# Electromagnetic Source Direction of Arrival Estimation via Virtual Antenna Array

Meiling Yang, Shuguo Xie, Yilong Zhu

*Abstract*—Nowadays, due to diverse electric products and complex electromagnetic environment, the localization and troubleshooting of the electromagnetic radiation source is urgent and necessary especially on the condition of far field. However, based on the existing DOA positioning method, the system or devices are complex, bulky and expensive. To address this issue, this paper proposes a single antenna radiation source localization method. A single antenna moves to form a virtual antenna array combined with DOA and MUSIC algorithm to position accurately, meanwhile reducing the cost and simplify the equipment. As shown in the results of simulations and experiments, the virtual antenna array DOA estimation modeling is correct and its positioning is credible.

*Keywords*—Virtual antenna array, DOA, localization, far field.

## I. INTRODUCTION

IN several decades, electronic products make our life more automatic and convenient, however, electromagnetic radiation, electromagnetic leakage and other problems caused by the complex electromagnetic environment will directly or indirectly affect our lives and work even worse would cause great harm [1]. Therefore, the localization and troubleshooting of the electromagnetic radiation source is very important.

According to the distance between the electromagnetic radiation source and the receiving antenna, we can roughly divide the antenna region into two regions [2], the far field and the near field. For example, UAVs have caused many accidents and troubles recently for airports, whose positioning is very necessary. In this paper, we focus on the far field. In the study of far field signal source localization, spatial spectrum estimation is a new signal processing theory developed in the last 30 years [3]. The method of spatial spectrum estimation direction finding (SSEDF) can simultaneously estimate several signals' directions of arrival, which is more suitable to solve the disturbance of the same frequency and multi-path problem. Among them, the multiple signal characterization [4] method (MUSIC) belongs to the high-resolution subspace method, its performance is excellent, and the algorithm is less computational.

The existing SSEDF systems are mostly based on multi-channel array structures. The system is complex, bulky and expensive. So localization radiation source via simple structure and low cost is urgently needed to be addressed.

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Therefore, this paper proposes a single antenna radiation source localization method that is a single antenna motion to form a virtual antenna array, acquisition signal phase at each virtual array element then phase compensation, combined with SSEDF localization algorithm, at last positioning accurately, which achieves the expected goal of simplifying the equipment and reducing the cost.

## II. THEORY

### A. Direction of Arrival

DOA is a classical localization method. Take uniform linear antenna array [5] (ULA) for example; below is DOA principle on condition of far field.

Hypothesis: the uniform linear antenna array composed of M array elements with equal interval d, each element places a receiving antenna connected to a signal receiving channel, namely multi-channel antenna array receiving system. There is a stable signal emitted by the radiation source. On the condition of far field, ULA wave-front signal can be approximated as a plane wave. The signal is incident at θ angle which indicate the normal angle between the incident direction of the signal and the antenna array, as shown in Fig. 1.
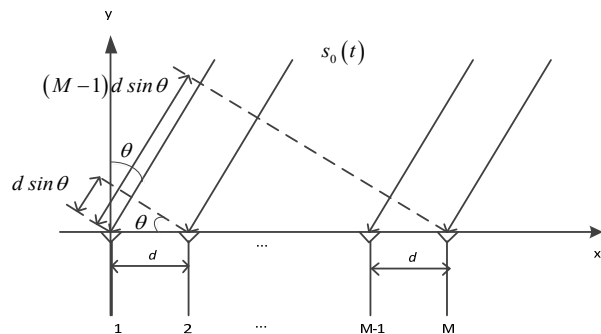


Fig. 1 Uniform linear array receiving plane wave signals

Suppose the first array element is the reference, the received signal is $S_0(t)$, the amplitude is $A_0$, and the angular frequency is $\omega_0$, so the received signal at first array element is expressed as:

$$x_1(t) = s_0(t) = A_0 \cos(\omega_0 t) \qquad (1)$$

Since wave path difference, there is a time difference between the signals received by each array element. With the first element as the reference, the received signal of the kth

element can be expressed as:

$$x_k(t) = s_0(t + \tau_k) = A_0 \cos[\omega_0(t + \tau_k)] \qquad (2)$$

The difference of the wave path between the kth element and the first element is:

$$\Delta r_k = (k-1)d \sin\theta \qquad (3)$$

The time difference corresponding is:

$$\tau_k = \frac{\Delta r_k}{c} = \frac{(k-1)d \sin\theta}{c} \qquad (4)$$

By substituting $\tau_k$, the received signal at the kth array element is:

$$x_k(t) = s_0(t + \tau_k) = A_0 \cos\left[\omega_0\left(t + \frac{(k-1)d \sin\theta}{c}\right)\right] \qquad (5)$$

In general, in order to facilitate theoretical derivation, the incident signal is written as a complex sinusoidal signal, as:

$$s_0(t) = A_0 \exp(j\omega_0 t) \qquad (6)$$

Then, the received signal of the kth array element can be expressed as:

$$\begin{aligned} x_k(t) &= s_0(t + \tau_k(\theta)) = A_0 \exp[j\omega_0(t + \tau_k(\theta))] \\ &= A_0 \exp(j\omega_0 t)\exp(j\omega_0\tau_k(\theta)) \\ &= s_0(t)\exp(j\omega_0\tau_k(\theta)), (k = 1,2,\ldots,M) \end{aligned} \qquad (7)$$

The upper form is written in vector form:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} \exp(j\omega_0\tau_1(\theta)) \\ \exp(j\omega_0\tau_2(\theta)) \\ \vdots \\ \exp(j\omega_0\tau_M(\theta)) \end{bmatrix} s_0(t) \qquad (8)$$

We make $x(t) = a(\theta)s_0(t)$, then

$$a(\theta) = \begin{bmatrix} \exp(j\omega_0\tau_1(\theta)) \\ \exp(j\omega_0\tau_2(\theta)) \\ \vdots \\ \exp(j\omega_0\tau_M(\theta)) \end{bmatrix} \qquad (9)$$

The vector $a(\theta)$ reflects the spatial distribution of the signal energy, named as the direction vector of the signal source. Because the direction vector is comprised of the phase delay of each element, and the phase delay is decided by the array element position, direction of signal arrival and signal center

frequency, so the direction vector contains so many important information to the signal source location.

The above contents are based on the case that the number of signal sources is only one. It needs to broaden the preconditions to meet the realistic situation. Suppose that the number of signal sources is I, represented as $s_1(t), s_2(t),\ldots,s_I(t)$, and the normal angle between the incident direction of the ith signal source and the array antenna is $\theta_i$, so the direction vector of the ith signal can be expressed as $a(\theta_i)$. At this point, the received signal can be represented as [6]:

$$X(t) = AS(t) + N(t) \qquad (10)$$

$X(t) = [x_1(t), x_2(t),\ldots,x_M(t)]^T_{M \times 1}$ is the received signal vector, $S(t) = [s_1(t), s_2(t),\ldots,s_I(t)]^T_{I \times 1}$ is signal source vector, $N(t) = [n_1(t), n_2(t),\ldots,n_M(t)]^T_{M \times 1}$ is noise vector, $A = [a(\theta_1), a(\theta_2),\ldots,a(\theta_I)]_{M \times I}$ is the direction vector with $a(\theta_i) = [\exp(j\varphi_1(\theta_i)), \exp(j\varphi_2(\theta_i)),\ldots,\exp(j\varphi_m(\theta_i))]^T_{M \times 1}, 1 \le i \le I$

*B. Virtual Antenna Array*

In order to reduce cost and simplify equipment, a single antenna is adopted to form a virtual antenna array to realize the localization of radiation source.

Hypothesis: When the single antenna moves to each equivalent virtual array element, sampling is performed K times. The sampling time interval is $t_s$, and the time of the antenna moving from one element to another is $t_0$, and the distance between the two elements is d. The time delay between array elements in a single antenna virtual array system is different from that of uniform linear array. The delay is caused by three factors:

1) Factor of Sampling Time

As the receiving antenna sample at each array element, the time interval leads to the time delay of the received signal, thereby producing phase difference. Suppose the receiving antenna sample K times at each array element. If the receiving channel start sampling from the moment t, then the time required for the antenna to take sth sampling at an array is:

$$t_k = (k-1) \times t_s, 1 \le k \le K \qquad (11)$$

The signal sampled at this time is:

$$x_k(t_k) = s_0[(k-1) \times t_s] = A_0 \cos[\omega_0(t + (k-1) \times t_s)] \qquad (12)$$
$$1 \le k \le K$$

2) Factor of Moving Time

Since it takes time when the antenna moves from one virtual array element to another, there is time delay of the received signal, thereby resulting in phase difference. Considering the time of the antenna movement, the receiving channel begins to

sample from the moment t, so the time required for the antenna to move to the mth element for the kth sampling is:

$$t_{mk} = (k-1) \times t_s + (m-1) \times (K-1) \times t_s + (m-1) \times t_0 \qquad (13)$$
$$1 \le m \le M, 1 \le k \le K$$

The signal sampled at this time is:

$$x_{mk}(t_{mk}) = s_0\left[(k-1) \times t_s + (m-1) \times T\right]$$
$$= A_0 \cos\left[\omega_0\left(t + (k-1) \times t_s + (m-1) \times T\right)\right] \qquad (14)$$
$$1 \le m \le M, 1 \le k \le K$$

With T=$(k-1) \times t_s + t_0$, T represents the total time from the antenna begins to sample at one array until arrives at the next neighboring element.

3) Factor of Array Element Distance

The phase delay $\varphi_m$ due to the different distance between each element and the signal source is:

$$\varphi_m = 2\pi \times \frac{d\sin\theta}{\lambda} \qquad (15)$$

The signal sampled at this time is:

$$x_{mk}(t_{mk}) = A_0 \cos\left[\omega_0(t + (k-1)*t_s + (m-1)*T) + \varphi_m\right]$$
$$= A_0 \cos\left[\omega_0(t + (k-1)*t_s + (m-1)*T) + 2\pi * \frac{d\sin\theta}{\lambda}\right]$$
$$\text{With } 1 \le m \le M, 1 \le k \le K. \qquad (16)$$

Accordingly, when I radiation sources incident at the same time, the upper form is written in vector form:

$$X = \sum_{i=1}^{I} \boldsymbol{\Phi} \boldsymbol{a}_i \boldsymbol{s}_i + \boldsymbol{N} = \boldsymbol{\Phi} \boldsymbol{A} \boldsymbol{S} + \boldsymbol{N} \qquad (17)$$

With $\Phi$ is vector of time delay, A is vector of direction, S is vector of signal and N is vector of noise. Once A is got, we can know the location of the radiation source.

*C. Music*

With the completed single antenna modeling, in algorithm we use the method of multiple signal characterization (MUSIC) [7], [8] to locate the radiation source.

First, the compensated phase is reconstructed as complex sinusoidal signal to get the vector of received signal $X(t) = exp(j\Phi)$. Considering that the real received matrix is finite, the maximum likelihood estimate of the data covariance matrix is $R = E(X*X^T)$. Then the matrix R is Eigen-decomposed to obtain its eigenvalues $\lambda_i$ and eigenvectors $V_i$ (i=0, 1,…,M). Sort $\lambda_i$ in descending order, since $\frac{\lambda_n}{\lambda_{n+1}} \gg 1$, the number of radiation source is equal to n. Next taking the corresponding eigenvectors $V_i$(i=n+1,…,M) to form the noise subspace matrix N. Calculation of spatial spectrum power P(θ)= $\frac{1}{a^T * E * E^T * a}$ with a(θ)= [1,exp(-jwt_1),…exp(-jw$\sum_1^i$ t_i)]^T, t_i=d_isinθ/c,i=0,1,…,M, then search peak of P. Finally, θ that corresponds to the

maximum of P is the direction of the signal source. When there are several radiation sources, A= [a $(\theta_1)$, ..., a $(\theta_k)$] , P will have K maximum values corresponding respectively to K angles θ.

III. LOCALIZATION SIMULATION

Simulation experiments are conducted to verify the correctness of the model of single channel single antenna synthetic virtual antenna array and the performance of far field localization estimation.

Hypothesis of parameters: in the far field, a signal radiation source frequency 450 MHz, azimuth angle θ as 30°. In the simulation, plus random noise directly on the phase which is about 10% of the phase range [-π, π]. Suppose antenna moves along a straight line, every distance 0.75 m resides at once, the number of times of residence is 6, the movement time between the elements 5s, the number of snapshots per time is 5, and the sampling snapshot interval is 1 s. The simulated received signal phase, the compensation phase and the final positioning result are as follows:
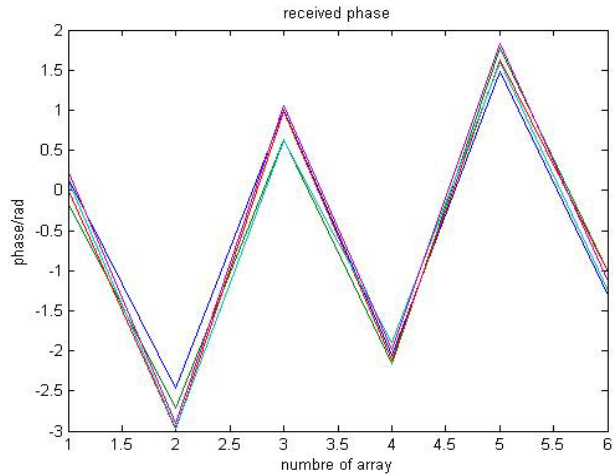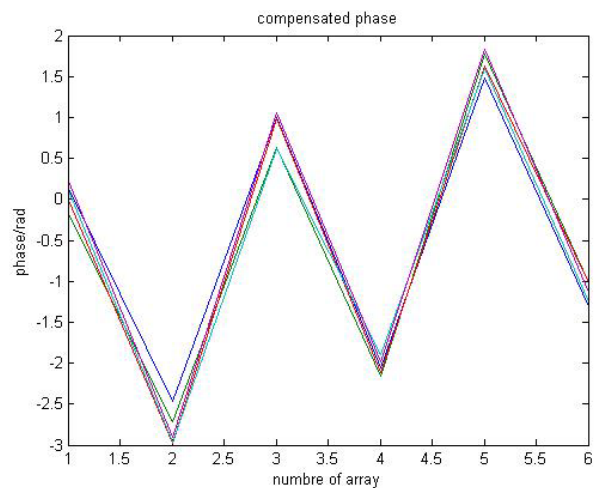


Fig. 2 Received phase
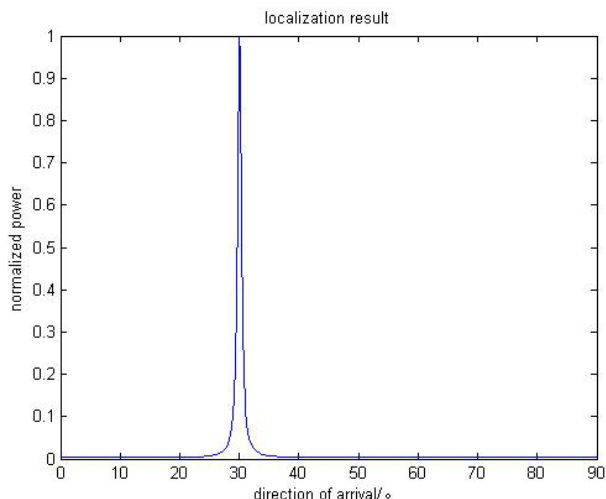


Fig. 3 Compensated phase

Fig. 4 Normalized power



Fig. 6 Compensated phase

The preset parameters of the simulation fit the real test data and can be implemented well. As shown in the simulation results, the localization result of the signal source is the theoretical value, which shows that the single antenna synthetic virtual array DOA positioning method model is constructed correctly, the spectrum peaks are clear and accurate, and the positioning are accurate.

## IV.  LOCALIZATION EXPERIMENT

For better verification, experiments were done in the real environment.

The experimental layout and settings: in a park, a signal source intercom 450 MHz continuous uniform signal, the intercom to the first array element is about 15 meters, the azimuth angle is about 30° - 40°. The moving path is a straight line, five array elements in total. The array element spacing measured by differential GPS on average about 0.75 m with movement time 5 s, snapshot number is one. The experimental received phase, the compensation phase and final positioning results are as follows:
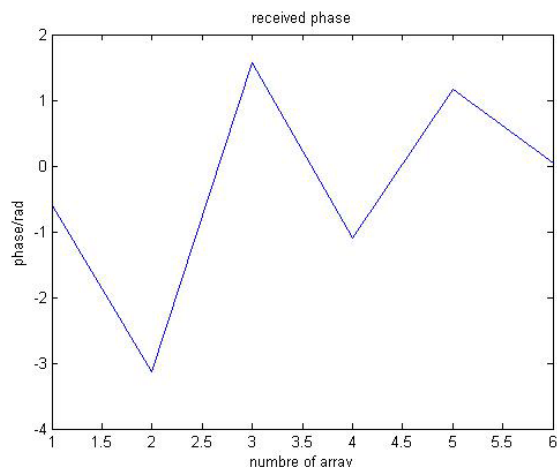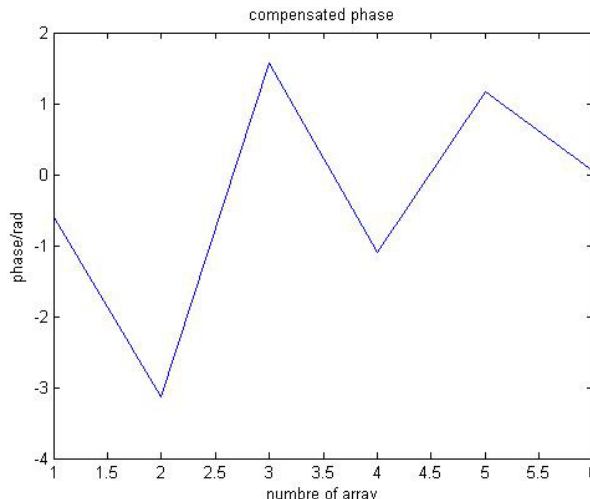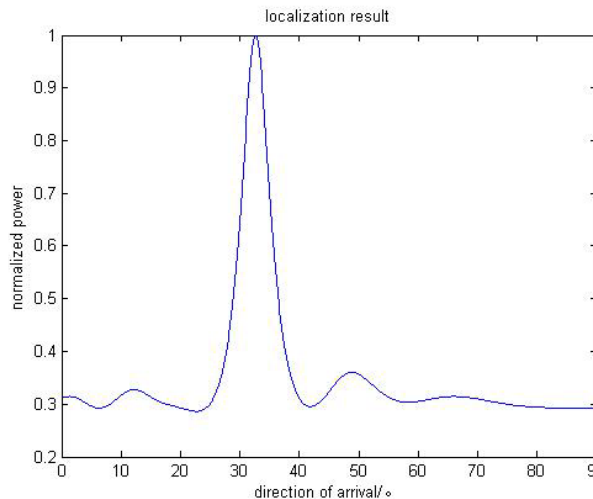


Fig. 7 Normalized power

The experimental results show that the localization result of the signal source is 32.6°, which conforms to the prediction range. Therefore, the experimental result of the single antenna virtual array DOA positioning is credible.

Many experimental results indicate that multipath problem in complex environment, imprecise test time and phase drift problem caused by single channel receiver would lead to error in the final positioning result about 5° - 10°. Therefore, in the future, we need to continue to improve the positioning accuracy, such as hardware accuracy.

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Fig. 5 Received phase

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