Selection of Material for Gear Used in Fuel Pump Using Graph Theory and Matrix Approach

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Abstract—Material selection is one of the key issues for the production of reliable and quality products in industries. A number of materials are available for a single product due to which material selection become a difficult task. The aim of this paper is to select appropriate material for gear used in fuel pump by using Graph Theory and Matrix Approach (GTMA). GTMA is a logical and systematic approach that can be used to model and analyze various engineering systems. In present work, four alternative material and their seven attributes are used to identify the best material for given product.

Keywords—Material, GTMA, MADM, digraph, decision making.

I. INTRODUCTION

MATERIAL selection is one of the key issues for the production of good products in industries. Materials and design are two very important subjects in their own right and their integration is critical to the development of successful new engineering products [1]. Numbers of material are available for a single product each having its own advantage and disadvantage. Availability of large number of materials makes the selection process a difficult task. During the selection of material, a number of attributes have to be taken into consideration. An attribute is factor which affects the material selection for a given problem. These attributes are: mechanical properties, electrical properties, magnetic properties, physical properties, chemical properties, manufacturing properties material cost, product shape, material impact on environment, performance characteristics, etc.

The selection of good and reliable material for any problem from given alternative materials and their attributes is a multiple attribute decision-making (MADM) problem.

During the past few decades various methods have been proposed to rank alternatives. Rao [2] have worked on selection of material for cryogenic storage tank for transportation of liquid nitrogen from the alternative material. He applied various MADM approaches such as SAW, WPM, graph Theory etc. Liao [3] presented a fuzzy multi criteria decision-making method for material selection. Giachetti [4] described a prototype material and manufacturing process selection system that integrates a formal multiple attribute decision model with a relational database. Rayg [5] used various MADM methods for selection of tool insert for finishing operation. Framework to represent and deal with the relationships between design variables of both materials parameters and system-level parameters was proposed by Raj [6]. Patel [7] used decision-making method to select robot in different industrial environments considering both qualitative and quantitative attributes. Nikam [8] used SAW and WPM for selection of cutting tool insert in turning operation.

GTMA is a logical and systematic approach that can be used to model and analyze various engineering systems. GTMA method has been employed for material selection [9] and selection of industrial robots [10]. Graph Theory and Matrix Approach (GTMA) helps in identifying different attributes of material and offer a better visual appraisal of the attributes and their interrelations [11]. GTMA has been employed in different fields by different authors:

- Evaluation of cutting fluids [13]
- Flexible manufacturing systems [14]
- Performance evaluation of technical institutions [15]
- Machinability evaluation of work materials [16]
- For contractor ranking [17]
- Quantification of risk mitigation environment of supply chains [18]
- Selection of locations of collection centers [19]
- Modeling and analysis of FMS performance variables [20]
- Disposition decisions in reverse logistics [21]
- Assignment of weightage to machining characteristics to improve overall performance [22]
- Selection of a power plant [23]
- Failure mode and effect analysis [24]
- Selection of cutting parameters in side milling operation [25]
- Barriers to green supply chain management in Indian mining industries [27]
- Disposition decisions in reverse logistics [28]
- Manufacturing system's effectiveness measurement [29]
- To optimize single-product flow-line configurations of RMS [30]
- Analyzing and modeling the performance index of ultrasonic vibration assisted EDM [31]
- Structural modeling and analysis of water resources development and management system [32]
- Evaluation of gas turbine power plant efficiency [33]

Influence of materials (characterized by a bending fatigue strength) of the safety factor for the bending transmission in

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gear was shown by [26].

II. METHODOLOGY

GTMA is a logical and systematic approach. Digraph had proved very useful in modeling and analyzing the various types of problems. If these digraph become complex then these digraph are analyzed with the help of Matrix method using a computer program. GTMA helps in identifying attributes, and offer a better visual appraisal of the attributes and their interrelations (Rao, 2007).

A. Material Selection Attributes Digraph

The material selection attributes and their interrelationships are graphically represented by a material selection attributes digraph. Fig. 1 illustrates the material selection attributes digraph for the material selection problem. The material selection attributes are surface hardness(SH), Core hardness (CH), oxidation(O), magnetic flux(MF), case depth at pitch circle diameter(PCD) ,case depth at root(R), Millipore test value(M). The digraph consists of seven nodes representing the seven attributes SH, CH, O, MF, PCD, R and M respectively. Directed edges are drawn from one node to another and represent the inter-relationship among the attributes
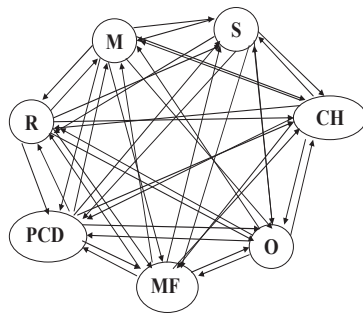

Fig. 1 Material Selection Attribute Digraph

The material selection attributes digraph enables a quick visual appraisal of the material selection attributes and their inter-relationships

B. Application of GTMA

Current problem is related with selection of a suitable material for a gear used in fuel pump. The material selection problem considers four alternative materials and seven attributes, and the data are given in Table I.

TABLE I
OBJECTIVE DATA OF ATTRIBUTES

Mat.	SH	CH	O	MF	PCD	R	M
1	61.5	32	.020	2	.83	.70	4.3
2	62	31.3	.0167	2	.62	.52	3.8
3	61	42	.0155	1.5	.65	.50	3.9
4	60	37	.0145	1	.75	.63	4.1

Mat. = Material. SH= Surface Hardness (HRC), CH= Core Hardness (HRC), O= Oxidation (mm), MF= Magnetic flux (Gauss), PCD= case depth at pitch circle diameter (mm), R=case depth at root (mm), M= Millipore test value (mg). Material 1= 16MnCr5, Material 2= SAE-8620, Material 3= 19MnCr5, Material 4= SCM-420HV.

C. Various Steps Used in Methodology

Step 1: In the current work, the attributes considered for material selection are surface hardness(SH), Core hardness(CH), oxidation(O), magnetic flux(MF), case depth at pitch circle diameter(PCD) ,case depth at root(R), Millipore test value(M). Objective data of attributes is given in Table I and normalized values of these attributes are given in Table II:

TABLE II
NORMALIZED VALUES

Mat.	SH	CH	O	MF	PCD	R	M
1	.992	.762	1	1	1	1	1
2	1	.745	.835	1	.747	.743	.884
3	.984	1	.775	.75	.783	.714	.907
4	.968	.881	.725	.5	.903	.9	.953

D. Permanent Function

A permanent function is a complete representation of the attributes of a problem and contains all possible information of the attributes and their interrelations. The permanent is similar to the determinant of a Matrix except in permanent all the terms are positive, no negative sign appears in the permanent function of a Matrix and hence no information will be lost. The permanent function for 'N' attributes Matrix when expanded, will have (N!) terms which may be arranged in (N+1) groups. The permanent function of Matrix 'R' can be calculated using (1):

$$
\text{per}(R) = \prod_{i=1}^{N} G_i
$$
$$
+ \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \ldots \sum_{N=t+1}^{N} (g_{ij}g_{ji}) G_k G_l G_m G_n G_o \ldots G_t G_N +
$$
$$
\sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} \ldots \sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{ki} + g_{ik}g_{kj}g_{ji})
$$
$$
G_k G_l G_m G_n G_o \ldots G_t G_N + \left[\sum_{i=1}^{N-3} \sum_{j=i+1}^{N} \sum_{k=i+1}^{N-1} \sum_{l=i+2}^{N} \sum_{N=t+1}^{N} \right.
$$
$$
(g_{ij}g_{ji})(g_{kl}g_{lk}) G_m G_n G_o \ldots G_t G_N +
$$
$$
\sum_{i=1}^{N-3} \sum_{j=i+1}^{N-1} \sum_{k=i+1}^{N} \sum_{l=j+1}^{N} \ldots \sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{kl}g_{li}
$$
$$
\left. + g_{il}g_{lk}g_{kj}g_{ji}) G_m G_n G_o \ldots G_t G_N \right]
$$
$$
+ \left[\sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} \sum_{l=1}^{N-1} \sum_{m=l+1}^{N} \ldots \ldots \sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{ki} \right.
$$
$$
+ g_{ik}g_{kj}g_{ji})(g_{lm}g_{ml}) G_n G_o \ldots G_t G_N +
$$
$$
\sum_{i=1}^{N-4} \sum_{j=i+1}^{N-1} \sum_{k=i+1}^{N} \sum_{l=i+1}^{N} \sum_{m=j+1}^{N} \ldots \ldots \sum_{m=t+1}^{N} (g_{ij}g_{jk}g_{kl}g_{lm}g_{mi}
$$
$$
\left. + g_{im}g_{ml}g_{lk}g_{kj}g_{ji}) G_n G_o \ldots G_t G_N \right]
$$
$$
+ \left[\sum_{i=1}^{N-3} \sum_{j=i+1}^{N-1} \sum_{k=i+1}^{N} \sum_{l=j+1}^{N} \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} \ldots \ldots \sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{kl}g_{li} \right.
$$
$$
\left. + g_{il}g_{lk}g_{kj}g_{ji})(g_{mn}g_{nm}) G_o \ldots G_t G_N \right.
$$
$$
+ \sum_{i=1}^{N-5} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^{N} \sum_{l=1}^{N-2} \sum_{m=l+1}^{N-1} \sum_{n=m+1}^{N} \ldots \ldots \sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{ki}
$$

$$+g_{ik}g_{kj}g_{ji})(g_{lm}g_{mn}g_{nl} + g_{ln}g_{nm}g_{ml})G_o \ ..... G_t G_N$$

$$+ \sum_{i=1}^{N-5} \sum_{j=i+1}^{N} \sum_{k=i+1}^{N-3} \sum_{l=i+2}^{N} \sum_{m=k+1}^{N-1} \sum_{n=k+2}^{N} ......$$

$$.. \sum_{N=t+1}^{N} (g_{ij}g_{ji})(g_{kl}g_{lk})(g_{mn}g_{nm})G_o \ ..... G_t G_N$$

$$+ \sum_{i=1}^{N-5} \sum_{j=i+1}^{N-1} \sum_{k=i+1}^{N} \sum_{l=i+1}^{N} \sum_{m=i+1}^{N} \sum_{n=j+1}^{N} ...$$

$$\sum_{N=t+1}^{N} (g_{ij}g_{jk}g_{kl}g_{lm}g_{mn}g_{ni}$$

$$+ g_{in}g_{nm}g_{ml}g_{lk}g_{kj}g_{ji})G_o \ ..... G_t G_N \qquad (1)$$

Relative weight of attributes is assigned using 5- point scale suggested by Chen and Hwang [12]. The relative weight of attributes is given in Table III.

TABLE III
RELATIVE WEIGHT MATRIX

	SH	CH	O	MF	PCD	R	M
SH	-	0.505	0.895	0.895	0.695	0.695	0.895
CH	0.495	-	0.895	0.895	0.695	0.695	0.895
O	0.105	0.105	-	0.505	0.295	0.295	0.695
MF	0.105	0.105	0.495	-	0.295	0.295	0.695
PCD	0.305	0.305	0.705	0.705	-	0.505	0.695
R	0.305	0.305	0.705	0.705	0.495	-	0.695
M	0.105	0.105	0.305	0.305	0.305	0.305	-

Step 2: Material selection attribute digraph shows the relative importance of each attribute upon each other. Relative weight Matrix of this digraph is shown in (1). Relative weight Matrix for material 1 is shown in Table IV:

TABLE IV
RELATIVE WEIGHT MATRIX FOR MATERIAL 1

	SH	CH	O	MFT	PCD	R	M
SH	0.992	0.505	0.895	0.895	0.695	0.695	0.895
CH	0.495	0.762	0.895	0.895	0.695	0.695	0.895
O	0.105	0.105	1	0.505	0.295	0.295	0.695
MFT	0.105	0.105	0.495	1	0.295	0.295	0.695
PCD	0.305	0.305	0.705	0.705	1	0.505	0.695
R	0.305	0.305	0.705	0.705	0.495	1	0.695
M	0.105	0.105	0.305	0.305	0.305	0.305	1

Relative weight Matrix for material 2 is shown in Table V:

TABLE V
RELATIVE WEIGHT MATRIX FOR MATERIAL 2

	SH	CH	O	MF	PCD	R	M
SH	1	0.505	0.895	0.895	0.695	0.695	0.895
CH	0.495	0.745	0.895	0.895	0.695	0.695	0.895
O	0.105	0.105	0.835	0.505	0.295	0.295	0.695
MFT	0.105	0.105	0.495	1	0.295	0.295	0.695
PCD	0.305	0.305	0.705	0.705	0.747	0.505	0.695
ROOT	0.305	0.305	0.705	0.705	0.495	0.743	0.695
M	0.105	0.105	0.305	0.305	0.305	0.305	0.884

Relative weight Matrix for material 3 is shown in Table VI:

TABLE VI
RELATIVE WEIGHT MATRIX FOR MATERIAL 3

	SH	CH	O	MF	PCD	R	M
SH	0.984	0.505	0.895	0.895	0.695	0.695	0.895
CH	0.495	1	0.895	0.895	0.695	0.695	0.895
O	0.105	0.105	0.775	0.505	0.295	0.295	0.695
MF	0.105	0.105	0.495	0.75	0.295	0.295	0.695
PCD	0.305	0.305	0.705	0.705	0.783	0.505	0.695
R	0.305	0.305	0.705	0.705	0.495	0.714	0.695
M	0.105	0.105	0.305	0.305	0.305	0.305	0.907

Relative weight Matrix for material 4 is shown in Table VII:

TABLE VII
RELATIVE WEIGHT MATRIX FOR MATERIAL 4

	SH	CH	O	MF	PCD	R	M
SH	0.968	0.505	0.895	0.895	0.695	0.695	0.895
CH	0.495	0.881	0.895	0.895	0.695	0.695	0.895
O	0.105	0.105	0.725	0.505	0.295	0.295	0.695
MF	0.105	0.105	0.495	0.5	0.295	0.295	0.695
PCD	0.305	0.305	0.705	0.705	0.903	0.505	0.695
R	0.305	0.305	0.705	0.705	0.495	0.9	0.695
M	0.105	0.105	0.305	0.305	0.305	0.305	0.953

The permanent function value of Matrix is calculated with the help of (1). The material selection index value of different material is given below:

Material 1(16MNCr5):   44.1314
Material 2(SAE-8620):   34.3993
Material 3(19MnCr5):   34.1925
Material 4(SCM-420HV):   32.2788

III. CONCLUSION

GTMA suggests the material designated as 1, i.e. 16MnCr5, as the right choice for the given problem of selection of a suitable material for a gear used in fuel pump.

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