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Comparison of Different Methods to Produce Fuzzy Tolerance Relations for Rainfall Data Classification in the Region of Central Greece

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***Abstract*—**The aim of this paper is the comparison of three different methods, in order to produce fuzzy tolerance relations for rainfall data classification. More specifically, the three methods are correlation coefficient, cosine amplitude and max-min method. The data were obtained from seven rainfall stations in the region of central Greece and refers to 20-year time series of monthly rainfall height average. Three methods were used to express these data as a fuzzy relation. This specific fuzzy tolerance relation is reformed into an equivalence relation with max-min composition for all three methods. From the equivalence relation, the rainfall stations were categorized and classified according to the degree of confidence. The classification shows the similarities among the rainfall stations. Stations with high similarity can be utilized in water resource management scenarios interchangeably or to augment data from one to another. Due to the complexity of calculations, it is important to find out which of the methods is computationally simpler and needs fewer compositions in order to give reliable results.

***Keywords*—**Classification, fuzzy logic, tolerance relations, rainfall data.

I. Introduction

IN recent years, the domain of fuzzy logic has made enormous progress and is taught in many countries around the world at both undergraduate and postgraduate levels, while its implementation has found fertile ground in many branches of engineering sciences [1]. Fuzzy logic is derived from the development of fuzzy sets theory of Lofti Zadeh [2]; it is well structured and performs well in ambiguous or uncertain environments. It is commonly accepted that the techniques based on classical logic have proved unsuccessful to approximate the procedures of common sense, learning from experience, etc. [3]. Classical (two-valued) logic deals with propositions that are either true or false. In many-valued logic, a generalization of the classical logic, the propositions have more than two truth values. Fuzzy logic is an extension of the many-valued logic in the sense of incorporating fuzzy sets and fuzzy relations as tools into the system of many-valued logic [4]. Zadeh in [5] proposed the principle of incompatibility, according to which: *as the complexity of a system increases, human ability to make precise and relevant (meaningful) statements about its behaviour diminishes until a threshold is reached beyond which the precision and the relevance become mutually exclusive characteristics. It is then that fuzzy statements are the only bearers of meaning*. A fuzzy relation between X and Y as a fuzzy subset of X×Y was proposed by Zadeh [2]. Zadeh later studied similarity relations in [6]. An ordinary tolerance relation is a reflexive and symmetric relation. Fuzzy tolerance relation in the name of resemblance appears in [7]. More recently, Chakraborty and Das in [8], [9] and Nemitz in [10] have studied fuzzy relations connected with equivalence and fuzzy functions.

This paper examines three methods that produce fuzzy tolerance relations for rainfall data classification. As is widely known, the similarities among rainfall stations are an interesting subject in hydrology science. For this reason, seven rainfall stations in the region of central Greece were classified using fuzzy logic. Fuzzy logic is appropriate for the examination of rainfall data because of its inherent ambiguity and uncertainty. The data obtained from the above-mentioned rainfall stations refer to 20-year time series of monthly rainfall height average. More specifically, the methods used to express these data as a fuzzy relation, are correlation coefficient, cosine amplitude and max–min method. The max-min composition method was used for all three methods to reform the specific fuzzy tolerance relation into an equivalence relation. The max-min composition method is the one used by Zadeh [5] in his original paper on approximate reasoning using natural language if-then rules. Finally, the rainfall stations were classified according to the level of confidence.

II. Fuzzy Logic

A. Fuzzy Tolerance Relation

This paper implements three different methods to produce fuzzy tolerance relation. These methods are:

1) Linear Correlation Coefficient, 2) Cosine Amplitude and 3) Max-Min method.

These methods make use of a collection of *n* data samples. If these data samples are collected, they form a data array, X, $X = \{x_1, x_2, \dots, x_n\}$. Each of the elements, x_i, in the data array X is itself a vector of length *m*, i.e., $x_i = \{x_{i_1}, x_{i_2}, \dots, x_{i_m}\}$. Hence, each of the data samples can be thought of as a point in *m*-dimensional space, where each point needs *m* coordinates to be completely described. Each element of a relation, r_i, results from a pairwise comparison of two data samples, say x_i and x_j, where the strength of the relationship between data

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sample x_i and data sample x_j is given by the membership value expressing that strength, i.e., $r_i = \mu_R(x_i, x_j)$. The relation matrix will be of size $n \times n$ and will be reflexive and symmetric – hence a tolerance relation [11]. Also, a binary relation that is reflexive and symmetric is usually called a compatibility relation or tolerance relation [12].

1) The linear correlation coefficient method calculates r_i in the following manner ($0 \le r_i \le 1$):

$$r_i = \frac{m\sum x_i x_j - \sum x_i \sum x_j}{\sqrt{m\sum x_i^2 - (\sum x_i)^2}\sqrt{m\sum x_j^2 - (\sum x_j)^2}} \quad (1)$$

where $i, j = 1,2, \dots, n$

2) The cosine amplitude method calculates r_i in the following manner ($0 \le r_i \le 1$):

$$r_i = \frac{|\sum_{k=1}^{m} x_{ik}\, x_{jk}|}{\sqrt{(\sum_{k=1}^{m} x^2_{ik})(\sum_{k=1}^{m} x^2_{jk})}}, \quad (2)$$

where $i, j = 1,2, \dots, n$

3) The max-min method is computationally simpler than the cosine amplitude method and is different from the max-min composition method. Calculate r_i in the following manner ($0 \le r_i \le 1$):

$$r_i = \frac{\sum_{k=1}^{m} \min(x_{ik}, x_{jk})}{\sum_{k=1}^{m} \max(x_{ik}, x_{jk})}, \quad (3)$$

where $i, j = 1,2, \dots, n$

B.Fuzzy Classification Methodology

The basic and necessary steps for achieving rainfall data classification are as follows:

Acquisition of rainfall data, 2) Implementation of one of the three above methods, 3) Then a relation $\tilde{R}_1$ is obtained, called tolerance relation, with the following properties:

- $\alpha_i \in [0,1]$
- $\alpha_{ii} = 1$ *(reflexivity)*
- $\alpha_i = \alpha_j$ *(symmetry)*

4) Relation $\tilde{R}$, which is called equivalence relation, is obtained from the tolerance relation $\tilde{R}_1$, and has the three following properties:

- $\alpha_{ii} = 1$ *(reflexivity)*
- $\alpha_i = \alpha_j$ *(symmetry)*
- $I\ \alpha_{i,j} = \lambda_1, \alpha_{j,k} = \lambda_2 \rightarrow \alpha_{i,k} = \lambda$ *w h*
  $\lambda \in [0,1]$, then for any λ should apply
  $\lambda \ge M\ (\lambda_1, \lambda_2)$ *(transitivity)*

Also, a fuzzy binary relation that is reflexive, symmetric and transitive is known as a fuzzy equivalence relation or similarity relation [5].

5) Every tolerance relation $\tilde{R}_1$ can be reformed into an equivalence relation $\tilde{R}$ by at most (n – 1) compositions with itself according to the following equation (where n is the cardinal number of the set defining $\tilde{R}$) [11]:

$$\tilde{R}_1^{n-1} = \tilde{R}_1 \circ \tilde{R}_1 \circ \cdots \circ \tilde{R}_1 = \tilde{R} \quad (4)$$

Fuzzy composition can be defined just as it is for crisp (binary) relations. Suppose $\tilde{R}$ is a fuzzy relation on the Cartesian space X×Y, S is a fuzzy relation on Y×Z and $\tilde{T}$ is a fuzzy relation on X×Z. Then, the fuzzy max-min composition is defined in terms of the set-theoretic notation and membership-function-theoretic notation in the following manner:

$$\tilde{T} = \tilde{R} \circ \tilde{S}$$
$$\mu_T(x, z) = \bigvee_{y \in Y} (\mu_R(x, y) \wedge \mu_S(y, z)) \quad (5)$$

6) Based on the equivalence relation $\tilde{R}$, α-cut sets (or α-cut R_α) R_α are defined, where $0 \le \alpha \le 1$. The R_α, which is a classical set is defined as follows:

$$R_\alpha = \{x \mid \mu_A(x) \ge \alpha\} \quad (6)$$

7) Set $[x_i] = \{x_i \mid (x_i, x_j) \in \mathbb{R}\}$, is defined as the equivalent class x_i on a universe of data, X and is contained in a special relation $\tilde{R}$, known as the equivalent relation. This class is a set of all elements related to x_i that have the following properties [13]:

- $x_i \in [x_i] \rightarrow (x_i, x_i) \in \mathbb{R}$
- $[x_i] \ne [x_j] \Leftrightarrow [x_i] \cap [x_j] = \emptyset$
- $\bigcup_{x \in X} [x] = X$

The first property is that of reflexivity, the second property indicates that equivalent classes do not overlap, and the third property simply expresses the fact that the union of all equivalent classes exhausts the universe X. Hence, the equivalence relation $\tilde{R}$ can divide the universe X into mutually exclusive equivalent classes, i.e.

$$X \mid \mathrm{R} = \{[x] \mid x \in X\} \quad (7)$$

where, $X \mid \mathbb{R}$ is called the quotient set. The quotient set of X relative to $\tilde{R}$, denoted $X \mid \mathbb{R}$, is the set whose elements are the equivalence classes of X under the equivalence relation $\tilde{R}$.

III.Case Study

A.Study Area and Rainfall Data

The study area is located in central Greece and specifically in the region of Karditsa. The rainfall stations with their locations and elevations are shown in Fig. 1 and values (monthly averages) are shown in Table I. In general, the climate is characterized as continental with the mountainous areas to the west and the lowland to the east. The maximum rainfall occurs in the period October-March. In contrast, during the months of April to September, the highest evaporation values occur and the lowest rainfall is observed.

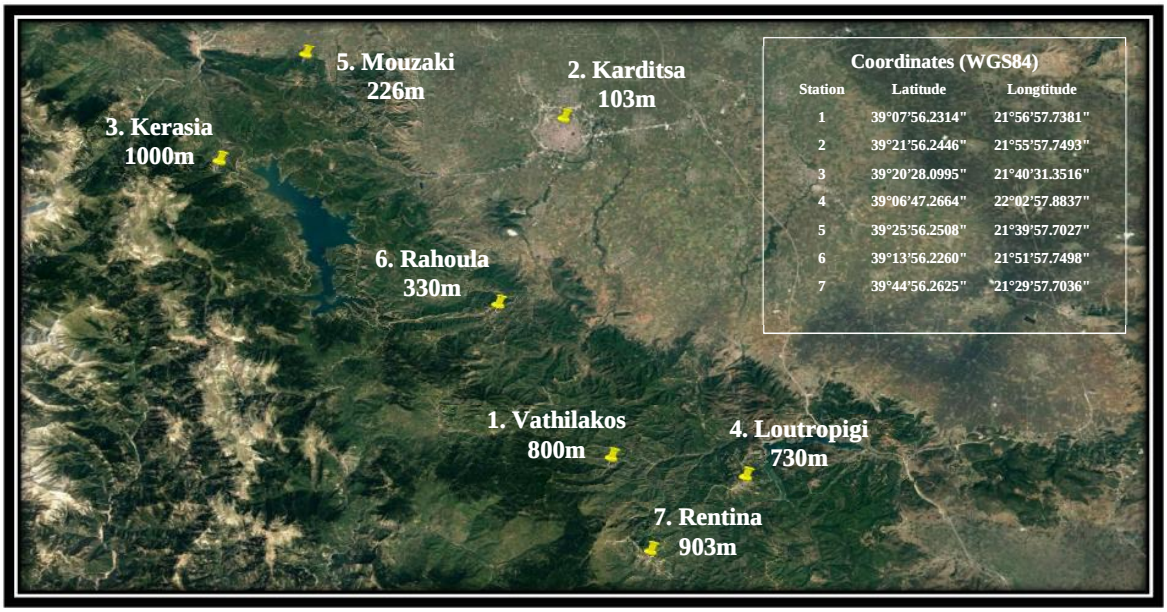

Fig. 1 Rainfall stations with corresponding elevations in the region of Central Greece

TABLE I
MONTHLY AVERAGES (MM)

Year	*Vathilakos*	*Karditsa*	*Kerasia*	*Loutropigi*	*Mouzaki*	*Rahoula*	*Rentina*
1981	80.6	25.3	126.2	104.9	80.3	85.0	99.2
1982	112.8	60.5	132.3	103.6	95.2	97.5	137.7
1983	86.6	47.7	83.5	78.5	48.9	82.0	85.5
1984	100.4	47.4	108.3	57.3	55.9	105.0	109.1
1985	92.3	41.0	140.7	81.4	59.6	97.4	120.8
1986	116.8	75.1	114.5	81.0	53.0	99.5	127.9
1987	84.5	58.5	132.0	30.6	75.3	84.9	104.3
1988	74.9	44.6	98.7	52.5	45.4	65.0	79.4
1989	76.4	29.8	103.6	53.8	33.6	58.3	63.2
1990	72.0	47.6	81.7	61.5	39.2	76.2	68.4
1991	78.6	51.1	91.2	66.1	36.7	85.9	78.1
1992	72.9	36.0	92.3	53.9	31.3	77.0	75.1
1993	83.1	38.3	89.6	72.2	61.1	83.4	89.2
1994	115.3	70.9	137.3	83.3	65.0	110.2	114.5
1995	90.4	53.9	120.5	60.5	34.1	87.1	96.1
1996	97.8	50.2	152.0	63.5	61.8	93.7	82.5
1997	122.7	42.9	131.5	79.0	79.8	117.1	85.7
1998	111.5	56.0	128.0	83.7	90.5	106.0	87.3
1999	97.0	44.8	116.3	77.0	42.0	92.2	80.8
2000	61.3	33.9	91.5	54.1	32.9	57.1	57.8
2001	78.7	38.1	79.5	53.7	27.0	77.4	70.9

B.Tolerance and Equivalence Relations

By applying (1)-(3) to rainfall data, the tolerance relations take the following form and satisfy the properties of reflexivity and symmetry:

- Linear correlation coefficient

$$\tilde{R}_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \begin{bmatrix} 1 & 0.653 & 0.654 & 0.540 & 0.653 & 0.906 & 0.673 \\ 0.653 & 1 & 0.359 & 0.135 & 0.306 & 0.556 & 0.616 \\ 0.654 & 0.359 & 1 & 0.356 & 0.679 & 0.628 & 0.584 \\ 0.540 & 0.135 & 0.356 & 1 & 0.575 & 0.508 & 0.514 \\ 0.653 & 0.306 & 0.679 & 0.575 & 1 & 0.640 & 0.613 \\ 0.906 & 0.556 & 0.628 & 0.508 & 0.640 & 1 & 0.647 \\ 0.673 & 0.616 & 0.584 & 0.514 & 0.613 & 0.647 & 1 \end{bmatrix} \end{array}$$

- Cosine amplitude

$$\tilde{C}_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \begin{bmatrix} 1 & 0.982 & 0.988 & 0.977 & 0.964 & 0.997 & 0.986 \\ 0.982 & 1 & 0.968 & 0.947 & 0.936 & 0.978 & 0.978 \\ 0.988 & 0.968 & 1 & 0.969 & 0.966 & 0.988 & 0.982 \\ 0.977 & 0.947 & 0.969 & 1 & 0.959 & 0.976 & 0.973 \\ 0.964 & 0.936 & 0.966 & 0.959 & 1 & 0.963 & 0.963 \\ 0.997 & 0.978 & 0.988 & 0.976 & 0.963 & 1 & 0.985 \\ 0.986 & 0.978 & 0.982 & 0.973 & 0.963 & 0.985 & 1 \end{bmatrix} \end{array}$$

- Max – Min

$$\tilde{M}_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{array} \\ \begin{bmatrix} 1 & 0.521 & 0.807 & 0.739 & 0.602 & 0.933 & 0.876 \\ 0.521 & 1 & 0.423 & 0.652 & 0.729 & 0.541 & 0.519 \\ 0.807 & 0.423 & 1 & 0.618 & 0.488 & 0.782 & 0.797 \\ 0.739 & 0.652 & 0.618 & 1 & 0.729 & 0.765 & 0.754 \\ 0.602 & 0.729 & 0.488 & 0.729 & 1 & 0.625 & 0.598 \\ 0.933 & 0.541 & 0.782 & 0.765 & 0.625 & 1 & 0.866 \\ 0.876 & 0.519 & 0.797 & 0.754 & 0.698 & 0.866 & 1 \end{bmatrix} \end{array}$$

where x_1=Vathilakos, x_2=Karditsa, x_3=Kerasia, x_4=Loutropigi, x_5=Mouzaki, x_6=Rahoula, x_7=Rentina.

By applying (4) for relations $\tilde{R}_1, \tilde{C}_1, \tilde{M}_1$ and using (5) max–min composition method, the tolerance relations are converted into equivalence relations. The tolerance relation of correlation coefficient method converted into equivalence relation after two compositions ($\tilde{R}_1^3 = \tilde{R}_1^2 \circ \tilde{R}_1 = \tilde{R}$), cosine amplitude after one composition ($\tilde{C}_1^2 = \tilde{C}_1 \circ \tilde{C}_1 = \tilde{C}$) and max-min after three compositions ($\tilde{M}_1^4 = \tilde{M}_1^3 \circ \tilde{M}_1 = \tilde{M}$), and thus, the final equivalent relations $\tilde{R}$, $\tilde{C}$ and $\tilde{M}$ for all methods are defined.

$$\tilde{R} = \begin{bmatrix} 1 & 0.6527 & 0.6543 & 0.5754 & 0.6543 & 0.9058 & 0.6730 \\ 0.6527 & 1 & 0.6527 & 0.5754 & 0.6527 & 0.6527 & 0.6527 \\ 0.6543 & 0.6527 & 1 & 0.5754 & 0.6786 & 0.6543 & 0.6543 \\ 0.5754 & 0.5754 & 0.5754 & 1 & 0.5754 & 0.5754 & 0.5754 \\ 0.6543 & 0.6527 & 0.6786 & 0.5754 & 1 & 0.6543 & 0.6543 \\ 0.9058 & 0.6527 & 0.6543 & 0.5754 & 0.6543 & 1 & 0.6730 \\ 0.6730 & 0.6527 & 0.6543 & 0.5754 & 0.6543 & 0.6730 & 1 \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} 1 & 0.9819 & 0.9882 & 0.9773 & 0.9664 & 0.9970 & 0.9858 \\ 0.9819 & 1 & 0.9819 & 0.9773 & 0.9664 & 0.9819 & 0.9819 \\ 0.9882 & 0.9819 & 1 & 0.9773 & 0.9664 & 0.9882 & 0.9858 \\ 0.9773 & 0.9773 & 0.9773 & 1 & 0.9664 & 0.9773 & 0.9773 \\ 0.9664 & 0.9664 & 0.9664 & 0.9664 & 1 & 0.9664 & 0.9664 \\ 0.9970 & 0.9819 & 0.9882 & 0.9773 & 0.9664 & 1 & 0.9858 \\ 0.9858 & 0.9819 & 0.9858 & 0.9773 & 0.9664 & 0.9858 & 1 \end{bmatrix}$$

$$\tilde{M} = \begin{bmatrix} 1 & 0.7287 & 0.8069 & 0.7650 & 0.7287 & 0.9331 & 0.8756 \\ 0.7287 & 1 & 0.7287 & 0.7287 & 0.7293 & 0.7287 & 0.7287 \\ 0.8069 & 0.7287 & 1 & 0.7650 & 0.7287 & 0.8069 & 0.8069 \\ 0.7650 & 0.7287 & 0.7650 & 1 & 0.7287 & 0.7650 & 0.7650 \\ 0.7287 & 0.7293 & 0.7287 & 0.7287 & 1 & 0.7287 & 0.7287 \\ 0.9331 & 0.7287 & 0.8069 & 0.7650 & 0.7287 & 1 & 0.8756 \\ 0.8756 & 0.7287 & 0.8069 & 0.7650 & 0.7287 & 0.8756 & 1 \end{bmatrix}$$

The relations $\tilde{R}_1^3$, $\tilde{C}_1^2$ and $\tilde{M}_1^4$ satisfy the properties of reflexivity and symmetry, but they also satisfy the property of transitivity that transforms them into equivalence relations. Continuation of compositions between relations $\tilde{R}_1^3$, $\tilde{C}_1^2$, $\tilde{M}_1^4$ and initial relations $\tilde{R}_1$, $\tilde{C}_1$ and $\tilde{M}_1$, respectively, will provide the same results [14]. This is another way of verifying the final equivalence relations.

C. α-Cut Sets

By taking α-cuts of fuzzy equivalent relations $\tilde{R}$, $\tilde{C}$ and $\tilde{M}$ at values $\alpha = 0.6730$, $\alpha = 0.9882$ and $\alpha = 0.8756$, respectively, as an example and using (6), the following defuzzified crisp equivalence relations are derived:

$$\tilde{R}_{0.6} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \tilde{C}_{0.9} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{M}_{0.8} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

D.Classification

The universe X contains the seven stations as:

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$$

Table II shows the final classification of rainfall data depending on α-cut levels for the three different methods.

TABLE II
CLASSIFICATION OF SEVEN RAINFALL STATIONS ACCORDING TO α-CUT LEVEL

α-cut Level	Classification
Correlation Coefficient	
0.5754	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
0.6527	$\{x_1, x_2, x_3, x_5, x_6, x_7\}[x_4]$
0.6543	$\{x_1, x_3, x_5, x_6, x_7\}[x_2][x_4]$
0.6730	$\{x_1, x_6, x_7\}\{x_2, x_5\}[x_2][x_4]$
0.6786	$\{x_1, x_6\}\{x_3, x_5\}[x_2][x_4][x_7]$
0.9058	$\{x_1, x_6\}[x_2][x_3][x_4][x_5][x_7]$
1.0	$[x_1][x_2][x_3][x_4][x_5][x_6][x_7]$
Cosine Amplitude	
0.9664	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
0.9773	$\{x_1, x_2, x_3, x_4, x_6, x_7\}[x_5]$
0.9819	$\{x_1, x_2, x_3, x_6, x_7\}[x_4][x_5]$
0.9858	$\{x_1, x_3, x_6, x_7\}[x_2][x_4][x_5]$
0.9882	$\{x_1, x_3, x_6\}[x_2][x_4][x_5][x_7]$
0.9970	$\{x_1, x_6\}[x_2][x_3][x_4][x_5][x_7]$
1.0	$[x_1][x_2][x_3][x_4][x_5][x_6][x_7]$
Max - Min	
0.7287	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
0.7293	$\{x_1, x_3, x_4, x_6, x_7\}\{x_2, x_5\}$
0.7650	$\{x_1, x_3, x_4, x_6, x_7\}[x_2][x_5]$
0.8069	$\{x_1, x_3, x_6, x_7\}[x_2][x_4][x_5]$
0.8756	$\{x_1, x_6, x_7\}[x_3][x_2][x_4][x_5]$
0.9331	$\{x_1, x_6\}[x_2][x_3][x_4][x_5][x_7]$
1.0	$[x_1][x_2][x_3][x_4][x_5][x_6][x_7]$

We can express the classification scenario described in Table II with a classification diagram, as shown in Fig. 2. It is also shown that the higher the α-cut value, the more rigorous the classification becomes. That is, each data point is assigned to its own class when α gets larger.

As is easily understood, computationally process is quite complex and many helpful algorithms can be used from [15].

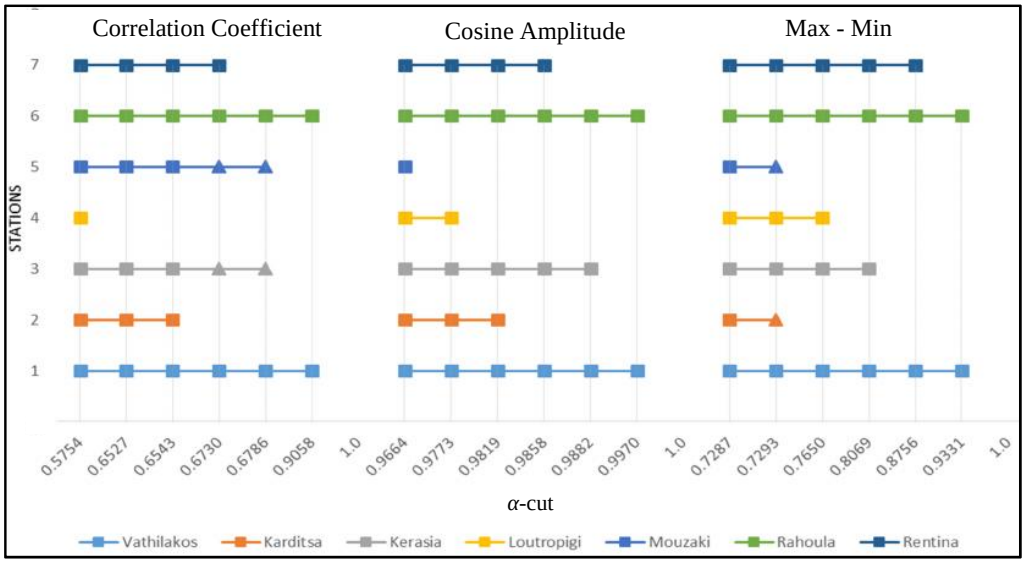

Fig. 2 Classification diagram for the three different methods

IV. Conclusion

According to the results, we can state that fuzzy classification of data can provide answers under a certain degree of confidence in complex problems characterized by ambiguity and uncertainty. In essence, the three methods under study terminated by giving the same results at the highest level of confidence for each method. On the other hand, there were fundamental differences on rainfall data classification for the rest α-cuts. For all methods the best classification obtained, was in the stations Vathilakos and Rahoula with a degree of confidence 0.9058 for correlation coefficient, 0.997 for cosine amplitude and 0.9331 for max-min. There is significant elevation difference between these two stations, but they have the best similarity. In general, we can say that the level of confidence varies significantly with the method. Specifically, the minimum level of confidence for correlation coefficient method is 0.5754, for cosine amplitude is 0.9664 and for max-min is 0.7287. In addition, the fact that the max-min method is computationally simpler than the other two methods, but needs more compositions to extract equivalence relation, is very important. Finally, it should be noted that the whole process is quite complex as far as computing is concerned, and the use of appropriate software such as MATLAB is considered necessary for reliable results.

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