Periodic Topology and Size Optimization Design of Tower Crane Boom

Wu Qinglong, Zhou Qicai, Xiong Xiaolei, Zhang Richeng

Abstract—In order to achieve the layout and size optimization of the web members of tower crane boom, a truss topology and cross section size optimization method based on continuum is proposed considering three typical working conditions. Firstly, the optimization model is established by replacing web members with web plates. And the web plates are divided into several sub-domains so that periodic soft kill option (SKO) method can be carried out for topology optimization of the slender boom. After getting the optimized topology of web plates, the optimized layout of web members is formed through extracting the principal stress distribution. Finally, using the web member radius as design variable, the boom compliance as objective and the material volume of the boom as constraint, the cross section size optimization mathematical model is established. The size optimization criterion is deduced from the mathematical model by Lagrange multiplier method and Kuhn-Tucker condition. By comparing the original boom with the optimal boom, it is identified that this optimization method can effectively lighten the boom and improve its performance.

Keywords—Tower crane boom, topology optimization, size optimization, periodic, soft kill option, optimization criterion.

I. INTRODUCTION

THE tower crane is an important equipment in the construction industry and the boom is its key component. Optimization of the boom can reduce the weight, material usage and even energy consumption during operation. Yang et al. proposed the two-level optimization strategy of the tower crane's lifting point and the member size, which chose the stress level of the member and the weight reduction of the boom as the purpose to establish the optimization mathematical model, then the optimization program was compiled. The optimized boom's weight was reduced and the stress distribution was more uniform [1]. Taking the bar section size as the design variable, the stress and the strain of the boom under extreme working condition as the constraints, Jia and Wan used fuzzy algorithm and genetic algorithm to optimize QTZ5010 tower crane boom for the purpose of gaining the minimum weight [2]. Zhang and Xu used the MATLAB toolbox and the genetic algorithm to optimize the cross sectional dimension and bar size of the truss-like boom of the deck crane [3]. Jin took the cross sectional sizes of the chord members and web members of the tower crane boom as the design variable, the weight of the boom as the objective function to do the optimized design under the constraints of strength and stiffness. The optimization was performed in the optimization module of the ANSYS software [4]. Aelmić et al. had optimized the cross section of the triangular tower crane boom and deduced the practical optimization formula [5]. Mijailović and Kastratović compared the triangular, trapezoidal, and rectangular cross sections of the tower crane boom. With the minimum weight as the target, the critical stress as the constraint, they optimized the cross section dimensions of the boom based on the Lagrange multiplier method. And they also gave the suggestions about the selection of boom cross section under different applications [6].

Different from the current research, a new method for continuum topology optimization combined with discrete truss optimization is proposed. Continuum topology optimization gets the optimized material distribution of the tower crane boom. Then, this optimized topology is transformed into discrete truss-like boom according to its principal stress path. Finally, the optimization criteria method is applied to get the optimized cross section size of the crane boom. In this way, a tower crane boom with optimized web member layout and section size is achieved.

II. PARAMETERS OF THE ORIGINAL TOWER CRANE BOOM

A. Basic Shape and Size Parameters

An existing QTZ63 tower crane is used as the optimization design case. The shape and outline size parameters of this QTZ63 tower crane boom are shown in Figs. 1 and 2. The cross section shape and size of the boom chord members and web members are shown in Table I.

TABLE I
CROSS SECTION SHAPE AND SIZE OF THE BOOM MEMBERS

Member type	Cross section	Size	Material
upper chord	hollow pipe	$\Phi108\times8$	20# steel
lower chord	12# channel steel		Q235C
side diagonal web	hollow pipe	$\Phi42\times4$	20# steel
side web	hollow pipe	$\Phi42\times4$	20# steel
underside web	hollow pipe	$\Phi38\times4$	20# steel
underside diagonal web	hollow pipe	$\Phi42\times4$	20# steel

B. Optimization Load Cases

The lifting performance curve of the chosen QTZ63 tower crane is shown in Fig. 3. Three different rated working conditions are chosen as the optimization load cases: Load case 1: maximum working range case with 1.3t hoisting load and 50 m working range); Load case 2: 3.4t hoisting load and 24-m working range; Load case 3: maximum hoisting load case with 6t hoisting load and 14.7-m working range. The three load

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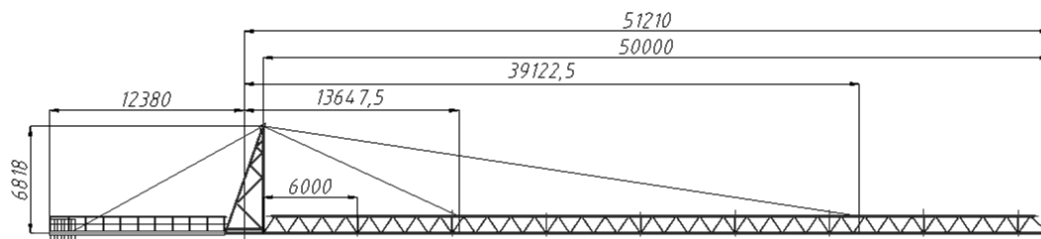
cases are shown in Table II.

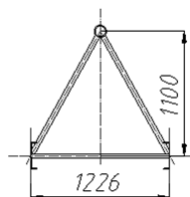Fig. 1 The overall shape and size of the QTZ63 tower crane boom

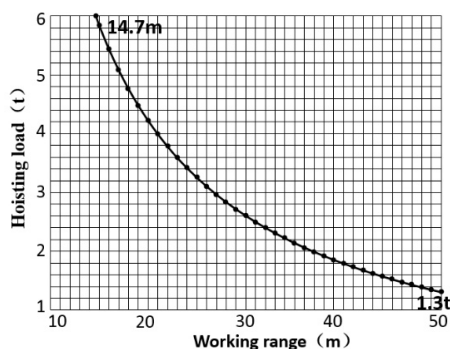Fig. 2 Shape and size of the boom cross section

Fig. 3 The lifting performance curve of the QTZ63 tower crane

TABLE II
THREE REPRESENTATIVE LOAD CASES

Load case	1	2	3
range (m)	50	24	14.7
load (t)	1.3	3.4	6

III. PERIODIC TOPOLOGY AND SIZE OPTIMIZATION OF TOWER CRANE BOOM

The process of discrete boom optimization method based on continuum topology optimization is shown in Fig. 4. Firstly, the optimization boom model is built. It is noted that the optimization field should be modeled using continuum element such as plane, shell, and solid element. Because the next step is to carry out topology optimization with SKO method, which is a continuum topology optimization method. After getting the optimization continuum topology, it is transformed into discrete boom through extracting its skeleton based on the principal stress transferring path. Finally, the section sizes of the boom web members are optimized using the optimization criterion, which is extracted through the Lagrange multiplier method and Kuhn-Tucker condition.
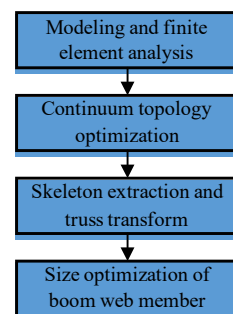

Fig. 4 Procedure of optimization

A. Optimization Model of the Tower Crane Boom

According to the parameters in Figs. 1 and 2, and Table I, the finite element model of the optimization boom is established, as shown in Fig. 5. The difference between the optimization boom and the original boom is that the side web members of the original boom are removed and replaced with web plates. The upper and lower chord members and the underside web members are reserved. The chord member and web member are simulated with BEAM188 element and LINK180 element respectively. The web plate is simulated with SHELL181 element.
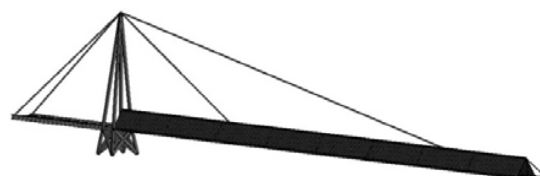

Fig. 5 Finite element optimization model of the tower crane boom

B. Periodic Topology Optimization of the Boom

1) SKO Topology Optimization Method

SKO is a kind of heuristic structural topology optimization method which was put forward firstly by the Karlsruhe Research Center in Germany by Baumgartner et al. based on the adaptive biological growth rule [7]. A virtual temperature indicator is introduced to represent the property of the material. The higher the temperature is, the softer the material is. Through iterative calculation, the material in the structure continuously softens or hardens with regard to the stress in the material. Then, materials with high temperature are removed and materials with low temperature form the optimized

structure. The core iterative function is shown in (1). And the details of SKO method could be seen in the work of Hou and Ding [8].

$$T_i^{(n)} = \overline{T}_i^{(n-1)} - s(\sigma_i^{(n-1)} - \xi\sigma_{ref}^{(n)}) \qquad (1)$$

where $T_i^{(n)}$ and $\overline{T}_i^{(n-1)}$ correspond to the temperature of element i in the n-th and (n-1)-th iteration. $\sigma_i^{(n-1)}$ is the stress of elelment i in the (n-1)-th iteration. $\sigma_{ref}^{(n)}$ is the reference stress of the n-th iteration. s is the step factor and ξ is the weight coefficient.

2) Description of Periodic Topology Optimization Method

The direct use of topology optimization methods cannot achieve a satisfied result since the tower crane boom is a long-narrow structure. Instead, a periodic optimization method can perform more efficiently, and the optimization result is more conducive to practical engineering applications, as discussed in the work of Jiao et al. [9]. As shown in Fig. 6, the long-narrow boom structure is divided into m optimal design sub-domains, and the shape, size, and meshing of them are totally identical. $T_{i,j}$ is the design variable in SKO method, i is the number of sub-domain and j denotes the element number in a sub-domain. As shown in (2), when using the SKO method in topology optimization, the element temperatures should be equal to each other in the same position of sub-domains. This ensures the identical optimization results of all subfields, which is periodic topology optimization.
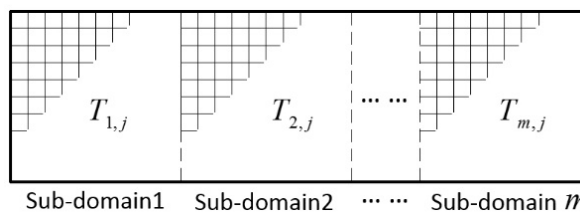

Fig. 6 Periodic topology optimization

$$T_{1,j} = T_{2,j} = \cdots = T_{m,j} = \min(T_{1,j}, T_{2,j}, \cdots, T_{m,j}) \qquad (2)$$

3) Optimization Result of the Tower Crane Boom

Fig. 7 shows three periodic topology optimization results of the tower crane boom under three different load cases respectively. Regardless of some details, they are similar to each other. We take the optimized topology of load case 1 as the final result, as shown in Fig. 8. Fig. 9 is the topology optimization result in one domain, it shows that the topology consists of three triangular configurations. Wind load and horizontal inertial load are taken into consideration, which leads to horizontal asymmetry load and makes the optimization results of two side slightly different.

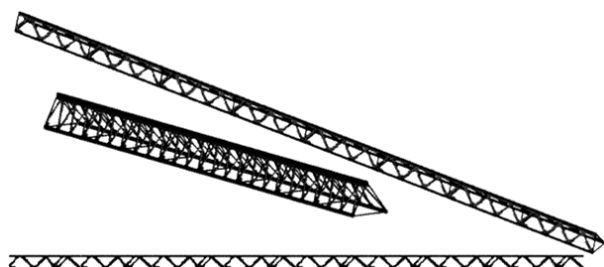Fig. 7 Topology optimization results of three load cases

Fig. 8 Web topology optimization result of the boom

C. Transformation of the Tower Crane Boom

Referring to the force transferring path thought in the bionic optimization discussed by Mattheck [10], the principal stress path of the optimized topology is calculated, as shown in Fig. 10. Based on this path, the skeleton of the optimized topology structure is extracted, and the truss-like boom is obtained according to the skeleton, as shown in Fig. 11. The final optimized tower crane boom is shown in Fig. 12. The details of this skeleton extraction method could be referred to the work of
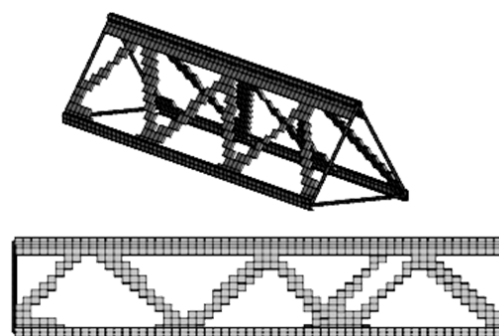
Wu et al. [11].

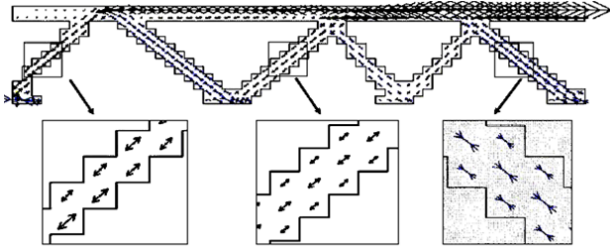Fig. 9 Topology optimization result in a periodic
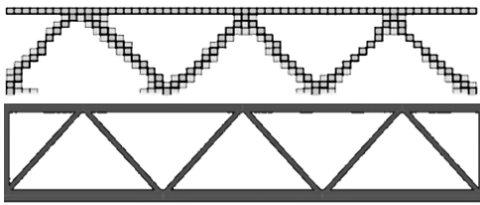
Fig. 10 Principal stress of the optimized topology
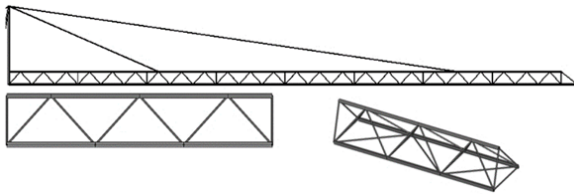

Fig. 11 Skeleton and the corresponding boom

Fig. 12 Final optimized tower crane boom

D.Size Optimization Method of the Boom Web Members

Taking the radius of the boon web members, the compliance of the boom and the material volume of the boom as design variables, objective and constraint respectively, the mathematical model is established. Based on Lagrange multiplier method and Kuhn-Tucker condition, the optimization criterion is derived according to this model.

1) Optimization Mathematical Model

The cross section size optimization mathematical model is shown in (3):

$$find \quad \boldsymbol{R} = (r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}, \ ... \ , r_{i,j}, \ ... \ , r_{n,1}, r_{n,2})$$

$$min \quad C(\boldsymbol{R}) = \boldsymbol{U}^T \boldsymbol{K} \boldsymbol{U} = \sum_{i=1}^{n} \boldsymbol{u}_i^T \boldsymbol{k}_i \boldsymbol{u}_i$$

$$s.t. \quad \boldsymbol{KU=F} \qquad i=1,2,...,n \qquad j=1,2 \qquad (3)$$

$$V = \sum_{i=1}^{n} \pi (r_{i,2}^2 - r_{i,1}^2) l_i \le V^*$$

$$r_{min} \le r_{i,1} < r_{i,2} \le r_{max}$$

where, $r_{i,1}, r_{i,2}$ is the inner and outer radius of web member i. R is a vector formed by all members' section radius. n is the number of web members in the boom. C and V are the volume and compliance of the boom. $\boldsymbol{K}$ is the stiffness matrix of the boom. $\boldsymbol{U}, \boldsymbol{F}, \boldsymbol{u}_i, \boldsymbol{k}_i$ are the displacement and force

vector of the boom and web member i. V^* is the volume constraints. r_{min}, r_{max} are the limits of the radius.

2) Derivation of the Optimization Criteria

Based on the Lagrange multiplier method, the Lagrange function of mathematic model (3) is shown in (4).

$$L=C+\boldsymbol{\lambda}_1^T (\boldsymbol{KU-F}) + \mu_1 (V-V^* + x_1^2) + \sum_{i=1}^{n} \mu_2 (r_{min} - r_{i,1} + x_2^2)$$
$$+ \sum_{i=1}^{n} \mu_3 (r_{i,1} - r_{i,2} + x_3^2) + \sum_{i=1}^{n} \mu_4 (r_{i,2} - r_{max} + x_4^2) \qquad (4)$$

where, $\boldsymbol{\lambda}_1, \mu_1, \mu_2, \mu_3, \mu_4$ are the Lagrange multipliers, $\boldsymbol{\lambda}_1$ is a vector, $\mu_1, \mu_2, \mu_3, \mu_4$ are scalars, x_1, x_2, x_3, x_4 are slack variables.

Considering that $\boldsymbol{R}$ gets the extremum $\boldsymbol{R}^*$, the corresponding Kuhn-Tucker condition of (4) is:

$$\begin{cases} \dfrac{\partial L}{\partial r_{i,j}} = \dfrac{\partial C}{\partial r_{i,j}} + \boldsymbol{\lambda}_1^T \dfrac{\partial \boldsymbol{KU}}{\partial r_{i,j}} + \mu_1 \dfrac{\partial V}{\partial r_{i,j}} \begin{cases} >0 & r_{i,1} = r_{min} \\ <0 & r_{i,2} = r_{max} \\ =0 & r_{min} < r_{i,1} < r_{i,2} < r_{max} \end{cases} \\ \qquad\qquad\qquad\qquad\qquad\qquad i=1,2,...,n \quad j=1,2 \\ \boldsymbol{KU=F} \\ \mu_1(V-V^*)=0 \quad , \quad \mu_1 \ge 0 \end{cases} \qquad (5)$$

According to the Kuhn-Tucker condition (5) and the element stiffness matrix of link element, the final section size optimization criterion of web member can be deduced. As shown in (6), the detailed deducing process could be seen in work of Zhou et al. [12].

$$r_{i,j}^{(k+1)} = \begin{cases} (f_i^{(k)})^{\delta} r_{i,j}^{(k)} & r_{min} < (f_i^{(k)})^{\delta} r_{i,j}^{(k)} < r_{max} \\ r_{min} & (f_i^{(k)})^{\delta} r_{i,j}^{(k)} \le r_{min} \\ r_{max} & (f_i^{(k)})^{\delta} r_{i,j}^{(k)} \ge r_{min} \end{cases} \qquad (6)$$

where f_i is the optimization criterion of web member i, as shown in (7). e_i is the strain energy of element i and $e_i = \dfrac{\boldsymbol{u}_i^T \boldsymbol{k}_i \boldsymbol{u}_i}{2}$. δ is a damping factor using stabilize the optimization process.

$$f_i = \dfrac{2e_i}{\mu_1 v_i} = 1 \qquad (7)$$

3) Periodic Size Optimization of the Boom

As shown in Fig. 5, the boom of the Tower crane QTZ63 is divided into eight sections. During the above optimization process, no periodic constraints are imposed and it will get different sizes of web member for every section of the boom. This can theoretically result in better solutions. However,

because of the diversity and the lack of regularity of the web members, the manufacturing becomes more difficult and the cost will increase. So, a periodic optimization method is proposed, which assumes that the eight sections of the boom of the Tower crane are exactly the same. It makes the manufacturing easier and reduces the cost. As shown in Fig. 13, $l_{p,q}$ represents the q-th web member in the p-th boom section. During the optimization process, a virtual boom x is built. Every single section of the crane boom has the same parameters with this virtual boom x. As shown in Fig. 14 and (8), the size of q-th web member in the virtual boom x is a function of the average value of the sizes of q-th web member in all boom section.
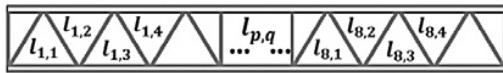

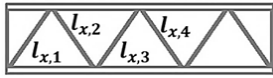Fig. 13 Diagram of the periodic optimization of the boom web member

Fig. 14 Diagram of the virtual boom x

$$r^{l_{1,q}} = r^{l_{p,q}} = r^{l_{8,q}} = r^{l_{x,q}} \quad q = 1,2,...,n \tag{8}$$

$r^{l_{p,q}}$ represents the radius of q-th web member in the p-th boom, $r^{l_{x,q}}$ represents the radius of q-th web member in the virtual boom x, n represents the number of web members that need to be optimized in every single boom.

$$r^{l_{x,q}} = \alpha \overline{r^{l_{p,q}}} \quad p = 1,2,...,8; q = 1,2,...,n \tag{9}$$

where α is the scaling factor.

In the periodic size optimization, 15 diagonal web members are chosen as the optimization objects within each optimization section. These web members are numbered as shown in Fig. 15. The eight sections are identical. After the respective optimization of the three load cases, three sets of data are obtained. Then, the enveloping process is performed to get the final size data of the boom web members. The enveloping process means to take the maximum value of the three sets for each particular member, as shown in (10).

Fig. 15 Number of web members within each section

$$r_i = \max\left(r_i^{LS1}, r_i^{LS2}, r_i^{LS3}\right) \tag{10}$$

where r_i denotes the radius of the *i-th* member after envelop. r_i^{LS1}, r_i^{LS2}, r_i^{LS3} denote the radius of member i under the three different load cases.

The final optimized diameters of web members within one section are listed in Table III.

TABLE III
OPTIMIZED SIZES OF WEB MEMBERS

No.	1	2	3	4	5	6	7
d_{in}	21.2	21.4	20.2	20.3	20.3	20.2	21.0
d_{out}	26.4	26.6	24.2	24.4	24.4	24.3	25.3
8	9	10	11	12	13	14	15
20.8	20.5	20.8	20.5	20.6	18.0	18.0	18.0
25.0	24.6	24.9	24.6	24.7	21.6	21.6	21.6

(d_{in}, d_{out} are the inner and outer diameters of the web members. The unit is *mm*.)

IV. OPTIMIZATION RESULT AND COMPARISON

The performance parameters of the original and optimized tower crane booms are shown in Table IV. The changes of performance after optimization can be directly seem from Fig. 16. Comparing to the original boom, the optimized boom achieves higher stiffness and lower stress with less steel usage and lighter weight. Fig. 17 shows the calculation pictures of the original and optimized tower crane boom, which demonstrates the deformation of the boom under the three load case and the corresponding maximum deflection and von Mises stress.

TABLE IV
PERFORMANCE OF THE ORIGINAL AND OPTIMIZED BOOMS

Performance		Original boom	Optimized boom	
			Value	Change
W		3053.7	2315.8	24.2%
	E	1363.6	1157.1	15.1%
Load case 1	Max U	0.273	0.239	12.5%
	Max S	93.5	81.9	12.4%
	E	921.8	860.9	6.6%
Load case 2	Max U	0.130	0.099	23.8%
	Max S	102	91.6	10.2%
	E	606.7	555.1	8.5%
Load case 3	Max U	0.125	0.094	24.8%
	Max S	104	93.5	10.1%

(W, E, U, S are the weight, strain energy, deflection and stress of the boom. The units of them are *kg, J, m, MPa*.)

As shown in Figs. 16 and 17, and Table IV, the optimization result of the tower crane boom is effective. The weight of the boom decreased by 24.2%. Moreover, the strain energy, the maximum deflection, and the maximum stress of the boom are notably reduced. It means that the stiffness of the optimized boom increased, the deformation and the stress level decreased. Since each section of the boom is exactly the same because of the periodic optimization, from the view of practical engineering application, it brings better manufacturability and can effectively control the manufacturing cost.
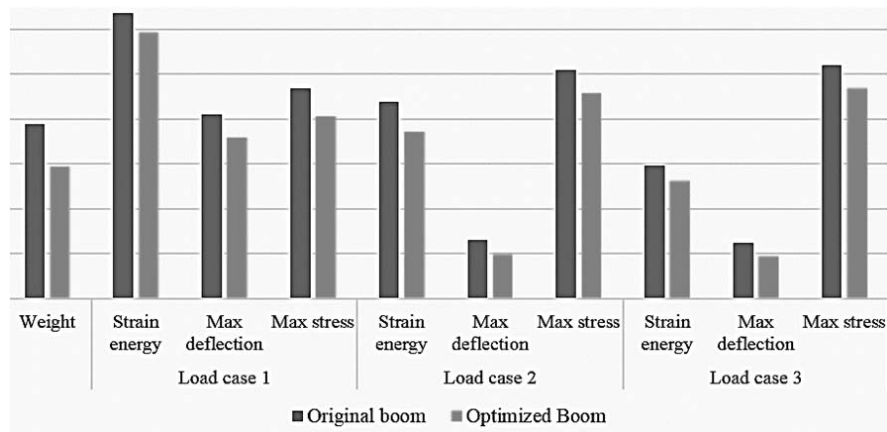
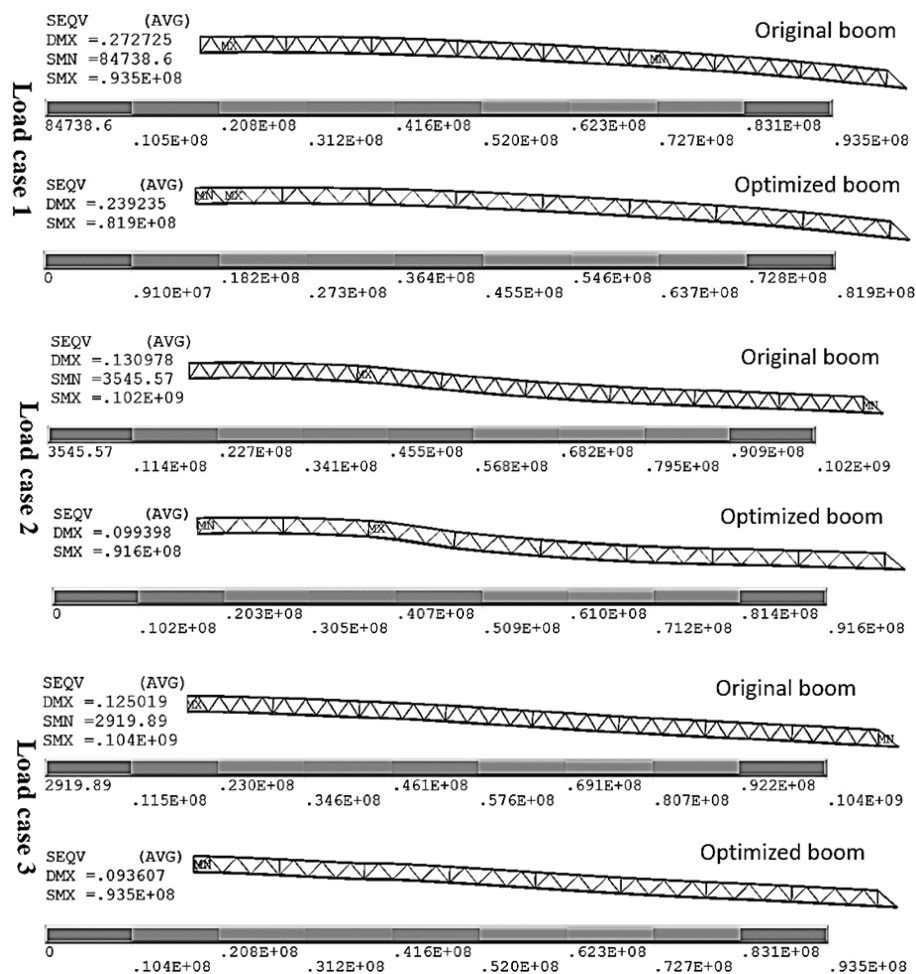Fig. 16 Performance comparison between the original and optimized boom

Fig. 17 Calculation pictures of the original and optimized boom

V. CONCLUSION

In general, with the using of the continuum topology optimization method and the size optimization criterion method, the proposed layout and size optimization method of the tower crane boom achieves an excellent result. Comparing to the traditional design method, it can get an optimized layout and section size of the crane boom web members with more reasonable load bearing. The optimized boom not only has a lighter weight, but also has a higher mechanical performance like the increased stiffness and reduced stress. The periodic

optimization method can overcome the problem where the slender boom is difficult to optimize and get a more practical structure. The effectiveness of the method has been proved by the detailed analysis data.

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