

# Numerical Solution of Transient Natural Convection in Vertical Heated Rectangular Channel between Two Vertical Parallel MTR-Type Fuel Plates

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**Abstract**—The aim of this paper is to perform, by mean of the finite volume method, a numerical solution of the transient natural convection in a narrow rectangular channel between two vertical parallel Material Testing Reactor (MTR)-type fuel plates, imposed under a heat flux with a cosine shape to determine the margin of the nuclear core power at which the natural convection cooling mode can ensure a safe core cooling, where the cladding temperature should not reach a specific safety limits (90 °C). For this purpose, a computer program is developed to determine the principal parameters related to the nuclear core safety, such as the temperature distribution in the fuel plate and in the coolant (light water) as a function of the reactor core power. Throughout the obtained results, we noticed that the core power should not reach 400 kW, to ensure a safe passive residual heat removing from the nuclear core by the upward natural convection cooling mode.

**Keywords**—Buoyancy force, friction force, friction factor, finite volume method, transient natural convection, thermal hydraulic analysis, vertical heated rectangular channel.

## I. INTRODUCTION

THE uses of the passive cooling mode like the free natural convection in the reactor technologies, are increased in the last decade, which can evacuate the reactor decay heat after a normal or accident shutdown efficiently without the need of any power supply, and this can satisfy some nuclear safety requirements and allowed us to overcome some nuclear safety issues.

For the open pool nuclear research reactor, the upward natural convection induced by the density difference between the core and the pool should be investigated to ensure that the cladding temperature will not reach 90 °C, which is considered like a safety limit, to avoid any undesired effect to the integrity of the fuel plat.

In this last decade, many authors are attracted by the features of the natural convection cooling mode, especially in nuclear field, mainly for their passive residual heat removal from the nuclear core.

Among the many published works, our whole interest goes toward the work done by Jo [1], which is considered like a reference of our study, for the application and the verification of our computer program. In their work, the authors carried out, by the both RELAP5/MOD3 and NATCON codes, a numerical simulation of plate type research reactors during the natural convective cooling mode in a hot spot of a fuel

assembly. Several convective heat transfer correlations are implemented into the simulations; then the coolant and cladding temperatures and ONB temperature margin, as a function of core power, are obtained from the simulations with a good agreement between the both used codes.

In the present study, a numerical solution of a transient upward natural convection of light water in a vertical channel between two parallel MTR-type fuel plates is carried out by the finite volume method to ensure a safe passive residual heat removal from the nuclear core. For this purpose, a computer program is developed to determine the distribution of the cladding and the coolant temperatures along the channel as a function of the core power.

## II. THE TRANSIENT NATURAL CONVECTION GOVERNING EQUATIONS

For the case of one-dimensional monophasic transient and free fluid flow, the momentum and the energy equations are respectively expressed by the two equations below [2].

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial z} = (\bar{\rho}_c - \rho_{pool})g - \frac{f}{2} \frac{(\rho_{in} v_{in})^2}{\rho D_h} \quad (1)$$

$$\rho A c_p \frac{\partial T}{\partial t} + \dot{m} c_p \frac{\partial T}{\partial z} = P_h q_{max} \cdot \cos\left(\frac{\pi \cdot z}{2 \cdot l_p}\right) \quad (2)$$

And for calculating the temperature distribution in the fuel meat and in the cladding, the heat equation is used.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) \quad (3)$$

The first and the second terms of the right side of (1), represent respectively the buoyancy and the friction forces, where  $\bar{\rho}_c$ ,  $\rho$ ,  $\rho_{in}$  and  $\rho_{pool}$  (kg/m<sup>3</sup>) are respectively the mean, the local, the inlet and the Pool coolant density.  $v$  and  $v_{in}$  (m/s) are respectively the local and the inlet coolant velocity,  $f$  is the friction factor,  $P_h$  (m) is the channel heated perimeter,  $l_p$  (m) is the extrapolated length.  $T$  (C°),  $C_p$  (J/kg s) and  $q_{max}$  (W/m<sup>2</sup>) are the coolant temperature, the coolant specific heat, and the surface heat flux generated in the hot channel.

### A. The Friction Factor Correlations

To calculate the friction factor in the rectangular heated channel more accurately two corrections are introduced, the first one is given by the factor ( $\xi$ ) as follows [3]:

$$f = \xi f_d \quad (4)$$

where for water  $\xi = \left[\frac{\mu_w}{\mu_b}\right]^{0.85}$  and  $\mu_w, \mu_b$  are respectively the fluid dynamics viscosity for the temperature of the wall and the bulk temperature. The Darcy friction factor  $f_d$  is calculated according to the flow regime of the coolant where the three following cases are considered.

#### B. Laminar Fluid Flow

For laminar flow, the correlation used is valid only for Reynolds number less than 2000, and  $K_R$  represents the Reynolds correction for the non-circular channel [3]

$$f_d = \frac{64}{ReK_R} \quad (5)$$

where,

$$K_R = \frac{2}{3} + \frac{11}{24}\alpha(1-\alpha) \quad \alpha = \frac{\text{the channel width}}{\text{the channel length}}$$

#### C. Transient Fluid Flow

For the case of transient fluid flow where the Reynolds number varies between 2000 and 5000, the friction factor without taking into account the Reynolds correction for the non-circular channel, is evaluated by a linear interpolation as [4]

$$f_d = fl + \left(\frac{Re-2000}{3000}\right)(f_t - fl) \quad (6)$$

$fl$  is the friction factor for laminar flow for Reynolds number equal to 2000.  $f_t$  is the friction factor for turbulent flow for Reynolds number equal to 5000.

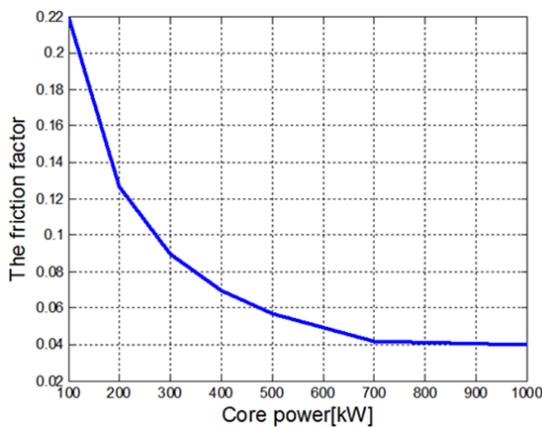


Fig. 1 The friction factor as a function of the core power

#### D. Turbulent Fluid Flow

For turbulent fluid flow in unheated channel and without taking into account the Reynolds correction for the non-circular channel, the correlation used is valid only for Reynolds number greater than 5000 [5].

$$\frac{2}{\sqrt{f_d}} = 1.7372 \ln\left(\frac{Re}{1.964 \ln(Re) - 3.8215}\right) \quad (7)$$

After the calculation of the friction factor as a function of the core power which depends only on the coolant velocity, when the coolant velocity increases with the core power, the friction factor is decreased as shown in Fig. 1.

#### E. The Coolant Velocity

The coolant velocity along the channel is determined by the discretization of (1), by the finite volume method over a control volume  $\Delta V = A dz dt$ . Then, the following algebraic equation is obtained.

$$a_p v_p^{n+1} + a_E v_E^n + a_W v_W^n = b \quad (8)$$

with,

$$a_p = \frac{\rho c_p A \Delta z}{\Delta t} + a_E + a_W$$

$$a_E = \max(-\dot{m}_e, 0); \quad a_W = \max(\dot{m}_w, 0)$$

$$b = \frac{\rho c_p A \Delta z}{\Delta t} v_m^n + (\rho_c - \rho_{pool})g A \Delta z - \frac{(\rho_{in} v_{in})^2}{2 D_h \rho_p} \bar{f} A \Delta z$$

where  $\dot{m}_e$  and  $\dot{m}_w$  are the control volume interfaces mass flow rates, which are calculated in this study as all the control volume interfaces parameters by a centered scheme.

#### F. The Coolant Pressure

The pressure distribution along the channel is calculated after a static pressure analysis by the following expression.

$$P(z) = P_{atm} + \rho g(H - z) - \frac{1}{2} \rho \vartheta^2(z) \quad (9)$$

### III. THE TEMPERATURE IN THE FUEL PLATE

#### A. The Coolant Temperature

To obtain the temporal and axial coolant temperature distribution along the channel active length, (2) is discretized by the finite volume method over the control volume  $\Delta V = A dz dt$ , then the following algebraic equation is obtained.

$$a_p T_p^{n+1} = a_W T_W + a_E T_E + b \quad (10)$$

with,

$$a_p = \frac{\rho_p c_{pp} A \Delta z}{\Delta t} + a_W + a_E$$

$$a_W = \max[\dot{m}_w c_{pw}, 0]; \quad a_E = \max[-\dot{m}_e c_{pe}, 0]$$

$$b = q'' p_h \Delta z + \frac{\rho_p c_{pp} A \Delta z}{\Delta t} T_p^n$$

#### B. The Cladding Temperature

The transient outer surface clad temperature is obtained throughout a combined operation, between the discretized heat equation (3) over the considered control volume and with the Newton's law, which is expressed by the simple equation below.

$$T_{cl}^{n+1}(z) = \frac{q_{max}}{h} \cos\left(\frac{\pi z}{2 l_p}\right) + T_c^{n+1} \quad (11)$$

So finally, the following algebraic equation is given.

$$a_p T_p^n + a_E T_E^n + a_W T_W^n = b \quad (12)$$

with,

$$a_p = \left(1 - \frac{(k_{cle} + k_{clw})\Delta t}{\rho_{cl} c_{cl} \Delta z^2}\right)$$

$$a_E = \frac{k_{cle} \Delta t}{\rho_{cl} c_{cl} \Delta z^2}; \quad a_W = \frac{k_{clw} \Delta t}{\rho_{cl} c_{cl} \Delta z^2}$$

$$b = \frac{q''}{h} + T_c^{n+1}$$

where  $h$  ( $W/m^2 C^0$ ) is the convective heat transfer coefficient. To calculate this coefficient, three different correlations of Nusselt number are analyzed.

- *Elnabass correlation* [5]:

$$Nu = \frac{1}{24} \frac{R_h}{L} (G_r P_r) \left(1 - e^{-2.4 \left[\frac{0.5 L}{R_h G_r P_r}\right]^{0.75}}\right)$$

$$G_r = \left[\frac{\rho^2 g \beta L^3 (T_{cl} - T)}{\mu^2}\right]^{0.75}$$

$$P_r = \frac{c_p \mu}{K}$$

- *Kreith & Bohn correlation* [3]:

$$Nu = \frac{R_a}{24} \left(1 - e^{-\frac{35}{R_a}}\right)^{0.75}$$

$$R_a = \frac{\rho g \beta C_p (T_{cl} - T) D_h^3}{K \nu}$$

- *McAdams correlation* [6]:

$$\begin{cases} Nu = 0.59 R_a^{0.25} & 10^4 < R_a < 10^9 \\ Nu = 0.129 R_a^{0.33} & 10^9 < R_a \end{cases}$$

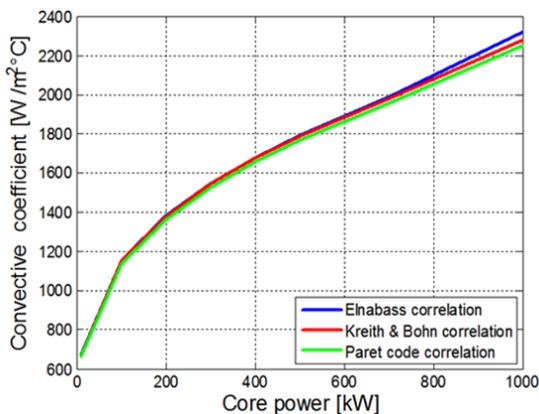


Fig. 2 The convective heat transfer coefficient for different correlations as a function of the core power

After we have evaluated the convective heat transfer coefficient by the three correlations as a function of the core power, the obtained results are presented in Fig. 2.

It is evident that the three correlations give almost the same values of the convective heat transfer coefficient. In our study, the McAdams correlation has been chosen because it is simpler and used in some validated nuclear codes.

### C. The Fuel Temperature

Also, by the same way, the transient fuel temperature is carried out by a combined operation with the electrical analogy expression of the clad-fuel exchanged heat flux, which is expressed by the simple equation

$$T_f^{n+1}(z) = T_{cl}^n + q_{\max} \cdot \cos\left(\frac{\pi \cdot z}{2 \cdot l_p}\right) \left(\frac{e_f}{k_f} + \frac{e_{cl}}{k_{cl}}\right) \quad (13)$$

So finally, the following typical algebraic equation is obtained.

$$a_p T_p^n + a_E T_E^n + a_W T_W^n = b \quad (14)$$

where,

$$a_p = \left(1 - \frac{(k_{fe} + k_{fw})\Delta t}{\rho_f c_f \Delta z^2}\right)$$

$$a_E = \frac{k_{fe} \Delta t}{\rho_f c_f \Delta z^2}; \quad a_W = \frac{k_{fw} \Delta t}{\rho_f c_f \Delta z^2}$$

$$b = q'' \left(\frac{e_f}{k_f} + \frac{e_{cl}}{k_{cl}}\right) + T_{cl}^n$$

where,  $e_f$  (m) and  $e_{cl}$  (m) are respectively the half fuel and the cladding thicknesses, while  $k_f$  ( $W/m^{\circ}C$ ) and  $k_{cl}$  ( $W/m^{\circ}C$ ) are respectively the nuclear fuel and cladding thermal conductivities.

## IV. RESULTS AND DISCUSSIONS

In this study, our computer program is applied to the same nuclear reactor core data of the work done by Jo [1], which is considered like a reference work of this study, where the main nuclear core and fuel element characteristics data used to perform our calculation are presented in Table I [1]. All the water properties are calculated as a function of pressure and temperature by the polynomial correlations of the work [7].

TABLE I  
THE MAIN CORE GEOMETRIC DATA

Parameter	Value
Pool depth	9m
number of assembly	18
number of fuel plates	21
number of channels	22
Plate thickness	1.27mm
Meat thickness	0.52mm
Meat width	62.1mm
Clad thickness	0.38mm
Channel thickness	2.35mm
Channel width	66.6mm
Plate length	660mm
Heated length	640mm
Unheated length	10mm

To carry out the distribution of the interested parameters related to the reactor core safety along the channel, an iterative process is established where it completely depends on the coolant inlet velocity. So, in this iterative process, we increase a guessed inlet velocity with a fixed increment until the difference between the buoyancy and the friction forces satisfies a convergence criterion as shown in Fig. 3.

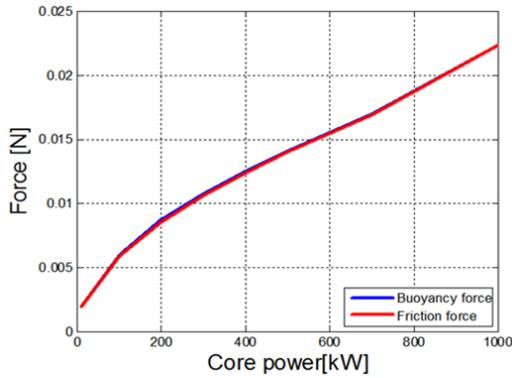


Fig. 3 The buoyancy and the friction forces after they satisfy the convergence criterion

The buoyancy and the friction forces are respectively calculated by mean of the following expressions [1]

$$F_B = (\rho_c - \rho_{pool}) A L g \quad (15)$$

$$F_F = \frac{(\rho_{in} v_{in})^2}{2} A \left[ \frac{1}{2\rho_{in}} + \sum_{i=1}^n f \frac{\Delta z_i}{\rho_p D_h} + \frac{1}{\rho_{out}} \right] \quad (16)$$

where  $\rho_{in}$ ,  $\rho_{out}$ , and  $\rho_p$  ( $\text{kg/m}^3$ ), are respectively the inlet, the outlet, and the local coolant densities.  $L$  [m] and  $A$  [ $\text{m}^2$ ], are respectively the channel length and cross section. In Figs. 4-7, we display the typical results that we obtained by mean of our simple and fast calculation computer program for 200 kW core power.

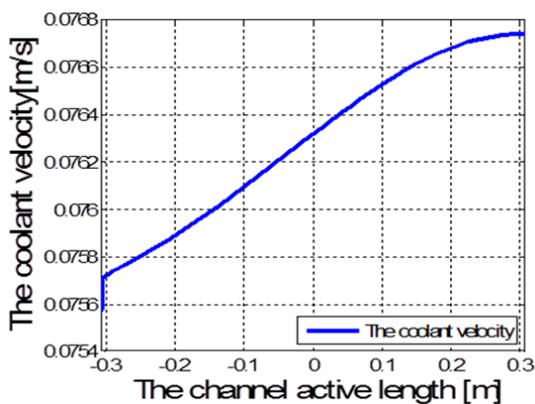


Fig. 4 The coolant velocity in the hot channel

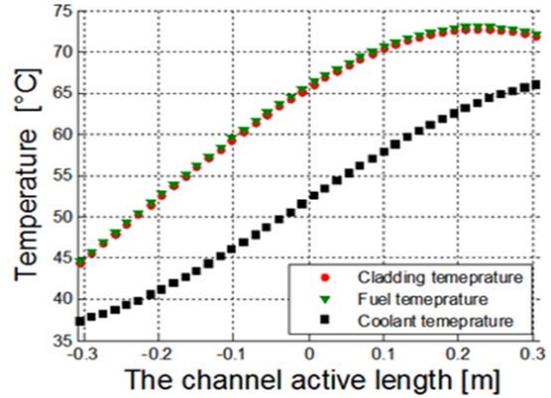


Fig. 5 The temperature distribution in the hot channel

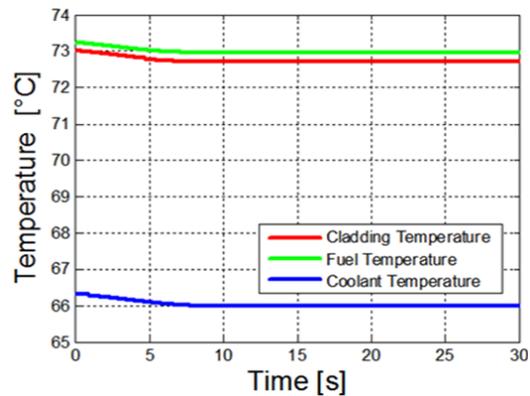


Fig. 6 The time variation of the fuel, cladding, coolant temperatures in the hot channel

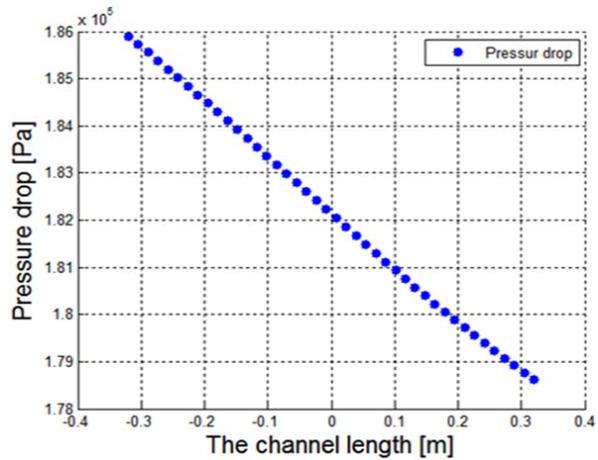


Fig. 7 The pressure distribution along the channel

In Fig. 8, we show the comparison of the coolant and cladding temperatures in the hot channel at 400 kW core power of both results obtained by our computer program and those published in the reference work.

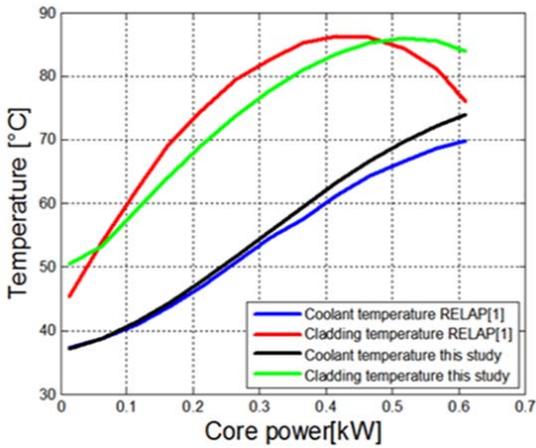


Fig. 8 The temperature distribution in hot channel for core power 400kW

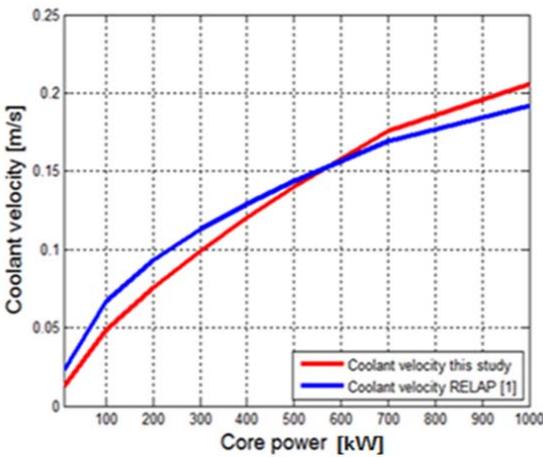


Fig. 9 The coolant velocity as a function of the core power

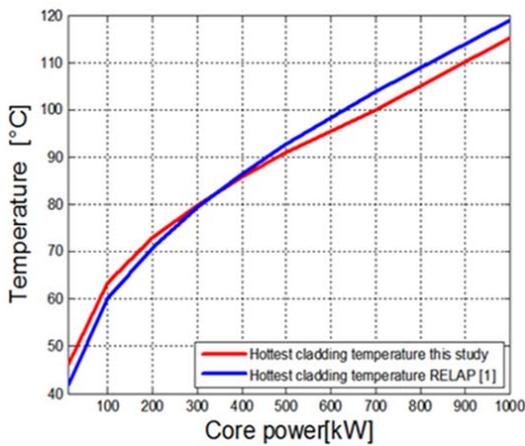


Fig. 10 The hottest cladding temperature as a function of the core power

In Figs. 9-11, we present respectively the coolant velocity, hottest cladding, and the outlet coolant temperatures which are compared always with the same reference work, in the hot

channel as a function of the core power. The nominal difference between the two calculation tools is more significant for the coolant velocity which can reach 20% for 100 kW and 200 kW core power. But, it is significantly less for the both outlet coolant and cladding temperatures where it does not exceed respectively the 10% and 2%. Finally, in Fig. 12, we present the outlet coolant pressure, where it is evident that it decreases implicitly with the increasing of the coolant velocity by the increasing of the core power.

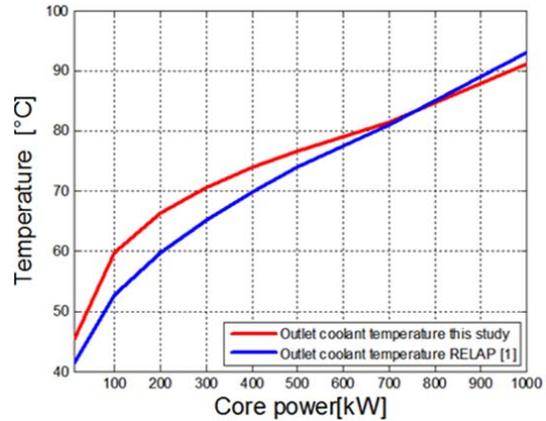


Fig. 11 The outlet coolant temperature as a function of the core power

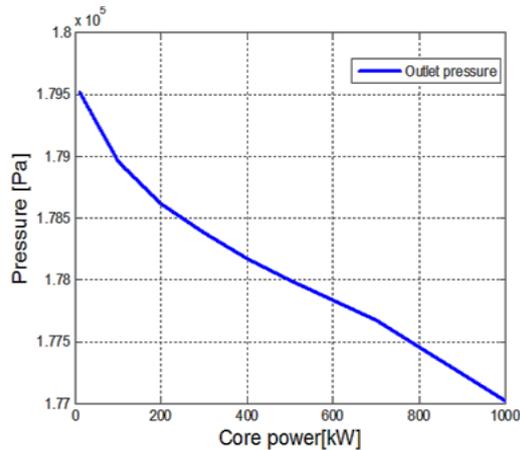


Fig. 12 The outlet pressure drop as a function of the core power

### V. CONCLUSIONS

In this study, we performed a thermal hydraulic analysis by solving numerically a transient natural convection in a heated rectangular channel between two vertical parallel MTR-type fuel plates, for the same core reactor of the reference work. The main goal of this study is to determine the core power margin to employ safely the natural convection cooling mode without reach a critical state, where the cladding temperature must stay below a specific safety limit (90 °C). For this purpose, a computer program is developed to calculate and determine the coolant and cladding temperature distributions in the hot channel of nuclear fuel element, as a function of the

core power. Then, our results are validating throughout a comparison against other published study. The core power should not reach 400 kW, to ensure a safe passive residual heat removal from the nuclear core by the upward natural convection cooling mode.

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