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Design of Bayesian MDS Sampling Plan Based on the Process Capability Index

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***Abstract*—**In this paper, a variable multiple dependent state (MDS) sampling plan is developed based on the process capability index using Bayesian approach. The optimal parameters of the developed sampling plan with respect to constraints related to the risk of consumer and producer are presented. Two comparison studies have been done. First, the methods of double sampling model, sampling plan for resubmitted lots and repetitive group sampling (RGS) plan are elaborated and average sample numbers of the developed MDS plan and other classical methods are compared. A comparison study between the developed MDS plan based on Bayesian approach and the exact probability distribution is carried out.

***Keywords*—**MDS sampling plan, RGS plan, sampling plan for resubmitted lots, process capability index, average sample number, Bayesian approach.

I. Introduction

STATISTICAL quality control is widely used in world high-tech industries; quality control methods attempt to reduce the waste of production as much as possible. If a manager wants to perform quality control in different parts of an organization, due to the high cost of control methods, he should use precise and suitable techniques. In the context of products quality control in all production stages, some measures are applied which are referred to as process capability analysis. An index called PCI is used for analyzing the process capability. By paying more attention to the wide application of these indices, selection of their estimators and probability distribution of these indices is very important; thus, a precise method like Bayesian statistical techniques is used to solve this problem. This technique specifies a prior distribution function for the given parameters and then in the next stage forms a posterior distribution function for these parameters using the collected data. One usage of PCI is to design an acceptance sampling plan to make decisions about a received lot from suppliers in production environments, so that the risk of producer and consumer adapts with the specified quality standards.

Among the classical methods of acceptance sampling plan, the variable sampling plan is very important due to the quantitative analysis of quality characteristics. Although the use of this plan is more difficult compared to the attribute sampling plan, it results in more precise measurements for decision making about lot and is less risky. The MDS sampling plan is one of the effective sampling plans methods which is based on a conditioned procedure and has been introduced by Wortham and Baker [1]. This sampling plan considers not only samples taken from the current lot, but also it considers sampling results of previous or future lots.

With attention to the importance of the variable sampling plan based on PCI, these types of sampling plans have been discussed in recent years. Examples include [2]-[6] and many others.

Recently, a new MDS sampling plan based on PCI has been proposed by Aslam et al. [7]. Also a new method of MDS sampling plan is presented by Balamurali and Jun [8] for the lot acceptance problem. Soundararajan and Vijayaraghavan [9] developed a procedure for scheming multiple dependent (deferred) state sampling plans. Vaerst [10] presented a plan to create MDS sampling plans. Wu et al. [11] proposed a novel lot sentencing method by variables inspection based on MDS. In addition, RGS plans based on the process capability are proposed by Sherman [12], Balamurali and Jun [8], Aslam et al. [7] and Aslam et al. [13]. Also, new methods of sampling plan for resubmitted lots are proposed by Govindaraju and Ganesalingam [14] and Aslam et al. [7].

In this paper, a variable MDS sampling plan is developed based on PCI using Bayesian approach. In this plan, it is supposed that the desired quality characteristic follows normal distribution function. In what follows, a double sampling model, sampling plan for resubmitted lots and RGS plan are developed based on Bayesian approach and then a comparison study is carried out between ASNs of introduced sampling plan. The main contributions of this research are as follows:

(i) Introducing a variable MDS sampling plan based on PCI and Bayesian approach.
(ii) Presenting different variable sampling plans (which includes SSP, DSP, RGS and Sampling plan for resubmitted lots) based on Bayesian approach.
(iii) Comparison of the developed variable sampling plans and determining the best plan.
(iv) Comparison of the MDS sampling plan based on two different approaches (which includes Bayesian approach and exact probability distribution) and selecting the suitable approach.

II. Process Capability Analysis

A. Estimation of PCI Based on the Multiple Samples

Suppose that the quality characteristic follows normal distribution with mean μ and variance σ^2. PCI is defined as:

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$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\}\frac{d-|\mu-M|}{3\sigma} \quad (1)$$

where $d=(USL-LSL)/2$ and $M=(USL+LSL)/2$ and LSL, USL are lower and upper specification limits. The average of the observations $(\overline{X})$ is used to estimate the mean value when the mean value of process is unknown.

Assume k subsamples have been gathered and the quality characteristic of the process is normally distributed, so that the sample size of i_{th} subsample is equal to n_i. We consider x_{ij} as the random observation from normal distribution with the mean μ and variance σ^2, $i=1,2,....,k$ and $j=1,2,....,n_i$. We consider that the process is under statistical control by $\overline{X}$ and S control charts. With this assumption, the statistics $\overline{X}_i$ and S_i^2 for each subsample will be defined as:

$$\hat{\mu}=\overline{\overline{x}}_i=\frac{1}{N}\sum_{j=1}^{n_j} n_i\overline{x}_i,$$
$$S_i^2=\frac{1}{n_i-1}\sum_{j=1}^{n_j}(x_{ij}-\overline{x}_i)^2, \quad (2)$$
$$and\ N=\sum_{i=1}^{m} n_i$$

where N is the total number of observations. Assuming $N_1=\sum_{i=1}^{k}(n_i-1)$, we use the average of samples mean as an unbiased estimator of μ and also accumulation of the samples variance as an unbiased estimator of σ^2, then the estimation of PCI based on the observations of multiple samples is as:

$$\hat{\mu}=\overline{\overline{x}}_i=\frac{1}{N}\sum_{j=1}^{n_j} n_j\overline{x}_j,$$
$$\hat{\sigma}^2=S_p^2=\frac{1}{N_1}\sum_{j=1}^{n}(n_i-1)S_i^2 \quad (3)$$

$$\hat{C}_{pk}^{*}=\min\left\{\frac{USL-\overline{\overline{x}}}{3s_p}, \frac{\overline{\overline{x}}-LSL}{3s_p}\right\}=\frac{d-|\overline{\overline{x}}-M|}{3s_p} \quad (4)$$

This estimator of PCI is mostly applied in practical and industrial applications.

B. Posterior Probability Distribution Based on the Multiple Samples

A Bayesian method proposed by Pearn and Lin [15] is applied to evaluate the probability distribution function of PCI, based on multiple samples. According to their Bayesian approach, the posterior probability distribution function of C_{pk} for specified constant values of w can be obtained as:

$$p=\Pr\{\text{the process is capable}\mid X\}=\Pr\{C_{pk}>w\mid X\} \quad (5)$$

Then, the probability of having a capable process will be as [16], [17]:

$$p=\Pr\{C_{pk}>w\mid X\}$$
$$=\int_0^{\infty}\frac{1}{\Gamma(\alpha)y^{\alpha+1}}\exp(-\frac{1}{y})\times\{\Phi[b_1(y)]+\Phi[b_2(y)]-1\}dy \quad (6)$$

where $\Phi(u)$ is the cumulative function of standard normal distribution, $\Phi(u)=\int_{-\infty}^{u}(2\pi)^{-1/2}\exp(-t^2/2)dt$ and the other parameters are defined as [11]:

$$\alpha=(N-1)/2, \qquad \delta=\frac{|\overline{\overline{x}}-M|}{s_p} \quad (7)$$

$$\gamma=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n_j}(x_{ij}-\overline{x}_i)^2/2}{\sum_{i=1}^{m}\sum_{j=1}^{n_j}(x_{ij}-\overline{\overline{x}})^2/2}=\frac{N_1 s_p^2}{N_1 s_p^2+\sum_{i=1}^{m} n_i(\overline{x}_i-\overline{\overline{x}})^2} \quad (8)$$

$$b_1(y)=3\sqrt{N}\left(\hat{C}_{pk}^{*}\times\sqrt{\frac{2\gamma}{N_1 y}}-w\right) \quad (9)$$

$$b_2(y)=3\sqrt{N}\left(\left(\hat{C}_{pk}^{*}+\frac{2}{3}\delta\right)\times\sqrt{\frac{2\gamma}{N_1 y}}-w\right) \quad (10)$$

III. Proposed MDS Sampling Plan

MDS sampling plan is a type of conditioned sampling methods which consider sampling results of past or future lots. For application of variable MDS sampling plan, the following assumptions should be valid.

(i) Lots are submitted from a process which has a constant proportion of non-conforming items.
(ii) The quality characteristic of interest follows a normal distribution.
(iii) There is no reason to believe that a particular lot is poorer than the preceding lots.
(iv) Optimal parameters of the proposed sampling plan are defined as follows:

n: Sample size, m: Number of preceding lots, k_1: The upper threshold of PCI for accepting the lot, k_2: The lower

threshold of PCI for rejecting the lot.

In order to display the proposed sampling plan for practical use, we explain the procedure of proposed MDS sampling plan using a real example. In various industries, the process capability of materials, products and systems needs to be compatible with the engineering tolerances. Here we consider a company with machine tools as a practical case. The mentioned company needs to keep machine tools within the desired tolerances. The engineers of the company consider a dimensional tolerance for different portions of the tool and also calculate the PCI for them. Finally, their decision about received lot based on MDS procedure and PCI is defined as follows:

Step 1. take a random sample of size n and compute PCI, $\hat{C}_{pk}$.

Step 2. if $\hat{C}_{pk} \geq k_1$, then accept the lot and if $\hat{C}_{pk} \leq k_2$, then reject it. If $k_2 \leq \hat{C}_{pk} \leq k_1$, then if the value of PCI in each of the samples taken from the m preceding lots is more than k_1 then accept the lot otherwise reject the lot.

It is noted that if $k_1 = k_2$, then proposed sampling plan converts to the single sampling plan.

The OC function of variable MDS sampling plan is obtained as [18]:

$$P_a(C_{pk}) = \Pr\{\hat{C}_{pk} \geq k_1 | X\} + \Pr\{k_2 \leq \hat{C}_{pk} \leq k_1 | X\}.[\Pr\{\hat{C}_{pk} \geq k_1 | X\}]^m \quad (11)$$

where $\Pr\{\hat{C}_{pk} \geq k_1 | X\}$ is the probability of accepting the lot based on a single sample. Also, the term $\Pr\{k_2 \leq \hat{C}_{pk} \leq k_1 | X\}.[\Pr\{\hat{C}_{pk} \geq k_1 | X\}]^m$ is the probability of accepting the lot based on the quality level of m preceding lots.

An optimization model is formulated to minimize ASN considering the constraints of first and second type error. Also the optimal value of m can be obtained by sensitivity analysis. The presented model is as:

$$\begin{aligned} &Minimize\ n \\ &subject\ to: \\ &C = C_{AQL} \Rightarrow P_A(C_{AQL}) \geq 1-\alpha \\ &and \\ &C = C_{LTPD} \Rightarrow P_A(C_{LTPD}) \leq \beta \end{aligned} \quad (12)$$

where C_{AQL} is the minimum acceptable value of PCI for the producer and C_{LQL} is the maximum unacceptable value of PCI for the consumer. The parameters α and β are the probabilities of first and second type error, respectively.

IV. Designing a Double-Sampling Plan

A double-sampling plan based on process capability is designed with the following parameters:

n_1 : Sample size of the first sample, n_2 : Sample size of the second sample, k_1 : The lower threshold of PCI for rejecting the lot based on the first sample, k_2 : The upper threshold of PCI for accepting the lot based on the first sample, k_3 : The upper threshold of PCI for accepting the lot based on the second sample

The decision-making method is as follows:

Step 1. Select n_1 observation in the first sample from the lot and compute $\hat{C}_{pk}$.

If $\hat{C}_{pk} \geq k_2$ then accept the lot and reject the lot if $\hat{C}_{pk} \leq k_1$ where $k_2 > k_1$ .if $k_1 < \hat{C}_{pk} < k_2$, then obtain a second sample with n_2 observations.

Step 2. Compute $\hat{C}_{pk}$ in the second sample. Accept the lot if $\hat{C}_{pk} \geq k_3$, otherwise reject the lot.

In double sampling plans, the general formula for the ASN, can be obtained as:

$$\textbf{min}\ ASN = n_1 + n_2 \left[\textbf{Pr}\ (C_{pk} > k_1) - \textbf{Pr}\ (C_{pk} > k_2)\right] \quad (13)$$

The constraint of producer risk is as:

$$C = C_{AQL} \Rightarrow Pr\ (\hat{C}_{pk} \geq k_1 | n = n_1) + Pr\ (\hat{C}_{pk} \geq k_3 | n = n_2) . Pr(k_2 \leq \hat{C}_{pk} \leq k_1 | n = n_1) \geq 1-\alpha \quad (14)$$

The constraint of consumer risk is as follows:

$$C = C_{LQL} \Rightarrow Pr\ (\hat{C}_{pk} \geq k_1 | n = n_1) + Pr\ (\hat{C}_{pk} \geq k_3 | n = n_2) . Pr(k_2 \leq \hat{C}_{pk} \leq k_1 | n = n_1) \leq \beta \quad (15)$$

Therefore, by solving the optimization model for specific values of C_{AQL} ,C_{LQL} and the different values of α and β , the decision parameters of double sampling plan can be obtained.

V. Designing a Variable RGS Model

The parameters of variable RGS plan are as follows:

n : Sample size, k_1 : The lower threshold of PCI for

rejecting the lot based on the single sample, k_2: The upper threshold of PCI for accepting the lot based on the first sample And its procedure is summarized as follows:

Step 1. Collect a sample with n observation from lot.

Step 2. If $\hat{C}_{pk} \geq k_2$ then accept the lot and if $\hat{C}_{pk} \leq k_1$ then reject it. If $k_1 < \hat{C}_{pk} < k_2$, then repeat step 1 and step 2.

The OC function of the variable RGS plan is as:

$$P_a(\hat{C}_{pk}) = \frac{P_a}{P_r + P_a} = \frac{\Pr(\hat{C}_{pk} \geq k_2)}{1 - \Pr(\hat{C}_{pk} \geq k_1) + \Pr(\hat{C}_{pk} \geq k_2)} \quad (16)$$

The objective function of the optimization model is the ASN and its constraints are producer risk, α and consumer risk, β that can be obtained as [18]:

$$Minimize\, T = \frac{n}{\Pr(\hat{C}_{pk} \leq k_1) + \Pr(\hat{C}_{pk} \geq k_2)}$$

$$subject\ to: \quad (17)$$

$$C = C_{AQL} \Rightarrow \frac{\Pr(\hat{C}_{pk} \geq k_2)}{1 - \Pr(\hat{C}_{pk} \geq k_1) + \Pr(\hat{C}_{pk} \geq k_2)} \geq 1 - \alpha$$

$$and$$

$$C = C_{LQL} \Rightarrow \frac{\Pr(\hat{C}_{pk} \geq k_2)}{1 - \Pr(\hat{C}_{pk} \geq k_1) + \Pr(\hat{C}_{pk} \geq k_2)} \leq \beta$$

VI. Designing a Variable Sampling Plan for Resubmitted Lots

The variable sampling plan for resubmitted lots is one of the most effective sampling plans. The parameters of this sampling plan are as: m: Number of resubmissions, n: Sample size, k_a: The lower threshold of PCI for accepting the lot based on the single sample. And its procedure is summarized as follows:

Step 1. Take a sample with n observation and compute PCI. If $\hat{C}_{pk} \geq k_a$ then accept the lot otherwise go to step 2.

Step 2. Repeat step 1 for *m* times. If the lot was not accepted after *m* repetitions of step 1 then reject the lot.

The OC function of the variables sampling plan for resubmitted lot will be as [14]:

$$P_A(\hat{C}_{pk}) = 1 - (1 - P_a)^m = 1 - \left(1 - P\left(\hat{C}_{pk} \geq k_a\right)\right)^m \quad (18)$$

The ASN of developed sampling plan is as [14]:

$$ASN = \frac{n(1 - (1 - P_a)^m)}{P_a} \quad (19)$$

Therefore, the optimization model is as:

$$Minimize\ ASN = \frac{n(1 - (1 - P_a)^m)}{P_a}$$

$$subject\ to:$$

$$C = C_{AQL} \Rightarrow P_A(C_{AQL}) \geq 1 - \alpha \quad (20)$$

$$and$$

$$C = C_{LQL} \Rightarrow P_A(C_{LQL}) \leq \beta$$

In order to obtain the plan parameters, we present methodology of how the mentioned plan parameters can be obtained. Then with considering the decision variables and assumption of plan, and with using a grid search, we can obtain the minimum ASN plans searching in the multi-dimensional grid formed setting m = 1 and 2, n = 2(1)100, k_1 = 1.4(0.001)2.4, k_2 = 0.7(0.001)1.3.

VII. Analyses and Discussions

A. Simulation Studies

The solutions of optimization model of the MDS plan for given values of producer risk $\alpha = 0.05$ and consumer risk $\beta = 0.10$ and fixed values of γ, δ and for the different values of C_{AQL}, C_{LQL} are determined. The results are denoted in Tables I and II. In Tables I and II, the value of the optimal parameters of the proposed MDS plan, including n, k_1, k_2 for different values of C_{AQL}, C_{LQL}, γ, δ are denoted. For example, in Table I, if $C_{AQL} = 2.0, C_{LQL} = 1.3$ and $\delta = 0.5, \gamma = 0.8, m = 1$, then the values of $n = 13$, $k_1 = 1.857, k_2 = 1.214$ are obtained as the optimal solution and the procedure of proposed MDS plan will be as follows:

Step 1. take a random sample of size $n = 13$ and compute $\hat{C}_{pk}$.

Step 2. if $\hat{C}_{pk} \geq 1.857$, then accept the lot and if $\hat{C}_{pk} \leq 1.214$, then reject it. If $1.214 \leq \hat{C}_{pk} \leq 1.857$, then if the value of PCI of $m = 1$ preceding lot is more than $k_1 (\hat{C}_{pk} \geq 1.857)$ then accept the lot, otherwise reject the lot. Also, it is observed that the case $m = 2$ has less sample size in most of the simulated cases, thus it is preferred to apply the case $m = 2$.

TABLE I
OPTIMAL VALUES OF PARAMETERS FOR DIFFERENT VALUES OF C_{AQL}, C_{LQL} AND $m=1$

$\delta=0.5,\gamma=0.8,m=1$					$\delta=0.1,\gamma=0.9,m=1$				
C_{AQL}	C_{LQL}	n	k_1	k_2	C_{AQL}	C_{LQL}	n	k_1	k_2
1.5	0.7	3	1.423	1.019	1.5	0.7	4	1.964	1.017
1.5	0.9	4	1.757	0.867	1.5	0.9	6	2.135	0.785
1.5	1.1	11	1.862	0.752	1.5	1.1	13	2.248	0.864
1.5	1.3	14	1.919	0.874	1.5	1.3	17	2.267	0.895
1.6	0.7	3	1.457	1.054	1.6	0.7	3	2.085	1.014
1.6	1	4	1.845	1.055	1.6	1	6	2.267	0.937
1.6	1.2	12	1.935	0.764	1.6	1.2	15	2.246	0.758
1.6	1.3	13	1.964	1.234	1.6	1.3	17	2.189	0.834
1.8	0.9	3	1.718	1.217	1.8	0.9	4	2.295	1.181
1.8	1.2	6	1.974	1.266	1.8	1.2	15	2.235	0.913
1.8	1.4	15	1.956	1.268	1.8	1.4	18	2.115	0.857
1.8	1.5	17	1.935	1.148	1.8	1.5	20	2.278	0.764
2.0	1.1	5	1.864	0.979	2.0	1.1	7	2.264	1.034
2.0	1.3	13	1.857	1.214	2.0	1.3	17	2.158	0.928
2.0	1.5	17	1.925	1.195	2.0	1.5	20	2.274	0.775
2.0	1.7	21	1.934	0.737	2.0	1.7	24	2.249	0.895
2.2	1	3	1.994	1.264	2.2	1	6	2.234	0.976
2.2	1.3	5	2.256	1.085	2.2	1.3	17	2.229	0.734
2.2	1.5	13	2.146	0.847	2.2	1.5	20	2.117	0.846
2.2	1.6	14	2.219	0.734	2.2	1.6	22	2.276	0.779

TABLE II
OPTIMAL VALUES OF PARAMETERS FOR DIFFERENT VALUES OF C_{AQL}, C_{LQL} AND $m=2$

$\delta=0.5;\gamma=0.8;m=2$					$\delta=1.0;\gamma=0.9;m=2$				
C_{AQL}	C_{LQL}	n	k_1	k_2	C_{AQL}	C_{LQL}	n	k_1	k_2
1.5	0.7	3	1.462	0.717	1.5	0.7	4	1.828	0.734
1.5	0.9	4	1.682	0.736	1.5	0.9	7	1.976	0.891
1.5	1.1	8	1.673	0.954	1.5	1.1	15	2.167	0.849
1.5	1.3	15	1.838	0.746	1.5	1.3	16	2.228	0.764
1.6	0.7	3	1.429	0.792	1.6	0.7	3	1.934	0.737
1.6	1	4	1.713	0.835	1.6	1	7	2.019	1.146
1.6	1.2	12	1.846	0.788	1.6	1.2	15	2.274	0.846
1.6	1.3	13	1.957	0.846	1.6	1.3	16	2.268	0.771
1.8	0.9	3	1.744	0.719	1.8	0.9	4	2.191	0.795
1.8	1.2	5	1.867	0.734	1.8	1.2	9	2.146	1.146
1.8	1.4	15	1.939	0.867	1.8	1.4	18	2.286	0.872
1.8	1.5	17	2.158	0.838	1.8	1.5	20	2.274	0.728
2.0	1.1	4	1.816	0.761	2.0	1.1	7	2.149	1.016
2.0	1.3	7	1.928	0.857	2.0	1.3	9	2.276	0.846
2.0	1.5	17	1.864	0.816	2.0	1.5	20	2.285	0.737
2.0	1.7	21	1.937	0.843	2.0	1.7	23	2.273	0.995
2.2	1	3	1.848	0.855	2.2	1	7	2.134	1.046
2.2	1.3	4	2.228	0.749	2.2	1.3	9	2.112	0.785
2.2	1.5	6	2.267	0.897	2.2	1.5	20	2.249	0.755
2.2	1.6	8	2.231	1.264	2.2	1.6	21	2.276	0.867

B. Comparisons of Plans

In this section, we compare the average sample number of different acceptance sampling plans. The results are denoted in Table III. For example, if $C_{AQL}=1.8$, $C_{LQL}=1.4$ and $\delta=0.5,\gamma=0.8$, then ASN of the single and double and the MDS sampling plan ($m=2$) are respectively, 23, 20.86 and 15. Thus, according to the results in Table III, it is observed that the ASN of the variable MDS plan is less than the ASN of other sampling plans. The variables RGS plan performs better than the sampling plan for the resubmitted lots and the double sampling plan. The variables single sampling plan has the worst performance in comparison with the other methods. Since the MDS sampling plan needs the historical data of m preceding lots, it may be inapplicable in some cases. Other

sampling methods do not need such data, and thus, the RGS sampling plan may be more appropriate in some practical problems.

TABLE III
RESULTS OF COMPARISON STUDY

$\delta = 0.5; \gamma = 0.8$						
				ASN		
C_{AQL}	C_{LQL}	Variable single sampling plan	Variable double sampling plan	Variable (RGS) plan	Variable MDS plan ($m = 2$)	sampling plan for resubmitted lots
1.6	0.7	31	23	7.40	3	11.74
1.6	1.3	32	27.27	14.62	13	19.86
1.8	1.2	27	29.57	14.44	5	16.22
1.8	1.4	23	20.86	16.15	15	19.74
2	1.1	34	15.60	6.33	4	12.22
2	1.5	47	28.32	22.24	17	23.70

TABLE IV
OPTIMAL VALUES OF PARAMETERS FOR DIFFERENT VALUES OF m

$\delta = 0.5, \gamma = 0.8, , \alpha = 0.05, \beta = 0.10$					
C_{AQL}	C_{LQL}	$ASN\ (m=1)$	$ASN\ (m=2)$	$ASN\ (m=3)$	$ASN\ (m=4)$
1.6	0.7	3	3	15	26
1.6	1.3	13	13	16	19
1.8	1.2	6	5	24	27
1.8	1.4	15	15	31	34
2	1.1	5	4	15	32
2	1.5	17	17	28	37

C. Sensitivity Analysis Based on the Parameter m

In order to investigate the effect of the parameter *m* on the proposed sampling plan, a sensitivity analysis is carried out. The results are denoted in Table IV. It is observed that the results of the MDS plan in the case of $m = 2$ performs better than the cases $m = 1$, m=3 , m=4 . For example, where $C_{AQL} = 1.8, C_{LQL} = 1.2$, the ASN of the MDS sampling plan in the case of $m = 2$ is equal to 5 and the ASN for the cases of $m = 1$, $m = 3$ and $m = 4$, is equal to 6, 24 and 27, respectively.

VIII. DESIGNING AN MDS SAMPLING PLAN BASED ON EXACT PROBABILITY DISTRIBUTION

The exact probability distribution of PCI can be determined using statistical techniques. Then we can design the acceptance sampling plan based on exact probability distribution of PCI. With considering the natural estimator of C_{pk} that is $\hat{C}_{pk}$, for a normally distributed process, a simple and exact form of the cumulative distribution function of the estimated parameter, $\hat{C}_{pk}$, can be determined. The cumulative distribution function is as follows (Pearn and Lin [15]).

$$F_{\hat{C}_{pk}}(Y) = 1 - \int_0^{b\sqrt{n}} G\left(\frac{(n-1)(b\sqrt{n}-t)^2}{9.n.y^2}\right) [\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\, dt \quad (21)$$

where $b = \frac{d}{\sigma}$ and b values can be rewritten as: $b = 3C_{pk} + |\xi|$ and $\xi = \frac{\mu - M}{\sigma}$. Also $G(.)$ is the CDF of the chi-square distribution, χ^2_{n-1} with n-1 degrees of freedom and $\phi(.)$ is the PDF of the standard normal distribution $N(0,1)$.

With regards to the OC function of the developed variable MDS sampling plan and exact probability distribution, the required sample size can be obtained using the optimization model (12).

IX. COMPARISON OF THE MDS SAMPLING PLAN BASED ON DIFFERENT APPROACHES

Now, we analyze the ASN of MDS sampling plan based on the Bayesian approach and exact probability distribution (exact approach). With considering the specified values of $\delta = 0.5, \gamma = 0.8$ and $\xi = 1$ for Bayesian approach and exact probability distribution function (PDF), it is observed that the ASN of the variable MDS plan under Bayesian PDF approach is less than the ASN of MDS sampling plan under the exact PDF approach. The results are denoted in Table V.

TABLE V
RESULTS OF A COMPARISON STUDY

		MDS sampling plan (Bayesian approach)	MDS sampling plan (exact approach)
C_{AQL}	C_{LQL}	$\delta = 0.5, \gamma = 0.8, m = 2$	$\xi = 1, m = 2$
1.6	0.7	3	7
1.6	1.3	13	16
1.8	1.2	5	11
1.8	1.4	15	17
2	1.1	4	10
2	1.5	17	21

When some historical data of the process is available, applying the Bayesian inference approach is preferred; however, when we do not have access to such data we must apply exact probability distribution.

X. Conclusion

In this paper, a variable MDS sampling plan based on PCI has been developed where the required probabilities are obtained by the Bayesian approach and exact probability distribution. The optimal parameters of the developed sampling plan are obtained based on the constraints of consumer risk and producer risk. Also, the procedure of MDS sampling plan has been explained with a real example. Then two comparison studies are performed; first, we compare the ASN of the developed sampling plan with other classical sampling methods. Second, we compared the ASN of MDS plan based on the Bayesian approach and exact probability distribution. It was concluded that the developed MDS plan is more economical than other sampling plans. Also, the Bayesian MDS plan performs better than MDS plan under exact probability distribution.

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