An Improved Single Point Closure Model Based on Dissipation Anisotropy for Geophysical Turbulent Flows

A. P. Joshi, H. V. Warrior, J. P. Panda

Abstract—This paper is a continuation of the work carried out by various turbulence modelers in Oceanography on the topic of oceanic turbulent mixing. It evaluates the evolution of ocean water temperature and salinity by the appropriate modeling of turbulent mixing utilizing proper prescription of eddy viscosity. Many modelers in past have suggested including terms like shear, buoyancy and vorticity to be the parameters that decide the slow pressure strain correlation. We add to it the fact that dissipation anisotropy also modifies the correlation through eddy viscosity parameterization. This recalibrates the established correlation constants slightly and gives improved results. This anisotropization of dissipation implies that the critical Richardson's number increases much beyond unity (to 1.66) to accommodate enhanced mixing, as is seen in reality. The model is run for a couple of test cases in the General Ocean Turbulence Model (GOTM) and the results are presented here.

Keywords—Anisotropy, GOTM, pressure-strain correlation, Richardson Critical number.

I. INTRODUCTION

MODELS which predict turbulent mixing due to rotation, shear and other sources in flows with vertical density gradients are of huge importance in the turbulence modeling of marine environment. Through this turbulence modeling, attempts are made to resolve the vertical structure of water column. Hence, these types of models have received a lot of importance during the last few decades, resulting in numerous such mixing models in the field of geophysical flows. Canuto et al. [1], Rodi [2], Mellor and Yamada [3] performed some of the important two parameter models. The mixing process with vertical density gradient is a complex physical process [4]. Reviews of such models for geophysical flows have been made by Umlauf and Burchard [5] Burchard and Bolding [6], Burchard and Peterson [7], etc.

The physics of this ocean mixing dynamics is well included in Navier-Stokes equations and molecular tracer equations of temperature and salinity. One of the bottle necks that we encounter is that small scale turbulence in these large scale processes is more or less unresolvable. This leads the ocean modeler to make assumptions for these smaller scale processes which become critical in order to achieve an ocean model that is practical for all applications. In conformity with the mainstream ocean turbulence modeling, statistical closure has

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been implemented in this present work. Statistical models are based on Reynolds decomposition. It decomposes scalar and momentum fields into mean and fluctuating terms. In the last few decades, there have been many solutions to this problem, taking into consideration higher statistical correlations to solve the heat and salt equations patterned in the same format as Reynolds momentum equations. The major achievement of turbulence modeler has been to suggest closure for this system of equations. This can be achieved at various levels of sophistication. Some milestones in this process were the works of Launder and colleagues [8], [9] and the works of Mellor and Yamada [3]. Until now, numerous modifications to these works have been suggested (e.g. [3], [10]-[12], [1]). This shows that till date entirely satisfying solutions have not been found.

In most of the flows of engineering interest, like a turbulent boundary layer, or channel flow or different wall bounded flows, there is a strong influence of anisotropy in the momentum equations. When the flow is subjected to strain or rotation the above phenomenon still stands true. This anisotropic distribution gives rise to many difficult problems while handling turbulence modeling. We have dealt with this problem in the present work. In ocean flows, anisotropy of dissipation becomes important in the mixed layer and coastal boundary layers, lesser so in thermocline and negligible for deeper waters. If the dissipation rate components are not calculated exactly (by not including anisotropy), in these boundary layers, the pressure strain term of turbulence will not model the inter-directional energy transfer, and this leads to erroneous conclusion about model constants. This is evident in literature from experimental determination of pressure strain correlation. It was also shown that anisotropy of dissipation rate was comparable in magnitude to Reynolds stress tensor for low to moderate Reynolds numbers [13], hence cannot be neglected [14]. Oberlack [15] derived a new transport equation for the length scale tensor and subsequently for the anisotropic dissipation rate tensor and developed a new model for the rapid pressure strain correlation of turbulence. This was followed by the work of Panda et al. [21] where they modeled the anisotropy of length scale in laboratory flows. Speziale and Gatski [13] included the anisotropic flow physics in the dissipation rate equation. Various researchers have tried to incorporate the dissipation anisotropy in the formulation of the return to isotropy term of the pressure strain correlation; Warrior et al. [16] and Panda et al. [21] tried to incorporate the missing flow physics to increase the fidelity of their models.

In the present study, anisotropy of dissipation rate has been introduced in the pressure strain correlation (instead of assuming an isotropic dissipation, as current ocean models do) and has been incorporated into GOTM to simulate some known test cases. The basic structure of Canuto et al. [1] has been retained as such, and the new formulation has been added to it to recalibrate the model constants of Canuto et al. [1].

The new formulation introduced here is from the numerical studies of Warrior et al. [16]. In that paper, an improvement in turbulence prediction was observed by including anisotropic dissipation term for laboratory experiments. In this paper, the same formulation is being implemented for geophysical flows, i.e. in an oceanic model.

II. BASIC EQUATIONS

While solving for turbulence in oceans, one needs to solve for mean variables e.g. mean velocity (U) and mean temperature (T). The dynamic equations for these variables are as:

$$\frac{DU_i}{Dt} = -(g_i + \rho^{-1}P_{,i}) - \tau_{ij,j} \qquad (1)$$

$$\frac{DT}{Dt} = -h_{i,i} + (c_p\rho)^{-1}I_{,z} \qquad (2)$$

In (2), I represents the solar radiation and g is the acceleration due to gravity. The turbulence enters through Reynolds stresses τ_{ij} and heat flux h_i. These variables are defined as:

$$\tau_{ij} = \overline{u_i u_j}, \; h_i = \overline{u_i \theta} \qquad (3)$$

The dynamic equation of these variables can be derived from Navier-stokes equations. The process is available in many literatures such as Lumley and Khajeh-Nouri [17], Pope [18], Lumley [19], Shih and Shabbir [20]. These equations can be defined as follows:

Reynolds stresses;

$$\frac{D\tau_{ij}}{Dt} + D_f(\tau_{ij}) = -\tau_{jk}U_{i,k} - \tau_{ik}U_{j,k} + \lambda_i h_j + \lambda_j h_i - \Pi_{ij} - \frac{2}{3}\varepsilon\delta_{ij} \quad (4)$$

Heat flux;

$$\frac{Dh_i}{Dt} + D_f(h_i) = -\tau_{ij}T_{,j} - h_jU_{i,j} + \lambda_i\overline{\theta^2} - \Pi_i^\theta \qquad (5)$$

In the above equations, the second term on the left hand side represents third order moments. These are closed by considering dynamic equations of these moments. This method is defined by Canuto et al. [1].

The other terms which require closure are fifth terms in (4) and (5). These are known as pressure-strain correlation and pressure-temperature correlation. Our main interest in this study is to develop the pressure-strain correlation. For the pressure-temperature correlation, we accept the closure of Canuto et al. [1]. Closure is also required for the sixth term which is the dissipation term in (4). Apart from (4) and (5), there are some more important equations which are as follows:

Temperature Variance:

$$\frac{D\overline{\theta^2}}{Dt} + D_f(\overline{\theta^2}) = -2h_iT_{,i} - 2\varepsilon_\theta \qquad (6)$$

where dissipation of potential energy (ε_θ) is:

$$\varepsilon_\theta = \tau_\theta^{-1}\overline{\theta^2} \qquad (7)$$

Turbulent kinetic energy:

$$\frac{DK}{Dt} + D_f(K) = \tau_{ij}U_{i,j} + \lambda_i\overline{u_i\theta} - \frac{1}{2}\Pi_{ii} - \varepsilon \qquad (8)$$

The problem addressed in this study is closure of the pressure-strain correlation. We have added anisotropic dissipation to the existing pressure-strain correlation. This is discussed in more detail in the next section.

III. PRESSURE-STRAIN CORRELATION

Extensive literature is available on this topic (e.g. [18]-[20], [2], [1] etc.). Parameterization of pressure-strain correlation is necessary in order to provide closure to the second order moments. In the literatures presented till date, we observe that the pressure-strain correlation consists of five terms: the return to isotropy (Rotta term or slow term), the mean shear interaction (Rapid part), the buoyancy contribution, the anisotropic shear production, and the vorticity.

This formulation though is not complete. In the classical paper of Lumley [19], it has been stated that the slow part of pressure-strain correlation depends upon both anisotropic Reynolds stresses and dissipation rate of turbulence. Later this was applied to engineering flows by Warrior et al. [16], and improved result was achieved. The models which have been developed until now have ignored this anisotropic dissipation rate tensor that we intend to apply. Hence, the proposed form of pressure-strain correlation is as follows:

$$\Pi_{ij} = C_1\tau_{pv}^{-1}b_{ij} + C_2KS_{ij} + C_3B_{ij} - C_4\Sigma_{ij} - C_5Z_{ij} + C_6\tau_{pv}^{-1}e_{ij} \quad (9)$$

In (9)

$$b_{ij} = \tau_{ij} - \frac{2}{3}K\delta_{ij} \; S_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right) B_{ij} = \lambda_i h_j + \lambda_j h_i - \frac{2}{3}\delta_{ij}\lambda_k h_k$$

$$Z_{ij} = V_{ik}b_{kj} + V_{jk}b_{ik} \; \Sigma_{ij} = S_{ik}b_{kj} + S_{jk}b_{ik} - \frac{2}{3}\delta_{ij}S_{kl}b_{kl} \quad (10)$$

Like other models, the dissipation anisotropy rate is assumed to have an algebraic relation with anisotropic Reynolds stress. It is somewhat similar to be having a direct proportionality between small scale turbulence and large scale processes in ocean. This relation is given as follows;

$$e_{ij} = 2f_s b_{ij} \qquad (11)$$

In (11), f_s is known as the blending function. This blending function relaxes the direct proportionality between small scale anisotropy and large scale isotropy.

There are many formulations of f_s available in literature. Early models had suggested that this blending function depends solely upon the turbulent Reynolds number [22], [23]. They assumed that anisotropic dissipation rate was dependent only on viscous effects. Later, the DNS results of Mansour et al. [24] and Durbin et al. [25] showed that the anisotropy in dissipation rate extends beyond the near-wall regions and it is due to non viscous blocking and eddy flattening effects.

Due to these effects, several authors ([26] etc.) expressed the blending function in second and third stress invariants. Warrior et al. [16] showed that, with the use of Gilbert and Kleiser [26] formulation of blending function, better result could be achieved in engineering flows. Hence, we use the same formulation and apply it in geophysical flows. The formulation is given as:

$$f_s = 1 - \sqrt{A} \qquad (12)$$

A is known as Lumley's flatness parameter and it is expressed as:

$$A = 1 - \frac{9}{8}\left(A_2 - A_3\right) \qquad (13)$$

A_2 and A_3 are the second and third order invariants respectively, which are expressed as:

$$A_2 = 4b_{ij}b_{ji} \text{ and } A_3 = 8b_{ij}b_{jk}b_{ki} \qquad (14)$$

Assuming the state of turbulence in oceans to be two components, we choose the values of the above-mentioned invariants from the Lumley's triangle. This is done to make sure that the values chosen make the state of turbulence realizable. Hence, after going through some simple algebra, we achieve from (13) that

$$e_{ij} = 0.1744 b_{ij} \qquad (15)$$

So now the proposed pressure-strain correlation in (11) can be rewritten as:

$$\Pi_{ij} = (C_1 + 0.1744C_6)\tau_{pv}^{-1}b_{ij} + C_2 KS_{ij} + C_3 B_{ij} - C_4 \Sigma_{ij} - C_5 Z_{ij} \qquad (16)$$

This formulation is structurally similar to that of Canuto et al. [1]. Now, the constants need to be calibrated.

IV. THE COMPLETE MODEL

For the model to be complete, we have to also consider the pressure-temperature correlation. In this study, we have taken the pressure-temperature correlation exactly from Canuto et al. [1]. This is because the pressure-strain correlation is complete in its sense in Canuto et al. [1]. The formulation of which is as:

$$\pi_i^\theta = \tau_{p\theta}^{-1}h_i + \gamma_1 \lambda_i \overline{\theta^2} - \frac{3}{4}\alpha_4 \left(S_{ij} + \frac{5}{3}V_{ij}\right)h_j, \qquad (17)$$

The procedure to obtain the complete model is a matter of algebraization. This is described in literature clearly. In this process we put (16) and (17) in (4) and (5) respectively. The resulting equations are complex and hence cannot be applied directly to the ocean models. To obtain the equations for variables of Reynolds stresses and heat fluxes, we adopt the algebraic Reynolds stress model method. The basic approach is to neglect time variations, advective and turbulent transport of Reynolds stresses and the fluxes and variance of temperature. The turbulent kinetic energy is multiplied by $\frac{2}{3}\delta_{ij}$ and is subtracted from (4). This is done to preserve the transport of kinetic energy. Then, further the left hand side of the resulting equation is set to zero. The xame procedure is done with (5), hence obtaining the following equations of Reynolds stresses and heat flux:

$$b_{ij} = -\beta_1 \tau K S_{ij} + \beta_2 \tau B_{ij} - \beta_3 \tau \Sigma_{ij} - \beta_4 \tau Z_{ij} \qquad (18)$$

$$A_{ik}h_k = -\left(K_h\right)_{ij}\frac{\partial T}{\partial x_j} \qquad (19)$$

The tensors A_{ik} and $(K_h)_{ij}$ are defined as follows:

$$A_{ij} = \lambda_5 \delta_{ij} + \lambda_8 \tau^2 \lambda_i T_{,i} + \lambda_6 \tau S_{ij} + \lambda_7 \tau V_{ij} \qquad (20)$$

$$\left(K_h\right)_{ij} = \tau\left(b_{ij} + \frac{2}{3}\delta_{ij}K\right) \qquad (21)$$

Equations (18)-(21) are exactly the same equations as of Canuto et al. [1]. Equation (18) is structurally the same with that of Canuto et al. model, but it is different in the values of its constants. These are expressed as:

$$\beta_1 = \frac{c_2}{c_1 + 0.1744c_6}\beta; \beta_2 = \frac{1-c_3}{c_1 + 0.1744c_6}\beta; \beta_3 = \frac{1-c_4}{c_1 + 0.1744c_6}\beta;$$

$$\beta_4 = \frac{1-c_5}{c_1 + 0.1744c_6}\beta; \beta = \frac{\tau_{pv}}{\tau} \qquad (22)$$

V. ASSUMPTION AND SOLUTION FOR REYNOLDS STRESS AND HEAT FLUX EQUATIONS

While solving (18) and (19) some assumptions are made. These assumptions are as follows: $\frac{\partial T}{\partial x_i} \to \frac{\partial T}{\partial z}\delta_{ij}$ and velocity is assumed to be neglected in the Z direction.

The shear and vorticity take form:

$$S_{ij} = \frac{1}{2}\begin{bmatrix} 0 & 0 & \frac{\partial U}{\partial Z} \\ 0 & 0 & \frac{\partial V}{\partial Z} \\ \frac{\partial U}{\partial Z} & \frac{\partial V}{\partial Z} & 0 \end{bmatrix} \qquad (23)$$

$$V_{ij} = \frac{1}{2}\begin{bmatrix} 0 & 0 & \frac{\partial U}{\partial Z} \\ 0 & 0 & \frac{\partial V}{\partial Z} \\ \frac{\partial U}{\partial Z} & -\frac{\partial V}{\partial Z} & 0 \end{bmatrix} \qquad (24)$$

Equations (18)-(21) acquire similar solution to that of Canuto et al. [1]. This is because the basic form of equations (18)-(21) is same to that of Canuto et al [1]. Hence, all the solutions thus obtained including turbulent momentum and heat diffusivities are structurally the same. Thedifference lies in the values of constants of (18). These constants need to be tuned toobtain better results. We made turbulence more anisotropic by the addition of dissipation term in pressure-strain correlation.

VI. MODEL CONSTANTS

A numerical ocean model, GOTM is used in the present work. The success of the model has been accepted by the fact that this model has been applied to various regions, specifications and scales, by modifying the various parameters of the code. Not only turbulence, but mean flow modeling can also be carried out easily. The turbulence inside GOTM is such that it can be integrated into atmospheric model or into various other 3-D models for vertical exchange. Observational data can be read into this model for many cases. At least one case from each member of turbulence models is present in GOTM. The model output is optional in ASCII or netCDF format.

The ocean domain is divided discontinuously from bottom to top, i.e. bottom being the first interval and surface being last. This discretization is not necessarily of equal distance. The different values of mean flow quantities like temperature, salinity, X and Y components of velocity represent average of an interval. Hence, they are located at mid points of each interval. On the other hand, the turbulent quantities like kinetic energy, length scales, eddy viscosity, heat diffusivity, etc. are located at the interface of each interval. The equidistant interval of grids allows second order approximation of vertical fluxes and tracers of momentum without averaging. Inferential treatment of diffusion and absence of advection allows time stepping to be equidistant, based on two time intervals. Fully constructive discretization of mean quantities leads each transport equation to a system of linear equations. This results in formation of tri-diagonal matrix which is solved by simplified Gaussian elimination method.

The model constants of the present model were calibrated against observational data, the model constants are presented in Table I.

TABLE I
CALIBRATED MODEL CONSTANTS

β_1	β_2	β_3	β_4	β
0.13	0.0048	0.083	0.093	5.3178

VII. RICHARDSON NUMBER (RI)

Richardson number is defined as the ration of mean shear to temperature gradient or mean shear to Brunt Vaisala frequency.

$$Ri = \frac{\Sigma^2}{N^2} \qquad (25)$$

In (25), $\Sigma^2 = \left(\frac{\partial U}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial Z}\right)^2$ and $N^2 = g\alpha\frac{\partial T}{\partial Z}$. the first term is mean shear term and second is temperature gradient term, also known as Brunt Vaisala frequency.

This is the measure of existence of turbulence in a flow. The earlier works of Miles [27] and Howard [28] used linear stability analysis and established that for linear stability to exist in a flow the sufficient conditions is:

$$Ri > \frac{1}{4} \qquad (26)$$

However, this was not a necessary condition. Some early turbulence modelers like Mellor and Yamada [3] showed from their model assumptions that turbulence ceases around Ri=0.25.

Equation (26) fails to say anything about the nonlinearities and hence turbulence. Yet most models had assumed this result in their model description. Nonlinear interactions were included in the study of Abarbanel et al. [29], who derived the necessary and sufficient condition for stability to be:

$$Ri > 1 \qquad (27)$$

In Canuto et al. [1], the first approach, i.e. (26), has been called "bottom-top" approach and the later approach, i.e. (27) is known as the "top-bottom' approach.

Some earlier studies [30] used laboratory data of Taylor in which they showed that turbulence mixing can go up to Richardson number greater than 1. Martin [31] who corrected

mixing layer depths at OWS Papa had to allow turbulence to get up to Richardson number 1. Later, Galperin et al. [32] in their studies for marine and atmosphere proved that Richardson number greater than 1 exists.The model developed by Canuto et al [1] was able to get Richardson number up to 1.

Due to the addition of dissipation rate of turbulence and recalibration of model constants, we have achieved that in our model turbulent does not cease till Richardson number 1.66, which is higher than 1. We assume this is due to the blending function used while modeling for dissipation rate. This actually allows a smooth transaction between small scale processes and large scale process in oceans.

The Richardson number at which turbulence ceases to exist is known as Critical Richardson number (Ri_c). As stated earlier, using a similar process as described in Canuto et al. [1], we obtain Critical Richardson number value of 1.66.

We define 'Y' as a function of mean shear stress. Since kinetic energy tends to zero as turbulence ceases to exist, Y will tend to infinity. This phenomenon is plotted in Fig. 1. This is because mean shear is inversely proportional to kinetic energy. The function 'Y' can be defined as:

$$Y = (\tau \Sigma)^2 \qquad (28)$$

Here, τ is the timescale which is defined as:

$$\tau = 2K\varepsilon^{-1} \qquad (29)$$

At Ri_{cr} stratification becomes too strong which does not allow flow to be turbulent. Thus, we can say that due to the novelty in the present model, we are able to achieve Ri> 1. This is in agreement with many studies like Galperin et al. [32], etc. The two structure functions S_m and S_h as defined by Canuto et al. [1] are plotted against Ri (Fig. 2). These structure functions (also known as stability functions) are structurally similar to those of Canuto et al. [1] but actually in values they are different. These can be obtained by solving (18) and (19), which have been explained in Section V.
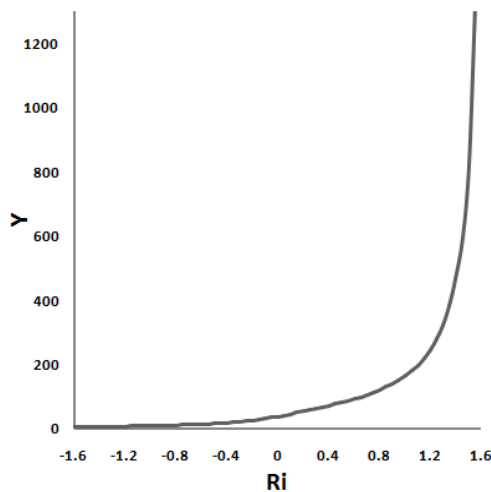

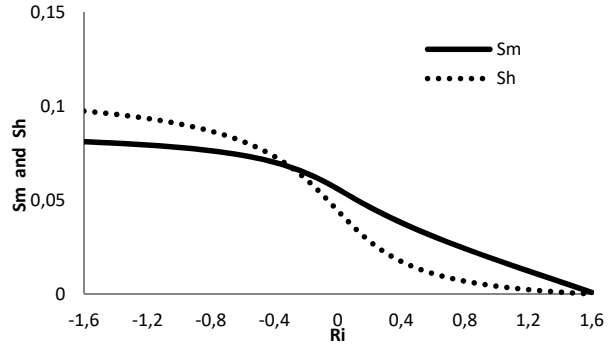Fig. 1 Variable 'Y' Vs the degree of stability Ri

Fig. 2 Variation of stability functions vs Richardson number

The relative position of the curves changes as one goes from negative Richardson number to positive Richardson number. As temperature gradient affects more the heat diffusivity than momentum diffusivity, the heat diffusivity is larger in the unstable case or negative Richardson number case. As one comes closer to positive Richardson number, moment diffusivity dominates.

VIII. RESULTS AND DISCUSSION

Before applying any model to ocean, testing the models reliability has been given huge importance in literatures. Hence, we pass our model through some reliability tests. These tests have also been performed by Canuto et al. [1]. We have plotted both the model of Canuto et al. and the proposed model in this study for these tests. The results and discussion on these results are explained elaborately in the following subsections.

A. Test for Pure Shear Case

This is the case of interest to the engineering flows. This case has been widely studied in [2]. Using the formula stated in Canuto et al. [1], we calculate the first order shear for our case and we get:

$$S_m = 0.065 \qquad (30)$$

The formulation of momentum diffusivity used in Canuto et al. [1] and this present study is:

$$K_m = 2S_m \frac{K^2}{\varepsilon} \qquad (31)$$

But the formulation used in [2] is:

$$K_m = c_\mu \frac{K^2}{\varepsilon} \qquad (32)$$

By comparing (31) and (32), we can obviously deduce that:

$$c_\mu = 2S_m \qquad (33)$$

Hence, from our present study, we get $c_\mu = 0.129$, whereas

Canuto et al. [1] got $c_\mu = 0.11$, both values agree well with the study of Rodi [2].

B. Test Results for Turbulent Prandtl Number Vs Ri

The experimental data of Webster [33] are available. It is based on the formulation:

$$\sigma_t(Ri) = \frac{K_m}{K_h} \qquad (34)$$

where

$$K_h = 2S_h \frac{K^2}{\varepsilon} \qquad (35)$$

This is known as heat diffusivity. Direct Numerical Simulation (DNS) results on stratified turbulent shear flow are available in Gerz et al. [34]. Canuto et al. [1] in their paper also plotted their model results with these data. In this study, we do the same in Fig. 4.

In Fig. 3, the turbulent Prandtl number has been non-dimentionalised using turbulent Prandtl number at Ri = 0.
The proposed or present model produces the data which agree well with Canuto et al. [1].

C. Test Results for Mixing Efficiency

In some literatures, the heat diffusivity is expressed in terms of mixing efficiency.

$$K_h = \Gamma \varepsilon N^2 \qquad (36)$$

In the above formulation, Γ is known as mixing efficiency, it is represented as:

$$\Gamma = \frac{R_f}{1 - R_f} \qquad (37)$$
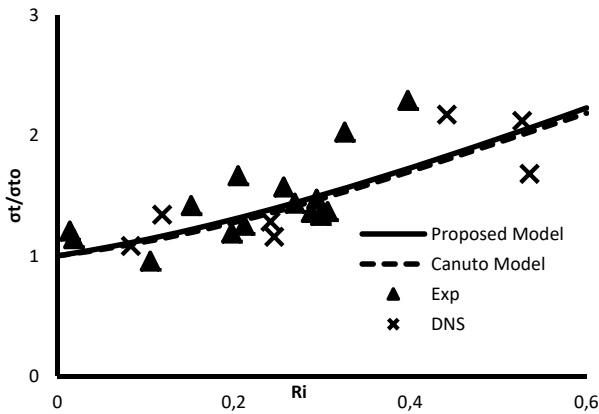
Fig. 3 Turbulent Prandtl number vs Richardson number

In (37), R_f is known as the flux Richardson number. The formulation presented by (37) is famously known as Osborne's formulation for turbulent diffusivity. This formulation assumes stationarity and has been shown to be an oversimplification of the mixing problem by Smyth et al. [35] and a number of recent studies like [36]. Still this formulation is used widely in literatures, hence we also use the same formulation.

The measured value of mixing efficiency $\left(\Gamma\right)$ is given as:

$$\text{For } R_f \geq 0.25 \; ; \; 0.12 \leq \Gamma \leq 0.48 \qquad (38)$$

In Fig. 4, we have plotted the values of mixing efficiency as calculated using the proposed model.
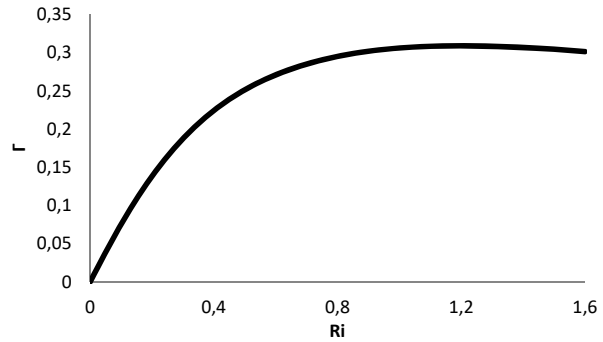

Fig. 4 Mixing efficiency Vs Richardson number

At $R_f = 0.25$, the proposed model predicts $\Gamma = 0.1646$ which is well within the measured range for the mixing efficiency. Then up to critical Richardson number, this study gives the values of mixing efficiency which are within the measured range.

D. Comparison with Ocean Data

After finding that the proposed model works well with the above mentioned tests, it can be concurred that our model is reliable. After the reliability of the proposed model is confirmed, we choose to apply our model to oceanic data available in GOTM.

1. Fladenground Experiment (FLEX)

This dataset has been used for the past several years as calibration for mixing parameterization. These data were collected during the measurements of the Fladenground experiment 1976 (FLEX'76). These measurements for meteorological forcing and temperature potential profiles were done in the spring of 1976 in northern North Sea. The water depth was about 145m and geographical location of $58^0 55$'N and $0^0 32$'E. The simulation was run from April 6 to June 7, 1976. These FLEX'76 data havr been used in several literatures in order to test different schemes [7].

Temperature variation along the depth has been taken for two random days of the experiment. This is shown in Figs. 5 and 6. It can be observed that, due to the recalibration of the model constants and addition of anisotropic dissipation rate,

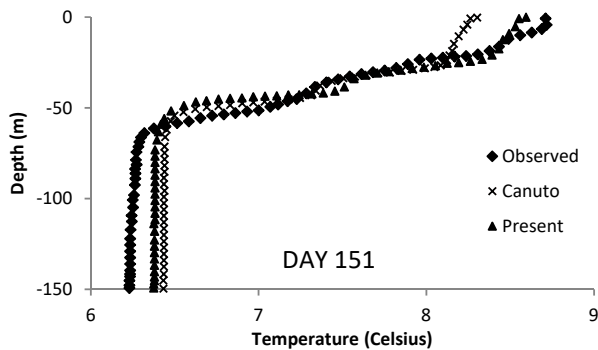the proposed model performs slightly better than Canuto et al. [1].

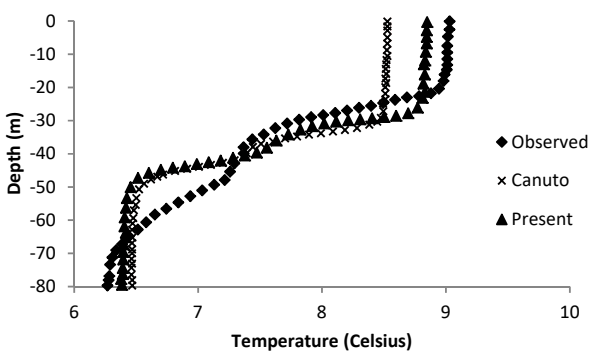Fig. 5 Temperature (Celsius) variation along the depth (m) on Day 151 of the FLEX data

Fig. 6 Temperature (Celsius) variation along the depth (m) of ocean for Day 157 of FLEX data

As can be observed in Figs. 7 and 8, there is a difference between eddy viscosity and heat diffusivity of the two models. It can be recalled that earlier it was stated that though structurally these are the same with Canuto et al. [1], but there is a significant difference due to the calibration of the constants.
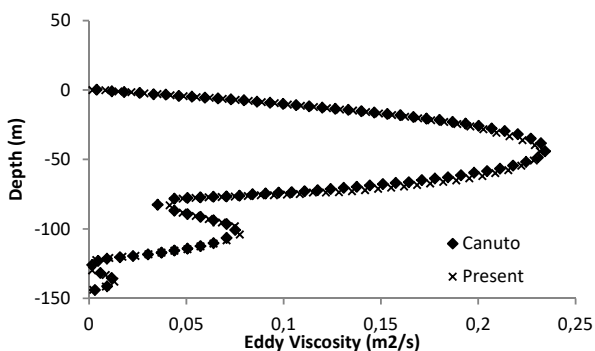
Fig. 7 Variation of eddy viscosity (m^2/s) along the depth (m) of the ocean
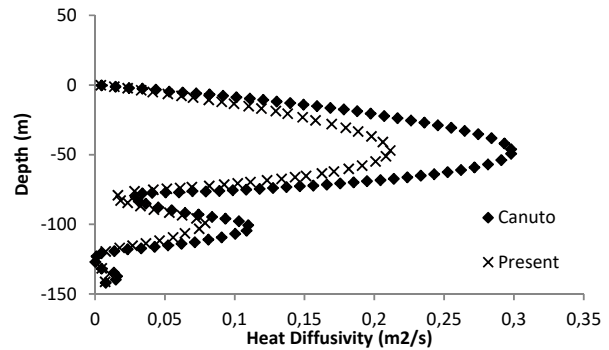
Fig. 8 Variation of heat diffusivity (m^2/s) along the depth (m) of the ocean

2. Gotland Deep Experiment

These simulations have been done for Central Eastern Gotland Sea of the Baltic Sea. Its geographical location is 20^0E and 57.3^0 N. with the water depth being 250 m. Initial conditions for forcing and temperature were taken from measurements. The simulated temperature has been compared with data from COMBINE program. The entire environmental monitoring within HELCOM, and the Baltic marine environment is carried out under COMBINE program. These data have been used for simulating the Gotland Deep ecosystem dynamics for the years 1981-1991 [37].

Fig. 9 shows the representation of sea surface temperature throughout the year for Gotland Deep. The present model predictions of temperature profiles are closer to the observed data and are better than the model of Canuto et al. [1]. In Figs. 10 and 11, the variation of temperature along the depth is presented. The present model seems to give better prediction of temperature.
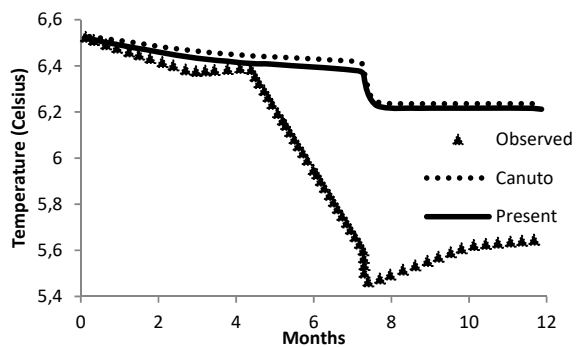

Fig. 9 Sea surface temperature (Celsius) variation throughout the year at Gotland deep

Figs. 12 and 13 represent the heat diffusivity and eddy viscosity. As predicted and also shown in the previous case, the values of heat diffusivity and eddy viscosity differ from Canuto et al. [1]. This is because of the incorporation of the missing flow physics in the formulation of the pressure strain correlation in terms of dissipation anisotropy.
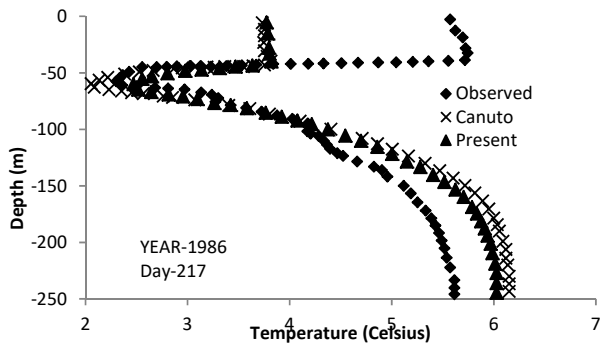
Fig. 10 Temperature (Celsius) variation (Day 217, 1986) along the depth (m) of the ocean for Gotland Deep data
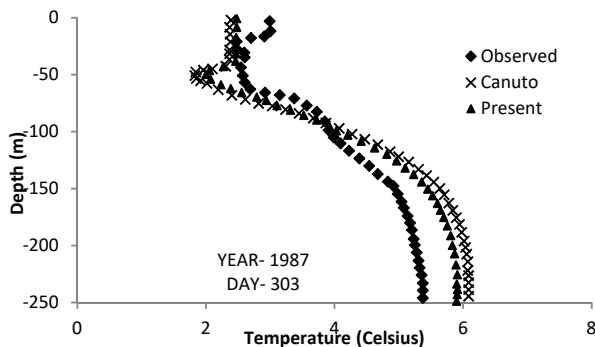

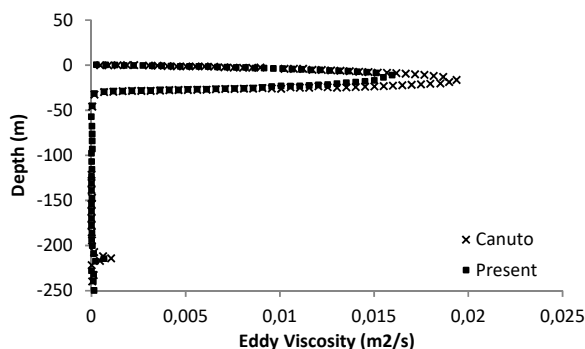Fig. 11 Temperature (Celsius) variation for year 1987 along the depth (m) of the ocean

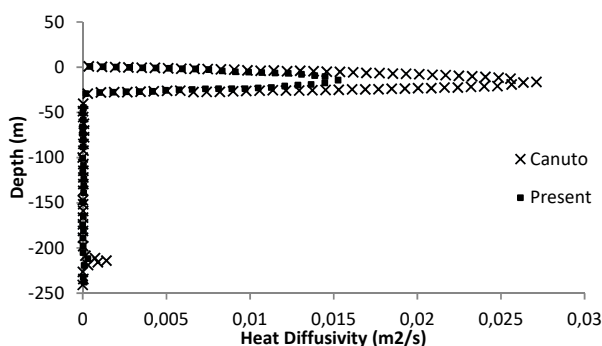Fig. 12 Variation of eddy viscosity (m^2/s) along the depth (m)

Fig 13: Variation of heat diffusivity (m^2/s) along the depth (m)

IX. Conclusion

In the current paper, we have tried to derive a new formula for the pressure strain correlation using the same structure as Canuto et al. [1]. The new formula incorporates the anisotropy in dissipation tensor by making the dissipation tensor a linear function of Reynolds stress anisotropy. More complicated models will include a non-linear relation with anisotropy of Reynolds stress and is not included here. This anisotropization of turbulence seems to give better results for both the test case results of FLEX and Gotland Deep. An increase in critical Richardson number to 1.66 is observed which indicates that, in real scenarios, the turbulence does not cease till Ri reaches 1.66. We feel that this new formulation for pressure strain correlation will improve the forecasts from ocean and atmospheric models and it is worthwhile looking into.

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