the radial stress $n(t)$ and displacement on the elastoplastic boundary, as well as from the conditions (10).

If we take into account the dependence of the elastic moduli on the temperature, then the functions in the equilibrium equation (8) take the form:

$$\xi = \mu, \zeta = (\mu r^{-1} + \mu_{,r}), \omega = -(\mu r^{-2} + \mu_{,r}r^{-1}), \beta = 2k_{,r} \quad (17)$$

We find the solution of (8), (17) by numerical method, and then the stresses (14) and irreversible deformations (15) are calculated. Fig. 1 shows the distribution of stresses in the material of a plate with a region of plastic flow.

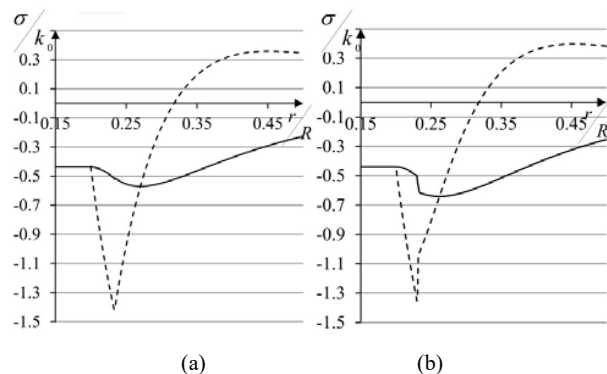


Fig. 1 Distribution of stresses in the material of the plate prior to the onset of unloading (a) - Elastic moduli are constant, (b) - elastic moduli depend on temperature

Unloading—When removing the heat source from the plate, the plate material cools. As a result, in the solutions obtained under the condition of constancy of the elastic moduli, the rate of growth of irreversible deformations decreases and the damping of the entire flow region follows. In the other words, the region $0 \leq r \leq n$ in which irreversible deformations developed e_r^p, e_ϕ^p, e_z^p , become an elastically deformable region with the presence of residual irreversible deformations p_r, p_ϕ, p_z , therefore the Duhamel-Neumann law takes the

form

$$\begin{aligned} \sigma_r &= \left(\frac{4\mu(\lambda + \mu)(u_{r,r} - p_r) + 2\lambda\mu(r^{-1}u_r - p_\phi) - 6K\mu\theta}{(\lambda + 2\mu)^{-1}} \right) \\ \sigma_\phi &= \left(\frac{2\lambda\mu(u_{r,r} - p_r) + 4\mu(\lambda + \mu)(r^{-1}u_r - p_\phi) - 6K\mu\theta}{(\lambda + 2\mu)^{-1}} \right) \end{aligned} \quad (18)$$

$$\begin{aligned} \xi &= 1, \quad \zeta = r^{-1}, \quad \omega = -r^{-2}, \\ \beta &= -0.5(\lambda + 2\mu)(r(\lambda + \mu))^{-1}(p_\phi - p_r) - p_{r,r} - 0.5\lambda(\lambda + \mu)^{-1}p_{\phi,r} - \\ &\quad - \theta(8\mu^2\lambda_{,r} + 2(3\lambda^2 + 4\lambda\mu + 4\mu^2)\mu_{,r})(\lambda + 2\mu)^{-2} - 6K\mu\theta_{,r}(\lambda + 2\mu)^{-1} \end{aligned} \quad (19)$$

Integrating the equilibrium equations (8), (19), there are displacements, which after substituting in (18) and thereby determining the stresses in the unloading region.

In accordance with (18), the functions in the equilibrium equation will be rewritten (8)

$$\begin{aligned} u_r &= 1.5K(\lambda + \mu)^{-1}r^{-1} \int_l^r \rho\theta(\rho)d\rho + 0.25r(\lambda + 2\mu)(\lambda + \mu)^{-1} \int_l^r \rho^{-1}(p_r(\rho) - p_\phi(\rho))d\rho + \\ &\quad + 0.75Kr^{-1}(\lambda + \mu)^{-1} \int_l^r \rho(p_r(\rho) + p_\phi(\rho))d\rho + 0.5rC_h + r^{-1}C_{h+1}, \\ &\quad l = 0, \quad h = 5, \\ \sigma_r &= -3K\mu(\lambda + \mu)^{-1}r^{-2} \int_l^r \rho\theta(\rho)d\rho - 1.5\mu Kr^{-2}(\lambda + \mu)^{-1} \int_l^r \rho(p_r(\rho) + p_\phi(\rho))d\rho + \\ &\quad + 1.5\mu K(\lambda + \mu)^{-1} \int_l^r \rho^{-1}(p_r(\rho) - p_\phi(\rho))d\rho + 3K\mu(\lambda + 2\mu)^{-1}C_h - 2\mu r^{-2}C_{h+1} \\ \sigma_\phi &= 3K\mu(\lambda + \mu)^{-1}r^{-2} \int_l^r \rho\theta(\rho)d\rho + (\mu^2(\lambda + \mu)^{-1} - 3\mu)\theta + \\ &\quad + 1.5\mu Kr^{-2}(\lambda + \mu)^{-1} \int_l^r \rho(p_r(\rho) + p_\phi(\rho))d\rho + 1.5\mu K(\lambda + \mu)^{-1} \int_l^r \rho^{-1}(p_r(\rho) - p_\phi(\rho))d\rho - \\ &\quad - 3K(\lambda + \mu)^{-1}p_r + 3\mu K(\lambda + 2\mu)^{-1}C_h + 2\mu r^{-2}C_{h+1} \end{aligned} \quad (20)$$

The integration coefficients C_1, C_2, C_5, C_6 , are found from the boundary conditions simulating the continuity of the radial stress and displacement to the boundaries, and from the conditions (10). In calculations that take into account the relationship between the elastic moduli and temperature, the attenuation of the plastic flow occurs in the center of the plate,

a unloading elastoplastic boundary is formed $m(t)$, which, with the spread of temperature, moves along the border $n(t)$. In the region of unloading $0 \leq r \leq m(t)$ the function in the equation of equilibrium (8) takes the form:

$$\begin{aligned} \xi &= 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}, \\ \zeta &= 4(\mu^2\lambda_{,r} + \mu_{,r}(\lambda^2 + 2\lambda\mu + 2\mu^2))(\lambda + 2\mu)^{-2} + 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}r^{-1}, \\ \omega &= 2(2\mu^2\lambda_{,r} + \lambda^2\mu_{,r})(\lambda + 2\mu)^{-2}r^{-1} - 4\mu(\lambda + \mu)(\lambda + 2\mu)^{-1}r^{-2}, \\ \beta &= -\theta(8\mu^2\lambda_{,r} + 2(3\lambda^2 + 4\lambda\mu + 4\mu^2)\mu_{,r})(\lambda + 2\mu)^{-2} - 6K\mu\theta_{,r}(\lambda + 2\mu)^{-1} + \\ &\quad + -0.5(\lambda + 2\mu)(r(\lambda + \mu))^{-1}(p_\phi - p_r) - p_{r,r} - 0.5\lambda(\lambda + \mu)^{-1}p_{\phi,r}. \end{aligned} \quad (21)$$

The solutions of the equilibrium equation (8) (21) in the case of a connection between the elastic moduli and the temperature are found numerically. After the obtained results are substituted in (18) and the stresses in the material of the plate are determined.

Repeated plastic flow - Repeated plastic flow is possible only in calculations with constant elastic moduli. In the material of the plate, about the boundary $r = n$ at the temperature distribution and cooling, the plastic flow condition begins to be satisfied $2\sigma_\phi - \sigma_r = -4k$. Here two

divergent boundaries are formed $w(t)$ and, $s(t)$ at that $w(t) \leq n \leq s(t)$. In the regions $0 \leq r \leq w(t)$ and $s(t) \leq r \leq R$. The stresses and displacements are found from relations (20) and (12), (13). In the plastic flow region $w(t) \leq r \leq s(t)$, the Duhamel-Neumann law, taking into accounts the developing irreversible deformations e_r^p, e_ϕ^p, e_z^p and the existing ones p_r, p_ϕ, p_z , takes the form

$$\begin{aligned}\sigma_r &= (4\mu(\lambda + \mu)(u_{r,r} - e_r^p - p_r) + 2\lambda\mu(r^{-1}u_r - e_\phi^p - p_\phi) - 6K\mu\theta)(\lambda + 2\mu)^{-1}, \\ \sigma_\phi &= (2\lambda\mu(u_{r,r} - e_r^p - p_r) + 4\mu(\lambda + \mu)(r^{-1}u_r - e_\phi^p - p_\phi) - 6K\mu\theta)(\lambda + 2\mu)^{-1}.\end{aligned}\quad (22)$$

Using the corollary of the associated law of plastic flow $e_r^p = -0.5e_\phi^p$ and the current plastic flow condition $2\sigma_\phi - \sigma_r = -4k$, we find an irreversible deformation

$$\begin{aligned}e_r^p &= (\mu(6\lambda + 10\mu)^{-1} - 0.5)(p_\phi - r^{-1}u_r) + \mu(3\lambda + 5\mu)^{-1}(u_{r,r} - p_r) \\ &- (\lambda + 2\mu)\mu^{-1}(3\lambda + 5\mu)^{-1}k + 1.5K(3\lambda + 5\mu)^{-1}\theta\end{aligned}\quad (23)$$

Equation (6), taking (22) and (23) into account, is written in the form (8), where the functions are

$$\xi = 1, \zeta = r^{-1}, \omega = -0.25r^{-2}$$

$$\begin{aligned}\beta &= -1.5K\theta_{,r} - 3/4\theta(\lambda + \mu)^{-1} + 1/3K^{-1}k + \\ &(2/3K^{-1} + 0.5\mu^{-1})r^{-1}k - (p_{r,r} + 0.5r^{-1}p_r) - \\ &- 0.5(p_{\phi,r} + 0.5r^{-1}p_\phi)\end{aligned}\quad (24)$$

We substitute the solutions (8), (24) in (22) and (23), we find expressions for stress calculations and irreversible deformations in the region of repeated plastic flow.

$$\begin{aligned}\sigma_r &= -2r^{-0.5} \int_{w(t)}^r \rho^{-0.5} k(\rho) d\rho + r^{-0.5} C_7 \\ u_r &= r^{-0.5} \int_{w(t)}^r \rho^{0.5} (p_r(\rho) + 0.5p_\phi(\rho)) d\rho - r^{0.5} (3\lambda + 5\mu)(3\mu K)^{-1} \int_{w(t)}^r \rho^{-0.5} k(\rho) d\rho + \\ &+ r^{-0.5} (3\lambda + 4\mu)(3\mu K)^{-1} \int_{w(t)}^r \rho^{0.5} k(\rho) d\rho + 1.5r^{-0.5} \int_{w(t)}^r \rho^{0.5} \theta(\rho) d\rho + (3\lambda + 5\mu)(12\mu K)^{-1} r^{0.5} C_7 - r^{-0.5} C_8, \\ e_r^p &= 0.5p_\phi - 0.5r^{-1.5} \int_{w(t)}^r \rho^{0.5} (p_r(\rho) + 0.5p_\phi(\rho)) d\rho + ((9K)^{-1} + (3\mu)^{-1})k + 0.5\theta + \\ &+ (\lambda + \mu)r^{-0.5} (2\mu K)^{-1} \int_{w(t)}^r \rho^{-0.5} k(\rho) d\rho - (1.5\lambda + 2\mu)r^{-1.5} (3\mu K)^{-1} \int_{w(t)}^r \rho^{0.5} k(\rho) d\rho + \\ &+ 0.5r^{1.5} C_8 - 0.125(\lambda + \mu)r^{-0.5} (\mu K)^{-1} C_7.\end{aligned}$$

The integration coefficients C_7 and C_8 (time functions), together with the coefficients C_1 , C_2 , C_5 , C_6 from the relations (12) and (13), (20), are found from the boundary conditions simulating the continuity of radial stress and displacement on the elastoplastic boundaries $w(t)$ and $s(t)$, as well as from conditions (10). To solve the elastic moduli taking into account the dependence on the temperature, there is no repeated plastic flow. At some point in time, the boundaries $m(t)$ and $n(t)$ are joined and the flow ceases to exist, there will be complete unloading.

Residual stresses - As the plate cools, in calculations with constant elastic moduli, the repeated plastic flow will fade. Irreversible deformations will cease to grow. In the region of repeated plastic flow, irreversible deformations formed in the course of this flow are added to the existing irreversible

deformations $p_r = p_r + e_r^p$.

With complete cooling of the stress in the material, the plates will create extremely irreversible deformations.

The functions ξ , ζ , ω , β in the equation of equilibrium with the presence of irreversible deformations are written in the form

$$\begin{aligned}\xi &= 1, \zeta = r^{-1}, \omega = -r^{-2}, \\ \beta &= -0.5(\lambda + 2\mu)(r(\lambda + \mu))^{-1}(p_\phi - p_r) - p_{r,r} - 0.5\lambda(\lambda + \mu)^{-1}p_{\phi,r}\end{aligned}\quad (25)$$

Integrating the equilibrium equations (8), (25), we determine the displacements, which we substitute in (18) with allowance for $\theta = 0$, we find the relations for the stresses.

$$\begin{aligned}u_r &= 0.25(\lambda + \mu)^{-1} \left((\lambda + 2\mu)r \int_0^r \rho^{-1} (p_r(\rho) - p_\phi(\rho)) d\rho \right) + 0.25(\lambda + \mu)^{-1} \left(3Kr^{-1} \int_0^r \rho (p_r(\rho) + p_\phi(\rho)) d\rho \right) + 0.5rC_1 \\ \sigma_r &= 1.5K\mu(\lambda + \mu)^{-1} \left(\int_0^r \rho^{-1} (p_r(\rho) - p_\phi(\rho)) d\rho - r^{-2} \int_0^r \rho (p_r(\rho) + p_\phi(\rho)) d\rho \right) + 3K\mu(\lambda + 2\mu)^{-1} C_1, \\ \sigma_\phi &= 1.5K\mu(\lambda + \mu)^{-1} \left(\int_0^r \rho^{-1} (p_r(\rho) - p_\phi(\rho)) d\rho + r^{-2} \int_0^r \rho (p_r(\rho) + p_\phi(\rho)) d\rho \right) - 3K\mu(\lambda + \mu)^{-1} p_\phi + 3K\mu(\lambda + 2\mu)^{-1} C_1 \\ C_1 &= (\lambda + 2\mu)(2R^2(\lambda + \mu))^{-1} \left(\int_0^R \rho (p_r(\rho) + p_\phi(\rho)) d\rho - R^2 \int_0^R \rho^{-1} (p_r(\rho) - p_\phi(\rho)) d\rho \right)\end{aligned}$$

The distribution of the residual stresses is shown in Fig. 2, and the distribution of irreversible deformations is shown in Fig. 3.

Results- If the cod condition is used, the solution will not be, since two conditions will immediately be fulfilled $\sigma_r = -2k(t)$ and $\sigma_\varphi = -2k(t)$, which contradicts the equation of equilibrium, from which it follows $\sigma_r = C$. The piecewise linear plastic potential of Ishlinsky-Ivlev makes it possible to solve the problem posed.

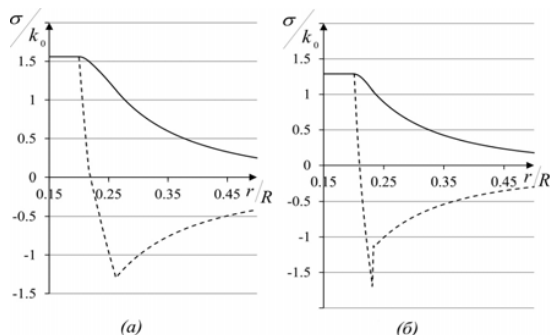


Fig. 2 Distribution of residual stresses in the material of the plate. (a) the solution obtained under the condition that the elastic moduli are constant; (b) the solution obtained under the condition that the elastic moduli depend on temperature

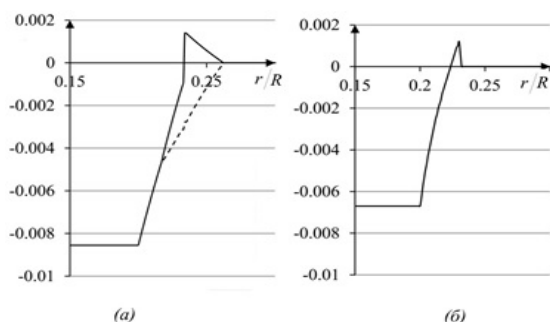


Fig. 3 Distribution of irreversible deformations in the material of the plate (a) the solution obtained under the condition that the elastic moduli are constant; (b) the solution obtained under the condition that the elastic moduli depend on temperature

As a result, one can single out that a repeated plastic flow is possible only in calculations with constant elastic moduli and an area of irreversible deformation is obtained more. The absolute value of the irreversible deformations is higher for the solution of the problem in which the elastic moduli are constant. There are also differences in the distribution and level of irreversible deformations of Fig. 3.

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