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Finite Time Symplectic Synchronization between Two Different Chaotic Systems

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Abstract—In this paper, the finite-time symplectic synchronization between two different chaotic systems is investigated. Based on the finite-time stability theory, a simple adaptive feedback scheme is proposed to realize finite-time symplectic synchronization for the Lorenz and Lü systems. Numerical examples are provided to show the effectiveness of the proposed method.

Keywords—Chaotic systems, symplectic synchronization, finite-time synchronization, adaptive controller.

I. Introduction

CHAOS is very interesting nonlinear phenomenon in physical science. The most important feature of chaotic systems is sensitive to its initial conditions. The synchronization of two chaotic systems with different initial conditions is of great important and has been intensively studied in the last few decades.

The basic idea of synchronization is to use the output of the drive system to control the response system, so that the trajectories of the response system will reach to the trajectories of the drive chaotic system asymptotically. Chaos synchronization has attracted more and more attention from various disciplines due to its potential application in many fields such as secure communication [2], neuroscience [3], chemical reaction[4], and complex networks [5], etc. During the past three decades, many kinds of synchronization have been proposed and investigated, such as complete synchronization [6], anti-synchronization [7], phase synchronization [8], lag synchronization [9], projective synchronization [10], function projective synchronization (FPS), etc. Among these, the generalized synchronization [11], [12], [13] has received much attention which refers to that the corresponding state variables of master-slave systems evolve in some functional relation. The generalized synchronization is noticeable because it is more applicable to secure communication than complete synchronization by introducing an additional function. More recently, the idea of symplectic synchronization [14] has been proposed and developed, which can be seen as an extension of generalized synchronization. Hence, symplectic synchronization can be expected to achieve better performance for applications than generalized synchronization.

In many practical applications, it is more proper to achieve a convergence in a given time than asymptotically. In recent years, some finite-time control techniques have been introduced to realize chaos synchronization. For example, Li proposed to use finite time control techniques to solve the synchronization problem of two chaotic systems so that two chaotic systems can be synchronized in finite time [15], [16]. Aghababa investigated the problem of finite-time chaos synchronization between two different chaotic systems with fully unknown parameter and introduced an adaptive sliding mode controller to ensure the convergence of the chaotic systems in a given finite time [17]. The proposed technique had finite-time convergence and stability in both the reaching and sliding mode phases. In [18], the authors showed that finite-time synchronization can be achieved with only a single control input based on the finite-time stability theory and adaptive control technique. In [19], based the Ito formula and Lyapunov stability theory, the finite-time synchronization of switched stochastic master-slave Rössler systems is studied. It is shown that the finite-time synchronization problem of stochastic Rössler systems can be achieved with a time-driven switching law.

Motivated by the above discussions, the symplectic synchronization between two nonidentical chaotic systems in finite time is investigated in this paper. Based on finite-time stability theory, an adaptive controller is designed to achieve finite-time symplectic synchronization. As numerical examples, the Lorenz chaotic system and the Lü system are taken as the target system and response system, respectively. Numerical simulations demonstrate the effectiveness of the proposed scheme.

The rest of this paper is organized as follows. Section II gives the description of the Lorenz system and the Lü system. In Section III, by employing finite-time stability theory and adaptive control theory, we obtain a sufficient condition for finite time symplectic synchronization between a Lorenz system and a Lü system. Numerical simulations are performed in section 4 to verify the effectiveness of the presented schemes, and concluding remarks are made in the final section.

II. Description of the Systems

The Lorenz system [20] is given by

$$\begin{cases} \dot{x_1} = a(x_2 - x_1) \\ \dot{x_2} = cx_1 - x_1x_3 - x_2 \\ \dot{x_3} = x_1x_2 - bx_3 \end{cases} \quad (1)$$

where x_1, x_2, x_3 are state variables, a, b, c are all positive real parameters. When the system parameters are $a = 10, b = 28, c = 8/3$, the system (1) has a chaotic attractor. The Lorenz chaotic system with initial conditions $(x_1(0), x_2(0), x_3(0)) = (-2, -1, 3)$ is depicted in Fig. 1.

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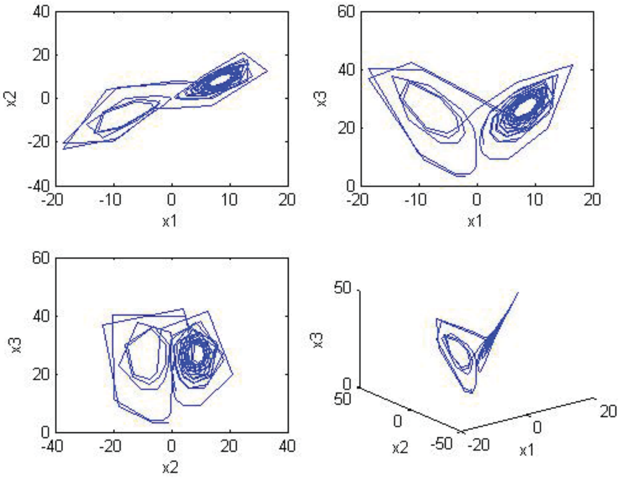

Fig. 1 Typical dynamical behaviors of Lorenz system

The Lü chaotic system is described by [21]:

$$\begin{cases} \dot{y_1} = a_1(y_2 - y_1) \\ \dot{y_2} = c_1 y_2 - y_1 y_3 \\ \dot{y_3} = -b_1 y_3 + y_1 y_2 \end{cases} \quad (2)$$

where y_1, y_2, y_3 are state variables, a_1, b_1, c_1 are all positive real parameters. When we selected the parameters as $a_1 = 36, b_1 = 3, c_1 = 20$, the system exhibits a chaotic behaviour, as shown in Fig. 2.

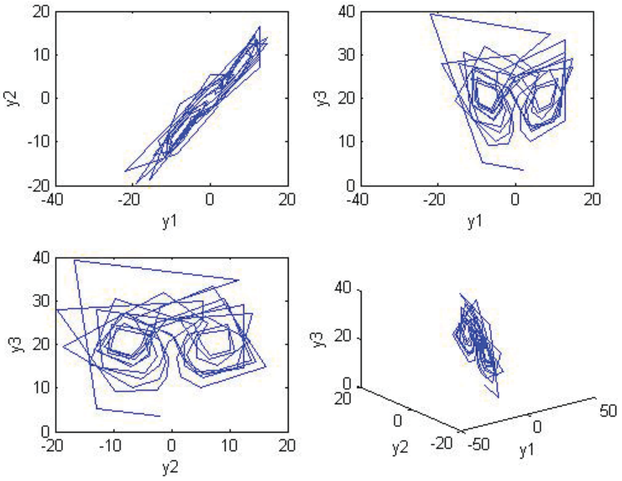

Fig. 2 Typical dynamical behaviors of Lü system

III. Finite Time Symplectic Synchronization between Lorenz System and Lü System

An illustration of the finite time symplectic synchronization is now presented. Consider a drive system

$$\dot{x} = f(x) \quad (3)$$

and the controlled response system

$$\dot{y} = g(y) + u(x, y, t) \quad (4)$$

where $x = (x_1, x_2, ..., x_n)^T$ and $y = (y_1, y_2, ..., y_n)^T$ are the state vectors of systems (3) and (4), respectively; $f, g : R^n \to R^n$ are two continuous nonlinear vector functions and $u(x, y, t)$ is is the controller to be designed.

For symplectic synchronization, the error system is defined as

$$e(t) = y(t) - H(x, y, t) - F(t) \quad (5)$$

where $F(t)$ is a given function of time in different form, such as a regular or a chaotic function.

Our control goal is to design the controller $u(t, x, y)$ for the response system (2), such that the error system (5) can be stable at the zero equilibriumin in a finite-time, i.e. $\lim_{t \to T} e(t) = 0$, and $e(t) \equiv 0$, if $t > T$. Note that when $H(x, y, t) = x$, (3) reduces to the generalized synchronization given in [12]. Therefore both complete synchronization and generalized synchronization can be seen as a special case of symplectic synchronization.

In order to observe finite time symplectic synchronization between the Lorenz and Lü systems, we assume that the Lorenz system is the drive system and the Lü system is the response system. Then the drive system is given in (1) and the response system can be described by follows:

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + u_1 \\ \dot{y}_2 = c_1 y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 = -b_1 y_3 + y_1 y_2 + u_3 \end{cases} \quad (6)$$

where $y_i (i = 1, 2, 3)$ are state variables, and $u_i (i = 1, 2, 3)$ are the controllers such that the two chaotic systems can be synchronized in the case that:

$$\begin{cases} \lim_{t \to T} (y_1(t) - H_1(x, y, t) - F_1(t)) = 0 \\ \lim_{t \to T} (y_2(t) - H_2(x, y, t) - F_2(t)) = 0 \\ \lim_{t \to T} (y_3(t) - H_2(x, y, t) - F_3(t)) = 0 \end{cases} \quad (7)$$

and $e_i(t) \equiv 0 (i = 1, 2, 3)$, if $t > T$.

In this study, we take $F_1(t) = sin(x_3(t))$, $F_2(t) = sin(x_1(t))$, $F_3(t) = sin(x_2(t))$. They are chaotic functions of time. $H(x, y, t)$ is choose as $H_i(x, y, t) = -x_i^2 y_i, i = 1, 2, 3$. So the error variables can be expressed as follows:

$$\begin{cases} e_1 = y_1(1 + x_1^2) - sin(x_3) \\ e_2 = y_2(1 + x_2^2) - sin(x_1) \\ e_3 = y_3(1 + x_3^2) - sin(x_2) \end{cases} \quad (8)$$

Then the detail error dynamics is written as:

$$\begin{cases} \dot{e_1}(t) = (1 + x_1^2)(a_1(y_2 - y_1) + u_1) + 2x_1 y_1 (a(x_2 - x_1)) \\ -cos(x_3)(x_1 x_2 - b x_3) \\ \dot{e_2}(t) = (1 + x_2^2)(c_1 y_2 - y_1 y_3 + u_2) + 2x_2 y_2 (c x_1 - x_1 x_3 - x_2) \\ -cos(x_1)(a(x_2 - x_1)) \\ \dot{e_3}(t) = (1 + x_3^2)(-b_1 y_3 + y_1 y_2 + u_3) + 2x_3 y_3 (x_1 x_2 - b x_3) \\ -cos(x_2)(c x_1 - x_1 x_3 - x_2) \end{cases} \quad (9)$$

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Before developing the design procedure of the proposed finite-time controller for synchronizing two different chaotic systems, we introduce two necessary lemmas.

Lemma 1 [22]. Consider the system

$$\dot{x} = f(x), f(x) = 0, x \in R^n \quad (10)$$

where $f : D \to R^n$ is continuous on an open neighborhood $D \subset R^n$. Assume that a continuous, positive-definite function $V(x)$, real numbers $p > 0$, $0 < \xi < 1$, satisfies the following differential inequality:

$$\dot{V}(x) + pV^{\xi}(x) \leq 0, \forall x \in D \quad (11)$$

Then, the origin of system (7) is a finite-time stable equilibrium.

Lemma 2 [23]. For any given $a_1, a_2, ..., a_n \in R$ and $0 < q < 2$, the following inequality holds:

$$|a_1|^q + |a_2|^q ... + |a_n|^q \geq (a_1^2 + a_2^2 + ... + a_n^2)^{q/2} \quad (12)$$

Theorem 1. The error system (9) can be stable at the zero equilibriumin in a finite time for any different initial condition with following adaptive controller

$$\begin{cases} u_1 = \frac{-2x_1y_1(a(x_2-x_1))+x_4)+cos(x_3)(x_1x_2-bx_3)-ksgn(e_1)|e_1|^r)}{(1+x_1^2)} \\ -(a_1(y_2-y_1)) \\ u_2 = \frac{-2x_2y_2(cx_1-x_1x_3-x_2)+cos(x_1)(a(x_2-x_1)))-ksgn(e_2)|e_2|^r}{(1+x_2^2)} \\ -(c_1y_2 - y_1y_3) \\ u_3 = \frac{-2x_3y_3(x_1x_2-bx_3)+cos(x_2)(cx_1-x_1x_3-x_2))-ksgn(e_3)|e_3|^r}{(1+x_3^2)} \\ -(-b_1y_3 + y_1y_2) \end{cases} \quad (13)$$

where $k > 0$ is a parameter and r is a constant and $0 < r < 1$.

Proof. Lyapunov function is constructed in the form of

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (14)$$

Then time derivative of V along the trajectory of the error system (9) is as follows

$$\begin{aligned} \dot{V} &= (e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3}) \\ &= e_1((1 + x_1^2)(a_1(y_2 - y_1) + u_1) + 2x_1y_1(a(x_2 - x_1)) \\ &- cos(x_3)(x_1x_2 - bx_3)) \\ &+ e_2((1 + x_2^2)(c_1y_2 - y_1y_3 + u_2) + 2x_2y_2(cx_1 - x_1x_3 - x_2) \\ &- cos(x_1)(a(x_2 - x_1))) \\ &+ e_3((1 + x_3^2)(-b_1y_3 + y_1y_2 + u_3) + 2x_3y_3(x_1x_2 - bx_3) \\ &- cos(x_2)(cx_1 - x_1x_3 - x_2)) \end{aligned} \quad (15)$$

Substituting (13) into (15), we have

$$\dot{V} = -k(e_1sgn(e_1)|e_1|^r) + e_2sgn(e_2)|e_2|^r) + e_3sgn(e_3)|e_3|^r)) \quad (16)$$

Using $sgn(e_i) = |e_i|/e_i$, we have

$$\dot{V} = -k(|e_1|^{1+r} + |e_2|^{1+r} + |e_3|^{1+r}) \quad (17)$$

From Lemma 2, we obtain

$$\dot{V} \leq -k2^{\frac{1+r}{2}}(e_1^2 + e_2^2 + e_3^2)^{\frac{1+r}{2}} = -k2^{\frac{1+r}{2}}V^{\frac{1+r}{2}} \quad (18)$$

Therefore, from Lemma 1, the errors $e_i(t) = 0 (i = 1, 2, 3)$ will converge to zero in the finite time. This completes the proof.

IV. Numerical Examples

In this section, we perform numerical simulations to demonstrate the effectiveness of the proposed symplectic synchronization scheme. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system. The system parameters are selected as $a = 10, b = 28, c = 8/3$, and $a_1 = 36, b_1 = 3, c_1 = 20$, such that the drive system (1) and the response system (2) are chaotic with no control applied. The initial values for the drive system and response system are given as $x(0) = (-2, 1, 4)$ and $y(0) = (2, -3, 2)$, respectively.

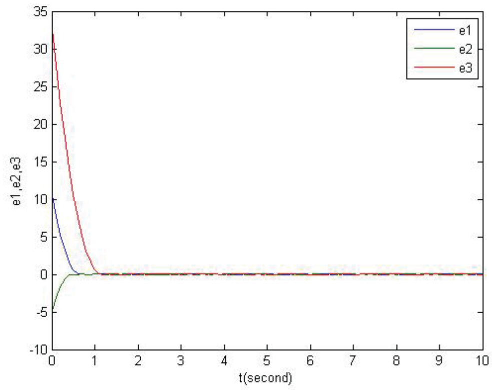

Fig. 3 Time response of the symplectic synchronization errors

Numerical results are displayed in Fig. 3 which shows the time response of error states for the error dynamical system (8). From Fig. 3 we can find that the errors tend to zero rapidly and and it is also proved that the proposed method can make the tracking errors converge to origin within the time t=1.2s, which implies that the finite-time symplectic synchronization of the systems (1) and (2) is indeed realized.

V. Conclusion

The problem of finite-time symplectic synchronization between two different chaotic systems has been discussed in this paper. Using Lyapunov stability theorem and the finite-time control theory, some sufficient conditions for finite-time symplectic synchronization of different chaotic systems are obtained. Numerical simulations are provided to illustrate the feasibility and effectiveness of the presented control technique. The proposed control method may be more valuable to be applied to the realization in engineering than symplectic synchronization technique.

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