International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:11, No:10, 2017

Modal Approach for Decoupling Damage Cost Dependencies in Building Stories

Haj Najafi Leila, Tehranizadeh Mohsen

Abstract**—**Dependencies between diverse factors involved in probabilistic seismic loss evaluation are recognized to be an imperative issue in acquiring accurate loss estimates. Dependencies among component damage costs could be taken into account considering two partial distinct states of independent or perfectly-dependent for component damage states; however, in our best knowledge, there is no available procedure to take account of loss dependencies in story level. This paper attempts to present a method called "modal cost superposition method" for decoupling story damage costs subjected to earthquake ground motions dealt with closed form differential equations between damage cost and engineering demand parameters which should be solved in complex system considering all stories' cost equations by the means of the introduced "substituted matrixes of mass and stiffness". Costs are treated as probabilistic variables with definite statistic factors of median and standard deviation amounts and a presumed probability distribution. To supplement the proposed procedure and also to display straightforwardness of its application, one benchmark study has been conducted. Acceptable compatibility has been proven for the estimated damage costs evaluated by the new proposed modal and also frequently used stochastic approaches for entire building; however, in story level, insufficiency of employing modification factor for incorporating occurrence probability dependencies between stories has been revealed due to discrepant amounts of dependency between damage costs of different stories. Also, more dependency contribution in occurrence probability of loss could be concluded regarding more compatibility of loss results in higher stories than the lower ones, whereas reduction in incorporation portion of cost modes provides acceptable level of accuracy and gets away from time consuming calculations including some limited number of cost modes in high mode situation.

Keywords**—**Dependency, story-cost, cost modes, engineering demand parameter.

I. Introduction

QUANTITATIVE performance-based earthquake engineering (PBEE) has received much attention in recent years as a new proficient method which provides measurable basis in assessing seismic performance of structures [1]. PBEE comprises performance in some quantitative metrics which are more relevant to stakeholders, namely, deaths (loss of life), dollars (economic losses) and downtime (temporary loss of application). One of very frequently used performance assessment procedures is the fully probabilistic quantitative performance evaluation methodology of Pacific Earthquake Engineering Research (PEER) Center which is subdivided into four basic stages accounting for the following: ground motion hazard of a site, structural response of a building, damage of building components and finally repair costs [2]. The fourth and final stage of this method which is the focused field of this research sets up decision variables (DVs) like economic loses based on fragility functions which are cumulative distribution functions prospecting structural response of a building to probability of being or exceeding particular levels of damage and could be served by stakeholders to make more conscious decisions.

The critical viewpoint of the fourth stage of PBEE is the highlighted field of investigations in some of the concerning studies. To respond to the mentioned inquiry, 30 designed modern buildings based on the modern codes have been evaluated in [3], [4]. The amounts of economic loss and number of injuries are high in these buildings despite of conformity to some modern design codes. Mean annual expected loss in the period of building life was evaluated equal to 1% of the whole building replacement cost which is too high and considering the period of the building life equal to 50 years and linear cost distribution in time (that is an underestimating assumption and in real situation the nonlinear distribution causes more costs), the cost of damage reaches to 50% of the building replacement cost in termination of 50 years that is very high amount regarding the conformity of design to some modern codes. In addition, the annual expected loss was estimated in range of 0.4% to 3.3%, representing very uncertain situation for the loss amounts.

The loss of entire building has been disaggregated to some subcategories to simplify evaluating the statistic parameters for loss. These subcategories have usually been considered in component level, some sets of performance group levels or story level. Probabilistic dependencies between these subcategories are known to be an important consideration in seismic loss estimation and risk analysis [5]-[7]; however, incorporating these correlations is not yet common. Some few works considered independency or full dependency situations for some damage states in component level (assuming the same demand level for all components) where partial correlation has not yet been formulated in a tractable manner. Baker and Cornell implemented partial correlation in damage, but only after “collapsing” by the discrete damage state prediction and induced financial loss distribution into a single continuous distribution of component economic loss for a given level of structural response [7], [8]; so, it could not be a general solution. Specifying some correlations among discrete damage states is the other proposed method; however, it is

L. Haj Najafi is with the Amirkabir University of Technology, No. 424-Hafez Ave., Tehran-Post code:15875-4413, Iran (corresponding author, phone: 98912-2049-302; fax: 9821-2217-6167; e-mail: lila_najafi@ aut.ac.ir).
M. Tehranizadeh is with Amirkabir University of Technology, No. 424-Hafez Ave., Tehran-Post code: 15875-4413, Iran (e-mail: dtehz@yahoo. com).

cumbersome for rapid implementation and thus likely impractical for general loss estimation approaches [9].

Dependency between the engineering demand parameters (EDPs) in different stories could be basically got by stability equilibrium equations. Discrepant levels of dependency between the EDPs based on different scaling levels and selected intensity measures for ground motion records could be observed [10]. Dependencies in damage fragility curves are the only ones considered by PACT which is the employed computer program for computing damage costs according to the probabilistic approach. However, this program considers only the dependencies of performance groups in one story and do not reflect dependencies between different stories. Dependencies of performance groups in one story comprise the great portion of involved dependencies and compatibility between the results derived by PACT, and also, the other proposed method by Backer [11] indicates covering almost all the dependencies as the dependencies in damage fragility curves. Dependencies for different performance groups are defined based on two classifications. The first is the situation of fully correlated performance groups which computes cost of damage in both of the considered performance groups in one, two, or all damage levels, and the cost of damage in one of them is double counted, and this causes some inaccuracies. The second is the situation of uncorrelated performance groups, and the cost of damage is individually computed for each of the two assumed performance groups in some or all of the EDP levels. Consideration or non-consideration of dependencies and their effects in precision of loss estimates is the subject of some researches like [6], [12]. Many studies have been performed accounting dependencies between damage and cost fragility functions by the purpose of evading 0% and 100% dependency conditions substituting matrixes of dependencies between damage fragility functions. It is evident that it is impractical to define dependencies between all fragility functions and to take out the amounts of these dependencies in computations since they alter by the levels of damage. These factors are correlated to each other, and each of them has aleatoric and modeling uncertainties where this study comprises only aleatoric uncertainties and does not include epistemic/modeling uncertainties.

Providing the details of the built cost because of the safekeeping situations of contractors is a great challenging issue. These costs are supplied in whole building level instead of component or story level. In this study, the initial costs of different building components derived from the 2014 RS Means Square Foot Costs [13]. For achieving costs in story or component level, two strategies could be followed; the first strategy that is also supplemented in this study is the estimation of building damage cost based on the replacement cost of whole building subjected to some damage states for each component. The second strategy is the estimation of component damage cost based on the amounts of expected costs by some contractors, which is a wide-ranging procedure and also it is very reliant on engineer's estimations [14]. In this study, the cost of each component derived from fragility specifications according to FEMA P-58-1 [12]. The values of mean and standard deviation and the best fitted distribution quantity plateau based on the 10%, 50% and 90% probability of occurrence have been presented in the utilized recommendation.

This paper proposed a new and very practical method founded on utilizing modal concepts which is called here modal cost superposition method for accounting damage cost dependencies in story level where there is no previously conducted study considering dependencies in this level. The principal of the proposed method of this paper is consideration of the cost of each story based on the stories' EDPs as a differential equation. Consequently, for each story, there is a differential equation, which is dependent on the other stories' equations where they should be solved as a system of equations by means of the presented "substituted matrixes of mass and stiffness".

II. The Proposed Method for Disaggregation of Dependencies

The amounts of damage costs for a simple supposed system could be disaggregated into three constituents (the damage cost associated with acceleration-dependent, velocity-dependent and displacement-dependent) costs expressing the equilibrium of all contributed costs in total damage cost. As EDPs are the function of ground motion records which are variable parameters by time and because of the function relations between costs and EDPs as each set of EDPs brings about a specific amount of loss subjected to a presumed level of probability, it could be concluded that costs of damage are correspondingly a function of time; however, it is a cumulative function of time and the cost of damage in time (t) remains in the system and aggregates to the cost of damage in ($t+t_0$). By omitting the velocity-dependent factor because of very few quantity and also scarce information about these components and their fragility functions, the cost equation can be obtained from (1):

$$C_a(t) + C_d(t) = C_t(t) \quad (1)$$

Since C_a is dependent on acceleration and C_d is dependent on displacement, these terms could be written in the form of $m\ddot{x}$ and kx respectively where m and k are the substituted factors of mass and stiffness for acquiring cost of damage. If $C_t(t)$ is assumed to be equal to 0, the cost equation of (1) will be satisfied and solved by a simple harmonic function as a general solution. Following this procedure, a system of equations has been obtained for multiple stories of a model where the quantities of ω^2 are the eigenvalues donating the square of the cost frequencies of the system and presented in (2). The vectors of $[\hat{x}]$ express the corresponding cost amounts known as the eigenvectors or mode shapes of damage costs.

$$[k - \omega^2 m][\hat{x}] = [0] \quad (2)$$

In brief, the principal of the proposed method of this paper is consideration of the costs of each story based on the stories'

EDPs as a differential equation. So, for each story, there is a differential equation, which is correlated to the other stories' equations. This system of equations is going to be solved by means of the proposed "substituted matrixes of mass and stiffness" presented in matrix form for a multistory building in (3).

$$[m][\ddot{x}]+[k][x]=[C_t] \tag{3}$$

where $[m]_{n\times n}$ is the substituted matrix of mass, $[\ddot{x}]_{n\times 1}$ is the matrix of acceleration, $[k]_{n\times n}$: is the substituted matrix of stiffness, $[x]_{n\times 1}$ is the matrix of displacement, and $[C_t]_{n\times 1}$ is the matrix of mean amounts of damage cost.
Disaggregating differential equations in the system equation of (3) has been conducted by application of orthogonality concept for modes of damage cost. In general, it is convenient to express the orthogonality conditions in terms of the mode-shape vectors, ϕ_n, exhibiting in (4). Thus, for systems in which no two modes have the same frequency, the orthogonality conditions could be applied to any two different modes and consequent independency of the costs' differential equations from each other.

$$\phi_m^T m\phi_n = 0 \qquad \phi_m^T k\phi_n = 0 \quad m \neq n \tag{4}$$

It can be verified that, for real, symmetric, positive definite substituted mass and stiffness matrices which pertain to stable structural systems, all roots of frequency equation will be real and positive. Once the collapse occurred, the factors of the substituted matrices of mass and stiffness could not be defined by real numbers and will be sent out of the field of this paper's discussions. Consequently, the collapse probability should be controlled individually and separately in the decision making stage and the ultimate estimated damage cost of building should be modified to capture damage probability in collapse condition. Although cost of damage is a positive scalar parameter, negative amounts could be validated as a declining term to the effects of the other cost modes in superposition process for a specific story.

In practice, cost analysis for the proposed modal procedure requires very slight efforts for decoupling dependencies and in most cases only a relatively small number of lower modes of cost need to be included in the superposition process. However, the main struggle of the proposed modal procedure is about computation of the terms involved in substituted matrixes of mass and stiffness. The substituted matrixes of mass and stiffness comprise probabilistic factors differing by altering the amounts of displacement and accelerations (EDPs) and thus by time as well as by proceeding behavior of the model in nonlinear zone.

In this study, some simplified assumptions have been assumed for definition of the substituted matrices of mass and stiffness in an easy and practical manner.

- Dependency of inter-story drift ratios (IDR) only in two consecutive floors.
- Independency of peak floor acceleration (PFA) parameters.

So, the substituted matrices of mass and stiffness could be defined for a typical 3-story model according to (5).

$$[k]=\begin{bmatrix} k_3 & -k_3 & 0 \\ -k_3 & k_2+k_3 & -k_2 \\ 0 & -k_2 & k_1+k_2 \end{bmatrix}, \quad [m]=\begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \tag{5}$$

where k_3 is the stiffness factor associated with the cost of damage for displacement control components in story 3, k_2 is the stiffness factor associated with the cost of damage for displacement control components in story 2, k_1 is the stiffness factor associated with the cost of damage for displacement control components in story 1, m_3 is the mass factor associated with the cost of damage for acceleration control components in story 3, m_2 is the mass factor associated with the cost of damage for acceleration control components in story 2, m_1 is the mass factor associated with the cost of damage for acceleration control components in story 1.

Factors of these matrices could be acquired according to the results from a cost analyses and subdivided these costs to the corresponding displacement or acceleration costs for each of the stories in addition to considering linear response characteristics in each step of cost calculation. The mode shapes of cost could be normalized to be orthogonal relative to substituted mass which satisfies the condition of (6):

$$[\phi^T][m][\phi]=I \tag{6}$$

where ϕ_i represents the i[th] mode vector, $[\phi]$ is the complete set of normalized mode shapes, and I is an identity matrix.

Normalized mode shapes could be supplied by subtracting each column of matrix of modes to the corresponding amounts of ($\sqrt{M_{ei}}$), where M_{ei} could be calculated by the means of (7):

$$M_{ei}=\phi_i^T[m]\phi_i \tag{7}$$

Providing M_{ei} for each mode, the mass participation factor (MPF_i) in addition to the effective substituted factor of stiffness for each mode (K_{ei}) could be gained through sets of relation in (8), where [m] is the substituted matrix of mass, [k] is the substituted matrix of stiffness, ϕ_i is the mode vector of cost for i[th] mode, i is the mode number, n is total number of modes, M_{ei} is effective mass for mode number i, M_t is total modal mass, MPF_i is mass participation factor for mode i, and K_{ei} is the effective stiffness factor for mode number i.

$$M_t=\sum_{i=1}^{n}M_{ei}\,, \quad MPF_i=\frac{M_{ei}}{M_t}\,, \quad K_{ei}=\phi_i^T[k]\phi_i \tag{8}$$

The factors of effective cost [C_{et}'], could be calculated based on (9). Also, containing C_{et}', M_{ei} and K_{ei}, each of the system's cost differential equations could be solved independently and the results could be aggregated by the means of (9).

$$[C'_{eti}] = [\phi_i]^T [C_t] \text{ , } [C_{normal \quad mt}] = [\phi_{normal}][C'_{et}] \qquad (9)$$

where in the above equations, $[\phi_i]$ is modal vector for mode i^t, $[C_t]$: is cost of damage vector for different stories, $[C_{et}']$ is the effective factor of damage cost vector considering modal effects for mode i, $C_{normal\ mt}$ is the modified amount of damage cost considering dependencies between occurrence probability of loss in sequential stories. Then, the final modified amount of loss including this aspect, C_{mt}, and the participation portion of each mode in entire damage cost of building (α_i) could be derived by application of (10) and (11), respectively. Here, n is the total number of modes, and $[C_{mt}]$ is the matrix of cost modified by consideration of dependencies between occurrence probability of loss in sequential stories.

$$[C_{mt}] = \begin{bmatrix} \frac{1}{MPF_1} \times C_{normal \quad mt,1} \\ . \\ . \\ \frac{1}{MPF_n} \times C_{normal \quad mt,n} \end{bmatrix} \qquad (10)$$

$$\alpha_i = [\emptyset_i]^T [C_{mt}] \qquad (11)$$

The procedure described above could be conducted to obtain an independent equation of cost for each mode, and thus, the use of the normal coordinates serves to transform the cost equations from a set of n simultaneous equations, which are coupled by the off-diagonal terms in substituted matrixes of mass and stiffness, to a set of n independent normal-coordinate equations. By this technique, the total cost could be divided into the costs induced by each mode in the first step (not costs of stories), and in the second step, by excluding the dependencies between these costs, the independent equation for cost of damage for each story could be derived. The proposed procedure in this study is called "modal cost superposition method" which is going to be more clarified through application of an example illustration.

III. Example Study

In this paper, various aspects of the proposed modal cost superposition method will be verified by comparison between the amounts of cost gained and modified by PACT and also through application of the proposed procedure referring to a typical three-story building model.

A. Selected Ground Motions

Ground motion selection and modification (GMSM) is the chief operative factor in predicting the median response of EDPs under prescribed seismic demands [15], [16]. Regarding the number of ground motions, typical practice in structural design is to use seven motions according to ASCE05-7 [17] and 11 ground motions according to ATC-58 [18] as it was employed in this research. The procedure of this paper for record selection is employment of random selection for records by consideration of minimizing deviations around the geometric mean of natural logarithmic spectral acceleration values to reduce the effects of record-to-record variations in structural responses. The efficiency of this record selection technique has been revealed by comparative assessment of deviations of EDPs subjected to some selected record groups by the addressed approach and some arbitrary picked out ones in an accomplished study by the same authors [19].

In this paper, one of very frequently utilized primarily sets of records has been employed. The 79 earthquake ground motions of this list have been carefully selected by Medina and Krawinkler from the PEER strong motion database. The earthquake magnitude in the list ranges in magnitude from 5.8 to 6.9 with the closest distance to rupture ranging from 13 km to 60 km. Recorded motions could be derived from a bin of ground motions from databases of PEER NGA database [20]. It is fine to mention that any arbitrary list of records could be substituted. All included ground motions were recorded on free-field sites which classified as site class D according to NEHRP seismic provision [21], where most of the codes like ASCE05-7 [17] and ATC-58 [18] permit application of this class of soil when the soil specification has not been studied; so, this list of records could be served when the site class has not been determined too. The target spectrum in the level of design earthquake level (DEL) representing 10% probability of earthquake occurrence in 50 years is going to be acquired according to ASCE05-7 procedure through calculating geomean between the design earthquake spectrums for each station. This paper employs a common method for record scaling based on the uniform hazard spectra (UHS) for a short-rise building by fundamental period equal to 0.769 s located in soil class D. This method, recommended by ASCE05-7 [17] and ATC-58 [18] in company with common provisions like IBC2006 and CBC2007 for application in nonlinear response history analysis of structures, suggests scaling record so that the average value of the 5 percent-damped response spectra for the record is not less than the target design spectrum over the period range from $0.2T_1$ to $1.5T_1$. The 11 selected records in company with the scale factors for soil class D are presented in Table I.

TABLE I
11 Selected Ground Motion Records

Record ID	Event	Year	Station	Scale factors
IV79e13	Imperial Valley	1979	El Centro Array #13	5.91
MH84g02	Morgan Hill	1984	Gilroy Array #2	6.45
PM73phn	Point Mugu	1973	Port Hueneme	5.52
PS86psa	N. Palm Spring	1986	Palm Springs Airport	4.89
WN87wat	Whittier Narrows	1987	Carson- Water St	4.93
SF71pel	San Fernando	1971	LA-Hollywood Store Lot	3.60
SH87pls	Superstition Hill	1987	Plaster City	3.32
BM68elc	Borrego Mountain	1968	El Centro Array #9	5.25
LP89slc	Loma Prieta	1989	Palo Alto- SLAC Lab	2.00
NR94del	Northridge	1994	Lakewood- Del Amo Blvd	5.13
CO83c05	Coalinga	1983	Parkfield- Cholame 5W	4.26

B. Description of Structural Systems Used for Evaluation

On account of the need for generality of the results, the assumed configuration of the model is not intended to represent a specific structure. For this purpose, 3-story building with one bay in width and one bay in length has been modeled where the height of each story and the length of each span were respectively deemed equal to 3.0 m and 6.0 m. Loading has been accomplished based on ASCE7-05 [17] by consideration of dead load equal to 600 kg/m^2 and live load equal to 200 kg/m^2 for story one and two and 150 kg/m^2 for the roof story. Also, the dead load relating to wall load was assumed equal to 650 kg/m in story one and two and 200 kg/m^2 in the roof story. The frame was modeled by the means of the open system for earthquake engineering simulation computer code [22]. The fundamental period of the model is equivalent to 0.769 s. Design has been accomplished based on AISC 2005 [23]. IPE and plate girders are utilized for beams, whereas BOX sections are used for columns as presented in Table II.

TABLE II
STORIES' DESIGNED SECTIONS

	Story 1	Story 2	Story 3
Column Section	Box 25×25×2	Box 25×25×2	Box 20×20×1.5
Beam Section	IPE300	IPE300	IPE270
Girder Section	IPE270	IPE270	IPE240

For modeling nonlinearity in structural responses, modified Ibarra-Krawinkler (MIK) model [24] has been employed with bilinear hysteresis behavior [25]. This model exhibited very acceptable compatibility between the derived results from analyses and experiments [26]. Modeling has been conducted by the means of Opensees through using concentrated plasticity in the end joints of each frame component [22]. Critical damping ratios in the first and second modes of vibration are assumed equal to 0.03. Modification of stiffness and damping has been done by consideration of modification factor equal to 10 based on studies conducted by Zareyian and Medina [27]. Geometric nonlinearity is considered through consideration of P-Δ effects [28]. Panel zone modeling has been conducted based on nonlinear behavior proposed by Gupta and Krawinkler composed up three linear fragments [29].

The selected EDPs in performance-based assessment are usually stories' IDR and PFA as well as in this paper. The EDPs have been usually classified in three subgroups; responses in near collapse, non-collapse, and responses from residual drift situations.

By former explanations about the non-applicability of the proposed modal approach for including the collapse condition, since the factors of the substituted matrices of mass and stiffness turned into unreal numbers, the assessments in the situations of collapse and residual drifts which both contribute to building demolishing have been excluded from the assessments of this study. This has been applied to the PACT model through assigning large spectral collapse value equal to $S_{a,C}$=2.5 g gained from incremental dynamic analysis of the model and small residual drift amounts equivalent to 0.001 for no consideration of these issues in evaluation.

For obtaining structural responses in non-collapse condition, nonlinear analyses have been served; all were conducted as Direct Integration Transient time history analyses using direct integration in Hilber-Hughes-Taylor's method, and the corresponding EDPs are extracted for each story of the models. Record scaling has been accomplished according to the scaling level of 1.5 DLE (Design level earthquake) equivalents to the level of MCE (maximum credible earthquake) to contribute the effects of nonlinear behavior. According to FEMA695 [30], the collapse probability in this scaling level is less than 10% for the models designed based on modern codes, which proves the considered assumptions about the collapse and residual drifts of the system. Nonlinear dynamic analyzing has been conducted according to the both N-S and E-W factors of the records. The dispersion amounts (β) of the EDPs are calculated by consideration of lognormal distribution of responses based on the procedure recommended in ATC-58 [18]. The ultimate calculated logarithmic dispersion amounts (β) for the model subjected to PFA and IDR in the scaling level of 1.5 DLE has been obtained equal to 0.31 and 0.38, respectively. The median amounts of EDPs subjected to both of the x and y factors of the scaled 11 records have been employed as EDP amounts.

C. Definition of Cost Model in PACT

PACT code is an electronic calculation tool accompanying databases which conforms to the most used performance-based recommendations like FEMA P-58 [13] and ATC-58 [18]. In order to account for many uncertainties inherent in affecting factors of performance, this program performs calculations based on the foundation of Monte Carlo simulation by generating a large number of simulations (or vectors) of demand per intensity level to develop a loss curve. Each try called a realization and represents one possible performance outcome for the building or for a specific story of a building considering a single combination of possible values for each of the uncertain factors. In this study, the number of realizations has been assumed equal to 200, and the utilized fragility functions are by default available based on FEMA P-58-1 or could be defined or altered based on any arbitrary fragility curves by user. The normative quantities for performance groups are developed based on the proposed normative quantities by FEMA P-58-1. The dependencies between the damage fragility curves of components are the only assumed dependencies based on FEMA P-58-1 requirements as also pointed in this paper.

The median amounts of loss for all stories of the model do not occurred simultaneously, or in the other words with equivalent probability; therefore, aggregating the loss amounts from different stories without considering the dependencies between costs of stories causes some inaccuracies. For a unique specific occurrence probability of loss for different stories of a building, different points in different loss fragility functions would be obtained and superposed probabilistically. Proceeding model's behavior in nonlinear section amplifies dependencies between EDPs of different stories as well as dissimilarities between the amount of loss gained in story level and the amount of loss evaluated on building level.

One straightforward but approximate procedure for modifying the gained amounts in story level according to the obtained amount of loss in entire building level is utilizing unique modification factor deduced by dividing calculated loss of building (evaluated on building level) to the amount of loss obtained from aggregation of each story loss (evaluated on story level). The amount of this modification factor differs by altering the amounts of EDPs and should be calculated for each inputted set of EDPs individually. The modal approach introduced in this paper considers these dependencies analytically and provides independent amounts regarding all the above discussions.

D.Structural Responses and Probable Amounts of Loss

After assigning the amounts of EDPs, the provided PACT code could be run based on the presumed fragilities and derived into the probability distribution of the system's loss in building level and also each of the story levels; however, the median amounts of damage cost are the amounts in emphasis of this study. The entire initial building cost has been evaluated considering the average amount of 250 $ per square foot for a building with special steel moment frame system and office occupancy through application of commonly applied cost code of RS Means Square foot [13]. Then, the entire initial cost for the studied example model is equivalent to 290477 $. Probable amounts of damage costs estimated by PACT have been presented in Table III.

TABLE III
THE AMOUNTS OF DAMAGE COST BY PACT (WITHOUT CONSIDERAION OF STORY-DEPENDENCIES)

	IDR	PFA (g)	Damage cost according to IDR/ Entire initial cost	Damage cost according to PFA/ Entire initial cost	Total damage cost/ Entire initial cost
Story3	0.018	0.493	7.16	1.56	8.72
Story 2	0.016	0.490	9.09	1.93	11.02
Story 1	0.008	0.485	2.45	2.78	5.23

Sum =24.97%

The median value of the total cost of damage anticipated by PACT is equal to 20.64% of the initial cost of the model; however, this value through aggregating the amounts of damage costs from the story cost is equal to 24.97% of the initial cost. So, the modification factor could be calculated for the total building costs equivalent to 20.64/24.97=0.827 for consideration of dependencies between different stories and some incompatibilities in statistical parameters for each story in comparison with the entire building damage cost where it has been simultaneously and similarly applied to the estimated damage costs of stories.

E. Cost Decoupling Based on the Proposed Modal Cost Method

The proposed substituted matrices of mass and stiffness could be generated according to the stories cost subjected to drift and acceleration from the mentioned amounts in Tables II and III as:

$$[C_t] = \begin{bmatrix} 8.72 \\ 11.02 \\ 5.23 \end{bmatrix},$$

$$k_1=(2.45/(0.008\times3))=102.083,$$

$$k_2=(9.09/(0.016\times3))=189.375,$$

$$k_3=(7.16/(0.018\times3))=132.593$$

$$m_1=(2.78/(0.485\times9.81))=0.5848,$$

$$m_2=(1.93/(0.490\times9.81))=0.4015,$$

$$m_3=(1.56/(0.493\times9.81))=0.3225.$$

Referring to (5), the substituted matrices of mass and stiffness could be estimated based on the computed factors of k and m. Then, cost modes could be resembled through discovering efficient vectors of the matrix of [k-mω^2]. These modes have been presented below as well as they could be exhibited schematically in Fig. 1.

$$[k]=\begin{bmatrix} 132.59 & -132.59 & 0 \\ -132.59 & 321.97 & -189.38 \\ 0 & -189.38 & 291.46 \end{bmatrix},$$

$$[m]=\begin{bmatrix} 0.3225 & 0 & 0 \\ 0 & 0.4015 & 0 \\ 0 & 0 & 0.5843 \end{bmatrix}$$

$$1^{st} \text{ mode of cost: } [\phi_1] = \begin{bmatrix} 0.6874 \\ 0.5831 \\ 0.4330 \end{bmatrix},$$

$$2^{nd} \text{ mode of cost: } [\phi_2] = \begin{bmatrix} -0.7865 \\ 0.0833 \\ 0.6120 \end{bmatrix},$$

$$3^{rd} \text{ mode of cost: } [\phi_3] = \begin{bmatrix} -0.4295 \\ 0.8187 \\ -0.3813 \end{bmatrix}.$$

These modes are normalized to be orthogonal relative to substituted mass matrix which satisfies the condition of (6).

M_1=3.9090, M_2=4.1312, M_3=4.0567 and $M_1+M_2+M_3$=12.0969

$$1^{st} \text{ normal mode of cost: } [\phi_{normal,1}] = \begin{bmatrix} 0.3477 \\ 0.2949 \\ 0.2190 \end{bmatrix},$$

2nd normal mode of cost: $[\phi_{normal,2}] = \begin{bmatrix} -0.3869 \\ 0.0410 \\ 0.3011 \end{bmatrix}$, 3rd normal mode of cost: $[\phi_{\text{normal},3}] = \begin{bmatrix} -0.2132 \\ 0.4062 \\ -0.1893 \end{bmatrix}$.

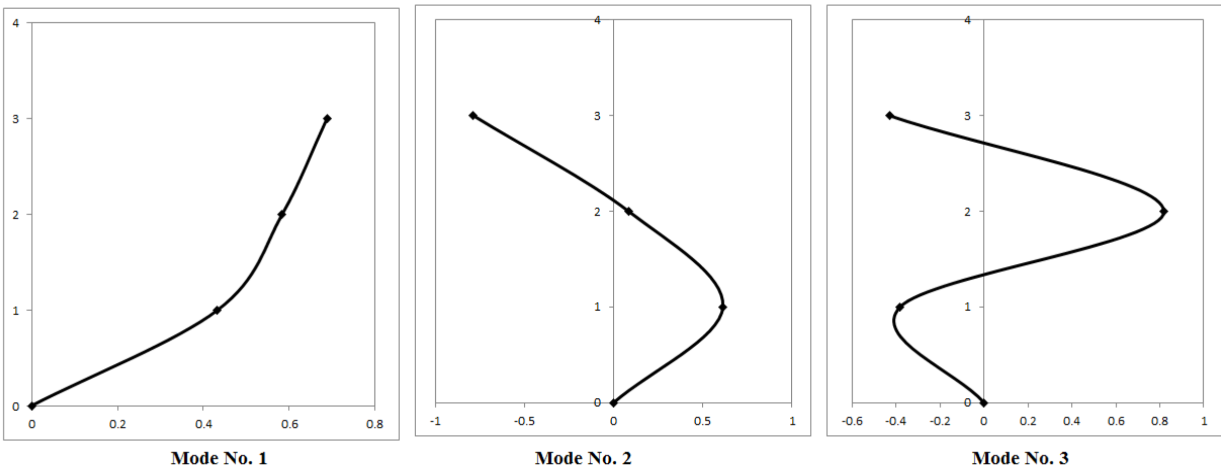

Fig. 1 Schematic diagrams of the modes of cost for the presumed model

Following the procedure which has been described formerly for getting decupled cost of each story through (7)-(10), the decoupled amount of loss for each story could be obtained:

$[\phi_{normal,1}]^T.[C_t] = 7.4273$, $[\phi_{normal,2}]^T.[C_t] = -1.3474$, $[\phi_{normal,3}]^T.[C_t] = 1.6298$.

$$7.4273 \times [\phi_{\text{normal},1}] - 1.3474 \times [\phi_{\text{normal},2}] + 1.6298 \times [\phi_{\text{normal},3}] = \begin{bmatrix} 2.7561 \\ 2.7977 \\ 0.9124 \end{bmatrix}$$

$$\begin{bmatrix} 2.7561 \times {12.0969}/{3.9090} \\ 2.7977 \times {12.0969}/{4.1312} \\ 0.9124 \times {12.0969}/{4.0567} \end{bmatrix} = \begin{bmatrix} 8.5295 \\ 8.1920 \\ 2.7206 \end{bmatrix},$$

$$[C_{mt}] = \begin{bmatrix} 8.5295 \\ 8.1920 \\ 2.7206 \end{bmatrix}$$

By aggregating stories' damage costs, the estimated amount of loss for entire building regarding the dependencies between occurrence probabilities of damage in different stories could be also obtained through application of the proposed modal technique in this paper equivalent to 19.44%, whereas the estimated amount of loss for entire building from PACT is equal to 20.64% and the estimated amount of loss for entire building from PACT by aggregating damage cost of different stories is equal to 24.97%. So, involving dependencies between occurrence probabilities of stories costs reduces the total damage cost of building gained from aggregation of story costs from which evaluated by PACT according to story level evaluation progress. The difference between the evaluated damage costs with and without consideration of the story-cost-dependencies for the entire building is equivalent to (24.97%-19.44%= 5.53%) of the entire initial cost of model, proving the prominence of considering stories dependencies in evaluations. While this reduction is not too considerable in the percentage of total damage cost of building in some cases, it could save great expenses particularly for the models with large areas and thus large initial costs assuming the amount of building cost in dollars. Also, the estimated damage costs for entire building from the proposed modal approach and the evaluated amount by PACT in building level prove close compatibility illustrating the correctness of the results from the proposed modal procedure. Table IV presents the incorporated amounts of cost for each story based on the proposed modal procedure explained in (7)-(10), evaluated amounts of damage cost in story level by PACT and also the modified amounts of cost from PACT by application of modification factor which had been explained formerly. The outcomes could be evaluated more explicably by the help of diagrams in Fig. 2.

The compatibility between the results has been more observed in higher stories than the lower ones concluding to more probability dependencies in lower stories and also because of more contribution portion of acceleration-dependent responses in top story which are assumed to be uncorrelated in this study through application of diagonal substituted matrix of mass. Comparing the results from the proposed method of this paper and the modified results from PACT illustrates insufficiency of applying simple modification factor for consideration of occurrence probability dependencies between stories because of different dependencies between damage costs of different stories; contrariwise, application of modification factor is a very computationally straightforward procedure.

Discussing portion of incorporation for each mode is very advantageous to comprehend the view of this new proposed method of dependency calculation and also supports explanation of cost modes meaningfully. These portions could be obtained through application of (11).

TABLE IV
THE PERCENTAGE OF DAMAGE COST FOR EACH STORY FROM PACT (WITH AND WITHOUT MODIFICATION) IN ACCOMPANY WITH THE AMOUNTS FROM THE PROPOSED MODAL APPROACH

Damage Cost	Damage costs evaluated by PACT in story level	Modified damage costs from PACT by applying simple modification factor (20.65/24.97)	Damage costs obtained from the proposed modal procedure
Damage Cost of Story 3	8.72	7.21	8.53
Damage Cost of Story 2	11.02	9.11	8.19
Damage Cost of Story 1	5.24	4.33	2.72
Sum %	24.97	20.65	19.44

Incorporation of the 1st mode of cost in total cost of model:

$$\begin{bmatrix} 0.6874 & 0.5831 & 0.4330 \end{bmatrix} \begin{bmatrix} 8.5295 \\ 8.1920 \\ 2.7206 \end{bmatrix} = 11.8179$$

Incorporation of the 2nd mode of cost in total cost of model:

$$\begin{bmatrix} 0.7865 & -0.0833 & -0.6120 \end{bmatrix} \begin{bmatrix} 8.5295 \\ 8.1920 \\ 2.7206 \end{bmatrix} = 4.3603$$

Incorporation of the 3rd mode of cost in total cost of model:

$$\begin{bmatrix} -0.4295 & 0.8187 & -0.3813 \end{bmatrix} \begin{bmatrix} 8.5295 \\ 8.1920 \\ 2.7206 \end{bmatrix} = 2.0060$$

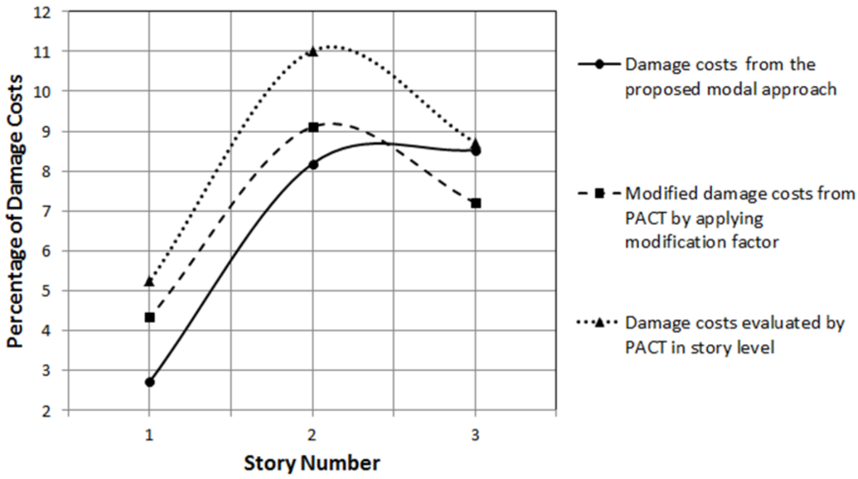

Fig. 2 Diagrams of the percentage of damage cost due to each story of the model from PACT (with and without modification) and the proposed modal approach

Further, the incorporation portion of each mode could be obtained as follow: the percentage of incorporation portion of 1st mode in total damage cost of building: 64.99%, the percentage of incorporation portion of 2nd mode in total damage cost of building: 23.98%, the percentage of incorporation portion of 3rd mode in total damage cost of building: 11.03%. The results illustrate that the 64.99% of the occurred damage cost are due to the pattern of first mode, and 23.98% and 11.03% are due to second and third mode patterns, respectively. Then, the incorporation portion decrease in higher modes of cost and for multi-story buildings with large number of stories, participation of some few modes could contribute to evaluations with acceptable level of accuracy and it is not required to encounter all modes which would be computationally an expensive task.

IV. CONCLUSION

This paper proposed a novel and very practical method for incorporating story-damage-cost dependencies in loss evaluations by the means of modal concept which is called here "modal cost superposition method". The principal of the proposed method of this paper is consideration of damage cost for each story based on the stories' EDPs as a differential equation. So, for each story, there is a differential equation, which is dependent on the other stories' equations and they should be solved as a system of equations by the means of the introduced "substituted matrixes of mass and stiffness". These matrices include probabilistic parameters varying by altering the amounts of EDPs and thus by time as well as by proceeding model's behavior in nonlinear zone. In addition, various aspects of the proposed modal cost superposition method have been inspected by reference to a typical instructive example study.

- The conducted reduction in the evaluated damage costs with and without consideration of the story-cost-dependencies for the entire building illustrates the prominence of including stories dependencies in evaluations. While this reduction is not too considerable in the percentage of total damage cost of building in some cases, it could save great expenses particularly for the models with large areas and thus large initial costs assuming cost of building in dollars.

- The estimated damage costs for entire building from the proposed modal approach and the evaluated amount by PACT in building level prove close compatibility illustrating precision of the results from the proposed modal procedure.
- Assessing the evaluated damage cost of stories, the compatibility between the results has been more observed in higher stories than the lower ones concluding to more probability dependencies in lower stories and also because of more contribution portion of acceleration-dependent responses in top story. Comparing the results from the proposed method of this paper and the modified results from PACT illustrates insufficiency of applying simple modification factor for considering occurrence probability dependencies between stories because of different amounts of dependencies between damage costs of different stories; contrariwise, application of modification factor is a very computationally straightforward procedure.
- The incorporation portion of cost decline in high mode situations; then, for multi-story buildings with large number of stories, participation of some few cost modes could result in evaluations with acceptable level of accuracy and it is not required to encounter all modes which is computationally an expensive task.

References

[1] Building Seismic Safety Council (BSSC), "NEHRP recommended provisions for seismic regulations for new buildings and other structures," FEMA 450, Federal Emergency Management Agency, Washington D.C, 2003.
[2] C. Ramirez and E. Miranda, "Building-specific loss estimation methods & tools for simplified performance-based earthquake engineering," Report No. 171, (Ph.D. dissertation), John A. Blume Earthquake Engineering Center: Stanford University; 2009.
[3] C. A. Goulet, C. B. Haselton, J. Mitrani-Reiser, J. L. Beck, G. G. Deierlein, K. A. Porter, J. P. Stewart, "Evaluation of the Seismic Performance of a Code-Conforming Reinforced Concrete Frame Building - from Seismic Hazard to Collapse Safety and Economic Losses," *Earth. Eng. Struc. Dyn.,* vol. 36, no.13, pp.1973-1997, 2007.
[4] C. B. Haselton, C. A. Goulet, J. Mitrani-Reiser, J. L. Beck, G. G. Deierlein, K. A. Porter, J. P. Stewart, E. Taciroglu, "An Assessment to Benchmark the Seismic Performance of a Code-Conforming Reinforced Concrete Moment-Frame Building," PEER Report No. 2007/12, Pacific Earthquake Engineering Research Center, College of Engineering: University of California, 2008.
[5] E. Miranda, H. Aslani and S. Taghavi, "Assessment of Seismic Performance in Terms of Economic Losses," *PEER 2004/05*, P. Fajfar and H. Krawinkler, editors. International Workshop on Performance-Based Seismic Design Concepts and Implementation, Bled, Slovenia: 2004. pp. 149-160.
[6] H. Aslani and E. Miranda, "Probabilistic Earthquake Loss Estimation and Loss Deaggregation in Buildings," Report No. 154, (Ph.D. dissertation), John A. Blume Earthquake Engineering Center: Stanford, CA; 2006.
[7] J. W. Baker and C. A. Cornell, "Uncertainty Propagation in Probabilistic Seismic Loss Estimation," *Struct. Saf.,* vol. 30, no. 3, pp. 236-252, 2008.
[8] J. W. Baker and C. A. Cornell, Uncertainty Specification and Propagation for Loss Estimation Using FOSM Method," PEER 2003-07. Pacific Earthquake Engineering Research Center, University of California at Berkeley: Berkeley, California, 2003.
[9] R. Lee and A. S. Kiremidjian, "Uncertainty and Correlation for Loss Assessment of Spatially Distributed Systems. Earth Spec. 2007; 23(4): 753-770.
[10] C. B. Haselton and G. G. Deierlein, "Assessing Seismic Collapse Safety of Modern Reinforced Concrete Frame Buildings," Technical Report No. 156. Stanford, CA: John A. Blume Earthquake Engineering Center, Stanford University; 2007.
[11] J. W. Backer, "Introducing correlation among fragility functions for multiple components," The 14th World Conference on Earthquake Engineering, October 12-17, Beijing, China: 2008.
[12] FEMA P-58-1. "Seismic Performance Assessment of Buildings, Applied Technology Council and Federal Emergency Management Agency," Report No. P-58-1, Washington D.C; 2012.
[13] Balaboni, B. *RSMeans Square Foot Costs 2014*, RSMeans Engineering Department, 35th ed. New York, United States; 2014.
[14] M. A. Touran and L. Suphot, "Rank Correlations in Simulating Construction Costs," *J. Constr. Eng. Manag.,* vol. 123, no. 3, pp. 297-301, 1997.
[15] J. W. Baker, "Measuring Bias in Structural Response Caused by Grand Motion Scaling," 8th Pacific Conference on Earthquake Engineering: Final program and book of abstracts, 5-7 December 2007, Technological University, School of Civil and Environmental Engineering: 2007, pp. 82–91.
[16] C. B. Haselton, A.S. Whittaker, J. W. Baker, J. Bray and D. N. Gray, "Selecting and Scaling Earthquake Ground Motion for Performing Response-History Analyses," In: Proceedings of the 15th world conference on Earthquake Engineering, Earthquake Engineering Research Institute; 2012, pp. 4207–4217.
[17] ASCE, "Minimum Design Loads for Buildings and Other Structures," ASCE/SEI 7-10. American Society of Civil Engineers, Reston, Virginia; 2010.
[18] Applied Technology Council, "Guidelines for Seismic Performance Assessment of Buildings," ATC-58-1 50% Draft, Washington D.C. (cited 2011 May 1). Available from: https://www.atccouncil.org/pdfs/ATC-58-50persentDraft.pdf/.
[19] L. Haj Najafi and M. Tehranizadeh, "Ground motion selection and scaling in practice," *Period. Polytech. Civ. Eng*., vol. 59, no. 2, pp. 233-248, 2015. DOI:10.3311/PPci.7808.
[20] PEER Strong Ground Motion Database. (cited 2016 May 05). Available from: http://peer.berkeley.edu/peer_ground_motion_database.
[21] NIST, "Soil-Structure Interaction for Building Structures, NIST/GCR 11-917-14. NEHRP Consultants Joint Venture for the National Institute of Standards and Technology, Gaithersburg, Maryl; 2011.
[22] OpenSees, Open system for earthquake engineering simulation, Pacific Earthquake Engineering Research Center, Berkeley, CA; 2009.
[23] AISC2005, "Specification for Structural Steel Buildings, American Institute of Steel Construction," One East Wacker Drive, Suite 700, Chicago, Illinois 60601-1802, Third Printing: April 2007.
[24] L. F. Ibarra, R. A. Medina and H. Krawinkler, "Hysteretic models that incorporate strength and stiffness deterioration, *Earth. Eng. Struct. Dyn*., vol. 34, no. 12, pp.1489–1511, 2005.
[25] D. G. Lignos and H. Krawinkler, "Sideway collapse of deteriorating structural system under seismic excitations," Report No. 177, (Ph.D. Dissertation), John A. Blume Earthquake Engineering Center, Stanford University, United State; 2012.
[26] D. G. Lignos, H. Krawinkler and A. S. Whittaker, "Prediction and validation of sideway collapse of two scale models of a 4-story steel moment frame," *Earthq. Eng. Struct. Dyn*., 2011; DOI: 10.1002/eqe.1061.
[27] F. Zareian and R.A. Medina, "A practical method for proper modeling of structural damping in inelastic plane Structural systems", *Computers Struct.* Vol. 88, no. 12, pp.45-53, 2010.
[28] Lignos D. G., "Modeling Steel Moment Resisting Frames with OpenSees," OpenSees Workshop, University of California, Berkeley, United States; 2014.
[29] A. Gupta and H. Krawinkler, "Seismic Demand for Performance Evaluation of Steel Moment Resisting Frames Structures," Report No. 132. Stanford, CA: John A. Blume Earthquake Engineering Center, Stanford University, 1999.
[30] FEMA P 695, "Quantification of Building Seismic Performance Factors," Building seismic safety council for the Federal Emergency Management Agency, Report No. FEMA P695," Federal Emergency Management Agencies, Washington DC; 2009.

Leila Haj Najafi was born in 1983, in Tehran, Iran. She obtained her BS degree in Civil Engineering from Sharif University of Technology, Tehran, Iran, in 2005, and her MS and PhD degrees in Earthquake Engineering in 2008 and 2015, respectively, from Amirkabir University of Technology, Iran. She has coauthored over 22 publications in related fields in conferences and

journals. Her research interests include: performance based evaluation, structural reliability, probabilistic risk assessment, hazard analyzing and ground motion selection.

Mohsen Tehranizadeh was born in 1954, in Tehran, Iran. He received his BS degree in Structural Engineering from Sharif University of Technology, Tehran, in 1975, and MS and PhD degrees in Structural Engineering and Structural Dynamics in 1978 and 1986, respectively, from the University of Southern California, USA. He is currently Professor at the faculty of Civil and Environmental Engineering, Amirkabir University of Technology, Tehran, Iran. His major interests lie in the application of seismic linear and nonlinear structural analysis and response to analyze and design of seismic resistant structures and rehabilitation of existing buildings. The most recent research deals with, the seismic evaluation of strengthening of existing structures, the effect of geology on ground motion, near fault features include directivity and polarisation effect, fault- fling, the soil structure interaction.