

Decision Making under Strict Uncertainty: Case Study in Sewer Network Planning

Zhen Wu, David Lupien St-Pierre, Georges Abdul-Nour

Abstract—In decision making under strict uncertainty, decision makers have to choose a decision without any information about the states of nature. The classic criteria of Laplace, Wald, Savage, Hurwicz and Starr are introduced and compared in a case study of sewer network planning. Furthermore, results from different criteria are discussed and analyzed. Moreover, this paper discusses the idea that decision making under strict uncertainty (DMUSU) can be viewed as a two-player game and thus be solved by a solution concept in game theory: Nash equilibrium.

Keywords—Decision criteria, decision making, sewer network planning, strict uncertainty.

I. INTRODUCTION

EVERYONE makes decisions of varying importance on each single day. A good understanding of what the decision making process involves and how to choose effective decision rules can help us to make better decisions and have higher probability of success. Based on decision maker's knowledge of the information and data, decision making problems are divided into different categories: decision making under certainty (DMUC), DMUSU and decision making under risk (DMUR) [1]. DMUC represents a situation where the true state is known to the decision maker and the consequence of an action can be predicted with accuracy; DMUSU implies that decision maker has no information about state of nature and the severity of uncertainty is immeasurable quantitatively; DMUR assumes that decision maker can assign probability distribution to each state of nature based on their own experience or historical frequencies.

Before going into the process of decision making, the decision maker needs to specify the relevant decision alternatives, states of nature and outcomes [2]. States of nature are the external factors which may affect the decision maker's preference among all the decisions. Decision alternatives are the possible choices available for the decision maker. Outcomes are the consequences of all the possible decisions under a given set of states of nature which ultimately help decision makers to figure out which decision to choose. Three concepts of decision making can be summarized in Table I, where d_1, d_2, \dots, d_m denote the “m” decision alternatives

available for the decision maker; the “n” possible states of nature are denoted by s_1, s_2, \dots, s_n ; the outcomes are denoted by a_{ij} , which is the consequence of taking decision d_i when s_j is the state. Note that a_{ij} are not necessarily numbers. In this paper, we assume that the decision maker can value them numerically, for example, a_{ij} can present the payoff or cost related to d_i under state s_j . In this way, outcomes form an $m \times n$ dimensional matrix $A = (a_{ij})_{m \times n}$ which is called as decision matrix, it helps decision maker to visualize the decision problem and makes the process of decision making more easily.

TABLE I
FORMALIZATION OF DECISION MAKING

Outcomes	States of Nature		
	s_1	s_2	s_n
d_1	a_{11}	a_{12}	a_{1n}
d_2	a_{21}	a_{22}	a_{2n}
Actions			
.			
.			
.			
d_m	a_{m1}	a_{m2}	a_{mn}

This paper focuses on DMUSU which depict a situation where the decision maker has no information about state of nature. He/she is not only unaware of the true states, but also, he/she cannot quantify his/her uncertainty in any way. He/she can only prepare an exhaustive list of the possibilities of the state of the world but the probability distribution over the possible state is unknown. Without any information about states of nature, the decision maker's attitude toward the unknown decides their behavior [3], [4]. Five classic criteria to find the optimal decision for DMUSU have been presented in literature [5], Wald [6], Savage [7], Hurwicz [8] and Starr domain [9]. Furthermore, Nash equilibrium (NE) in game theory is another criterion for DMUSU based on the link between DMUSU and a two-player game.

After the introduction on the theoretical part of DMUSU, one practical example of DMUSU in the field of sewer network planning is provided. Civil engineer of the city proposed four sewer network construction alternatives in order to direct more rainfall water of one particular area to the river. The city needs to make a decision to choose one alternative and implement it in this area. With the fact that the city has no information about the weather condition, this decision making problem belongs to DMUSU. With the existing data and analysis, decision matrix is generated and solved by five classic criteria and NE.

The rest parts of this paper are organized by: Section II

Zhen WU is with the Industrial Engineering Department of Université du Québec à Trois-Rivières, 3351 Boulevard des Forges, Trois-Rivières, QC, G9A 5H7 Canada (corresponding author to provide phone: 1-819-995-8728; e-mail: zhen.wu@uqtr.ca).

David Lupien St-Pierre and Georges Abdul-Nour are with the Industrial Engineering Department of Université du Québec à Trois-Rivières, 3351 Boulevard des Forges, Trois-Rivières, QC, G9A 5H7 Canada (e-mail: david.lupien-st-pierre@uqtr.ca, georges.abdulnour@uqtr.ca).

explains the definition of strict uncertainty within the field of decision making; Section III introduces five classic criteria in detail for DMUSU problem. Section IV shows that NE is one solution concept of solving DMUSU problem based on the link between DMUSU and two-player strategic game; Section V applies DMUSU in the field of sewer network planning and solves it by five criteria and NE; Section VI presents the conclusion of this paper.

II. STRICT UNCERTAINTY

Uncertainty is described as the lack of certainty, a situation of having imperfect or unknown information. The notion of uncertainty has been involved in many practical fields such as investigation in financial markets [10], weather forecasting [11], quantum Mechanics [12], metrology [13], and so on.

The concept of strict uncertainty is also named as severe uncertainty in the literature of info-gap decision theory [14], Knightian uncertainty in the literature of economics decision making [15], deep uncertainty in the literature of robust decision making [16]. It must be emphasized that under strict uncertainty, within the field of decision making, decision maker cannot say anything about the true state of nature, and the probabilities of the states of nature are immeasurable quantitatively.

III. FIVE CRITERIA FOR DMUSU

In decision making, the process of identifying the optimal decision is called decision criteria. It contains three parts: first, information is gathered about the decision alternatives and the environment; second, select one criterion that suits the decision maker's attitude and preference; third, make the choice. The research about criteria for DMUSU was actively discussed in the early 1950s, in which Laplace's insufficient reason criterion, Wald's maximin criterion, Hurwicz's criterion, Savage's minimax regret criterion, Starr's Domain criterion are the most classic ones.

Consider one DMUSU problem, where d_1, d_2, \dots, d_m denote the decision alternatives available for the decision maker, s_1, s_2, \dots, s_n represent the possible states of nature and a_{ij} is the outcome related to d_i under state s_j . To introduce the five classic criteria, assume that value $a_{ij} \in \mathbb{R}$ is the payoff value of choosing d_i under state s_j , hence, $A = (a_{ij})$ is a payoff matrix. Definitions and applications of each criterion have been summarized in [17].

A. Laplace's Principle of Insufficient Reason

Since decision makers know nothing about the true state of nature, Laplace suggested that they can consider them all having equal probability [18], hence this criterion assumes that the probabilities of the different possible states of Nature are all equal. Thus, for the i^{th} decision, his/her expectation is given by the average $(a_{i1} + \dots + a_{in})/n$, and he/she should choose the decision for which this average is maximized, i.e. Choose d_k such that

$$\frac{1}{n} \sum_{j=1}^n a_{kj} = \max_i \left\{ \frac{1}{n} \sum_{j=1}^n a_{ij} \right\} \text{ where } i = 1, \dots, m. \quad (1)$$

With this criterion, the problem shifts from strict uncertainty to risk which is a relatively simple to solve. However, with this assumption, the state space must be constructed in order to be amenable to a uniform probability distribution [19].

B. Wald's Maximin Criterion

With the i^{th} decision, decision maker's payoff will certainly be at least $\min_j a_{ij}$. The safest possible course of the action is therefore to choose a row for which $\min_j a_{ij}$ is maximized. i.e. Choose d_k such that

$$\min_j a_{kj} = \max_i \min_j a_{ij}, \text{ where } i = 1, \dots, m \text{ and } j = 1, \dots, n. \quad (2)$$

Wald's maximin is the rule which chooses the "best of the worst". It evaluates each decision by the minimum possible return associated with the decision. Then, the decision that yields the maximum value of the minimum returns (maximin) is selected. Hence, Wald's maximin is extremely conservative and may lead to exceedingly costly solutions resulting from over-protection against uncertainty.

C. Savage's Minimax Regret Criterion

Savage [20] defines $r_{ij} = \max_{k=1, \dots, m} a_{kj} - a_{ij}$ for all i, j , and a regret matrix $R = (r_{ij})$ that measures the difference between the payoff which could have been obtained if the true state of Nature had been known and the payoff which is actually obtained. Then, apply Wald minimax criterion to regret matrix R . That is, choose a row for which $\max_j r_{ij}$ is minimized, i.e. choose d_k such that

$$\max_j \{r_{kj}\} = \min_i \{ \max_j \{r_{ij}\} \}, \text{ where } i = 1, \dots, m \text{ and } j = 1, \dots, n. \quad (3)$$

Regret matrix only reflects the difference between each payoff and the best possible payoff in a column; hence, the disadvantage of Savage's minimax regret criterion is not considering the row differences.

D. The Pessimism-Optimism Index Criterion of Hurwicz

The Hurwicz's criteria presented in [21], [8] are defined as: Select a constant $0 \leq \alpha \leq 1$ which is a coefficient of the player's optimism. For each decision d_i , let a_i denote the smallest component and A_i the largest, then Hurwicz's measurement H_i is defined as:

$$H_i = \alpha A_i + (1 - \alpha) a_i \text{ where } i = 1, \dots, m. \quad (4)$$

Hurwicz suggests the decision rule: choose d_k such that

$$H_k = \max_i \{H_i\}, \text{ where } i = 1, \dots, m. \quad (5)$$

This criterion only considers the highest and the lowest payoff for each alternative. It does not take other non-extreme payoffs into account. Hence, two decisions get the same Hurwicz's measurement value when they have the same

smallest and biggest payoffs, even if one of them has many high outcomes and the other has many low outcomes [22].

E. Starr's Domain Criterion

Starr's Domain criterion suggested in [23], [9]: define the set D (the domain) of all possible probability distributions associated with the states of nature s_j , $j = 1, \dots, n$, as $D = \{p = (p_j) \in R_+^n | \sum p_j = 1\}$. This set is called the fundamental probability simplex (FPS). For any given distribution p , define the expected monetary value of the i^{th} decision:

$$E^p(d_i) = \sum_{j=1}^n p_j a_{ij}. \quad (6)$$

Then,

$$D_i = \{p \in D | E^p(d_i) \geq E^p(d_k) \forall k \neq i\} \quad (7)$$

is the set of all probability distributions p for which the i^{th} decision is optimal according to the Bayesian expected value criterion. Let $V(D_i)$ denote the volume of the set D_i . In Starr's criterion, the r^{th} decision is chosen if $V(D_r) \geq V(D_i) \forall i \neq r$. In the other words, Starr's criterion selects the decision that is most likely to have a higher expected payoff value than all the others.

When the number of states of nature $n \leq 3$, the volume can be computed by graphical method. For $n > 3$, otherwise one can use a Monte-Carlo sampling algorithm to approximate the volume. Reference [24] presents an algorithm which can find the exact volumes of convex polyhedral, also [9] proposes to use simulation with random sampling of points in the FPS. Although there are algorithms which can rapidly approximate the large dimension volume, it remains difficult for decision maker to clearly understand this approach. As such, the main drawback for the decision maker is the difficult to properly understand the criterion. Note that all the criteria are introduced with a payoff decision matrix. If the decision matrix is negative flow, e.g. cost matrix, when choosing the optimal solution, it is needed to find the minimum value instead of maximum.

IV. DMUSU AS A TWO-PLAYER GAME IN GAME THEORY

The link about DMUSU and two-player game in game theory has been made in literatures [17], [25]-[28]. A game in game theory is actually a mutual interdependent decision making problem, where the outcome of one player depends not only on what he or she acts but on what decisions that the other player makes. The basic conceptions of a two-player strategic game are: strategies for player 1, strategies for player 2 and payoffs of each player from possible strategy combination. These correspond, respectively, to the basic concepts of one decision making problem: alternative decisions, states of nature and outcome of each decision for each state of nature. Hence, one DMUSU problem can be considered as a two-player game; player 1 and player 2 can be referred as the decision maker and neutral nature separately, and decision alternatives and states of nature are the strategies of each player. Furthermore, it is also a non-cooperative and

non-zero-sum game since player 2 in this game is neutral nature. Non-cooperative game means that players cannot form and respect binding agreements between them, and non-zero-sum game implies that a gain by one player does not necessarily correspond with a loss by another, and the total benefit to all the player is not zero.

In game theory, if each player has chosen a strategy and no player has anything to gain by changing strategies, while the other players keep theirs unchanged, then the current strategy set choices and the corresponding payoffs constitute a NE [29], [30]. There are two types of NE: pure strategy NE where all players are playing pure strategies, and mixed strategy NE where at least one player is playing a mixed strategy. A pure strategy decides all one's moves during the game (and should therefore specify one's moves for all possible other players' moves). A mixed strategy is a probability distribution over all possible pure strategies (some of which may get zero weight).

References [31], [32] proposed one algorithm for computing the mixed strategy NE which is:

For each individual player:

- 1) Assign a variable to each strategy that denotes the probability of a player for choosing that strategy.
- 2) The total sum of the probabilities for each strategy of a player is 1.
- 3) Based on the randomization of the player's choice, the expected payoff for a player should be the same.
- 4) This creates a group of equations from which the probabilities of choosing each strategy can be computed.

Since a DMUSU problem can be considered as a two-player non-cooperative and non-zero-sum game, NE becomes one of the solution option for solving DMUSU problem.

V. APPLICATION OF DMUSU IN SEWER NETWORK PLANNING

The application of decision making appears widely in many fields. For instance, blood-bank inventory control [1], budget planning in production engineering [33], airport siting [34], medical screening [35], electric power generation [36], career choices [37], water resource management [38]. In this section, the application of DMUSU in one city's sewer networking planning is studied. Section V.A describes the sewer network planning project faced by the city and four available plans. Section V.B transfers city's plan-choosing situation into one DMUSU problem and generate the decision matrix. Section V.C applies five criteria and NE to it and shows the results. Section V.D summarizes and analyses all the results.

A. Problem Statement

A pumping station is located next to the river and north-west of Highway No. 40. This pumping station is receiving combined sewer water (rainfall and sanitary flow) from one certain area (see Fig. 1).

The local city would like to reduce the rainfall flow channeled to the pumping station in order to improve its capacity for the sanitary flow. To meet this goal, it wants to gather the rainfall water of the area and direct them to the river. In this way, there will be less rainfall water taking the space of the pumping station and more space for the sanitary

flows. The civil engineer department of the city has proposed four construction plans to build this new rainfall pipe:

- 1) Plan 1 is to build a new rainfall water pipe along Barkoff street from Boulevard des Ormeaux and it goes directly to the river. With this plan, rainfall water flows of this segment will be directed to the river. See black solid line in Fig. 2;
- 2) Plan 2 is to extend the existing rainfall water pipe along rue Vachon till the river, such that rainfall water of this
- 3) Plan 3 includes the construction of Plan 1. Furthermore, it will extend the rainfall pipe to north east till road du Parc. Plan 3 is black solid line and black dash line in Fig. 2;
- 4) Plan 4 includes the construction of Plan 2. Plus, it will extend the rainfall pipe to north east along road Morin and highway 40. Plan 4 includes gray solid line and gray dash line in Fig. 2.

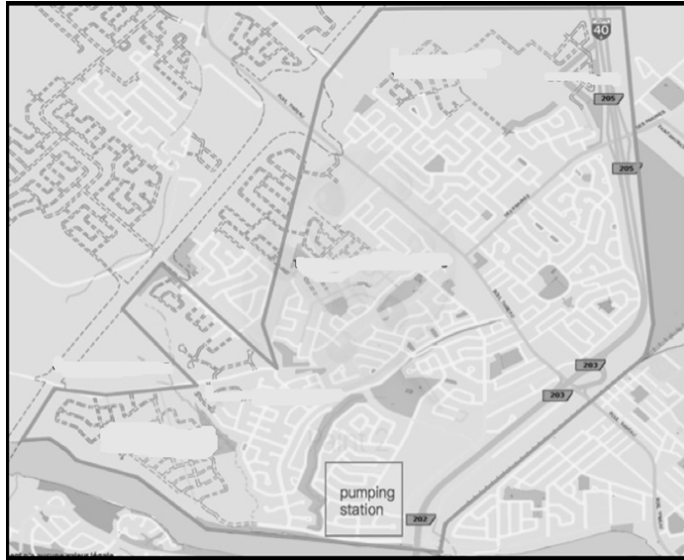


Fig. 1 Pumping station and its area

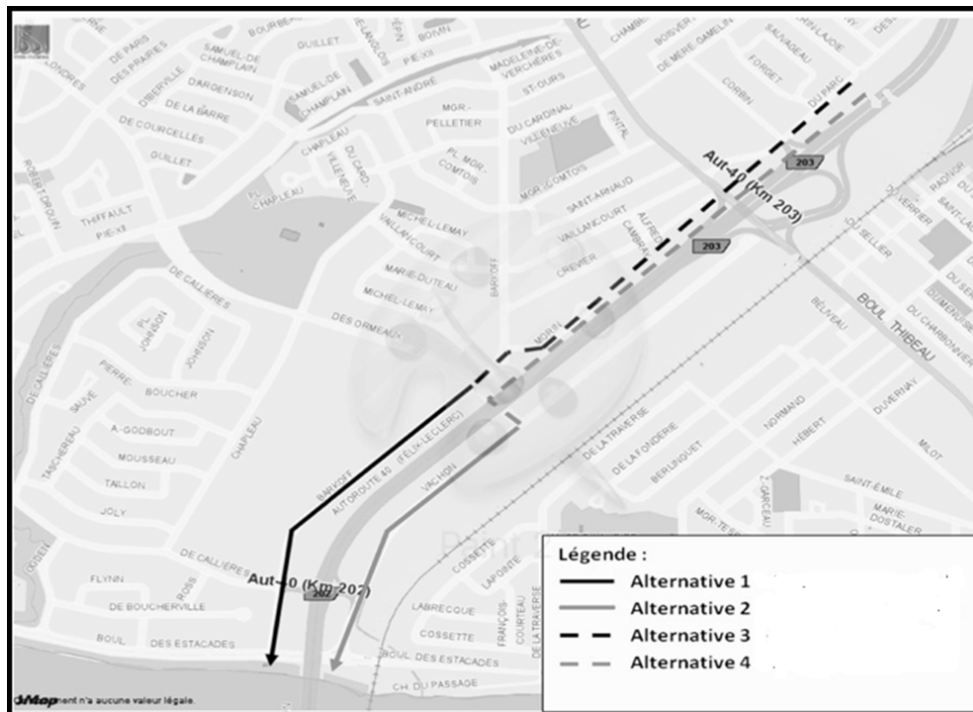


Fig. 2 Construction plans

The total cost for each construction plan is listed in Table II:

TABLE II TOTAL COST OF EACH PLAN	
plan	Total cost (CAD)
P1	1 884 753
P2	437 606
P3	4 127 967
P4	2 680 820

In order to evaluate how much rainfall water is relieved from the pumping station for each plan, civil engineers modeled the current sewer network of the area and the possible alternatives (Plan 1 to 4) using Sanitary and Combined Sewer Modeling Software (SewerGEMS), a fully-dynamic, multi-platform (GIS, CAD and Stand-Alone).

The process is: within SewerGems, first, set up the baseline rain: rain of 9 mm within a period of three hours. Second, execute the model of the current sewer network and each alternative respectively with this rainfall weather. Third, gather the value of the rainfall flow channeled to the pumping station per second for each model. Last, compare the different values.

The results are shown in Figs. 3-6 where the higher line indicates the rainfall flow channeled to the pumping station with the current existing sewer network, the lower line indicates the same value but for each individual plan, and the grey area is the reduced rainfall flow from the pumping station. These figures directly show the reduction of rainfall flows for each plan at the pumping station (the order of the reduced rainfall flow is Plan 3 > Plan 1 > Plan 4 > Plan 2) which also means how much capacity is improved for containing sanitary flow.

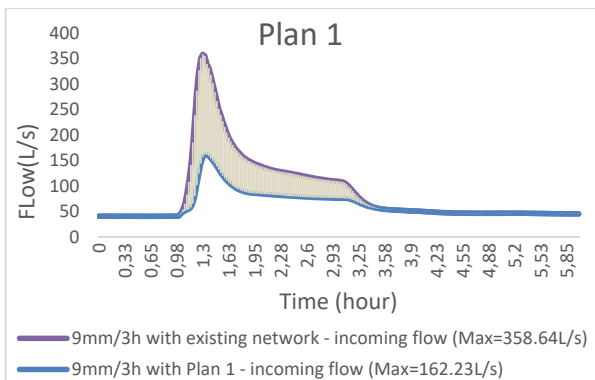


Fig. 3 Plan 1 VS Current sewer network with 9mm/3hrs rainfall

In reality, it is not always practical and beneficial to choose the plan with biggest reduction because of the cost per volume saved. Moreover, the first unit of volume saved is clearly of importance, yet the millionth might not be as important. Thus, a weighted sum of the volume saved is more representative for the need of the city. Also, from a pragmatic point of view, the functional level of pumping station should be considered.

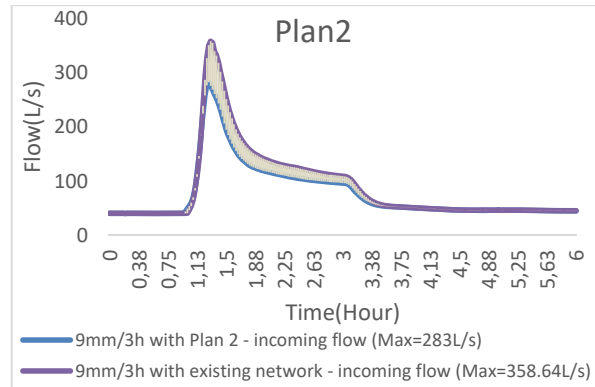


Fig. 4 Plan 2 VS Current sewer network with 9 mm/3 hrs rainfall

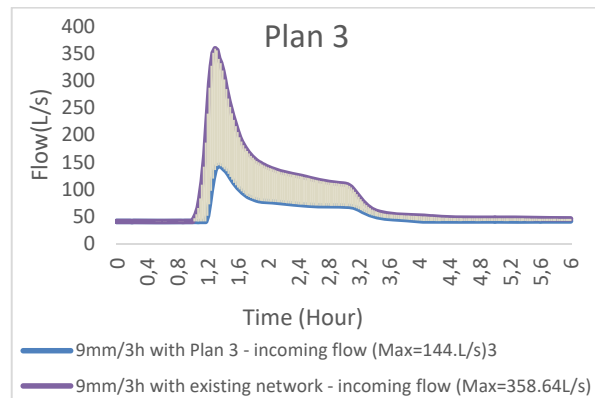


Fig. 5 Plan 3 VS Current sewer network with 9 mm/3 hrs rainfall

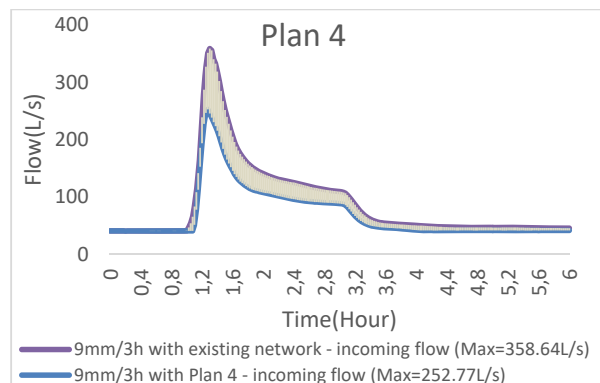


Fig. 6 Plan 4 VS Current sewer network with 9 mm/3 hrs rainfall

B. Conversion to a DMUSU Problem

In order to select one plan out of the four, the city is actually facing a DMUSU problem where weather conditions can be considered as states of nature. Decision maker (the city) has no information about their true states, and the probabilities of the states of nature are immeasurable quantitatively.

To form the DMUSU problem, three basic concepts (states of nature, decision alternatives and outcomes) should be specified. As mentioned before, states of nature are the rainfall weather which cannot be quantitative by decision maker, but a

list can be provided. Based on their preference, states of nature considered in this process are: $s_1=7.2\text{mm}$ over a period of 3 hours; $s_2=8.1\text{mm}$ over a period of 3 hours; $s_3=9\text{mm}$ over a period of 3 hours; $s_4=9.9\text{mm}$ over a period of 3 hours.

Clearly, decision alternatives are four construction plans: $d_1=\text{Plan 1}$; $d_2=\text{Plan 2}$; $d_3=\text{Plan 3}$; $d_4=\text{Plan 4}$.

Outcomes are the consequences of each plan under each rainfall weather which is the value that can encompass the cost, the amount of reduced rainfall water and the functional level of the pumping station. To do this, four steps are used to compute the outcomes of this DMUSU problem.

Step 1: Set up rainfall condition s_1, s_2, s_3, s_4 in SewerGems. Then, execute each decision (d_1 to d_4) respectively with each state of nature. After, gather the maximum of rainfall incoming flow channeled to the pumping station (liters per second) for each decision under each rainfall condition. See Table III.

States of nature	s_1	s_2	s_3	s_4
Plans				
d_1	107.2	133.11	162.23	195.01
d_2	176.36	226.25	283	342.41
d_3	92.12	116.13	144.3	175.29
d_4	152.03	198.44	252.77	307.13

Step 2: Set the rainfall incoming flow of the current sewer network under rainfall weather 9mm/3hrs: 358.64L/s as the base value. Compute the reduced rainfall incoming flow for each plan under each rainfall weather by the difference between the base value and the value in Table III. Results are presented in Table IV.

States of nature	s_1	s_2	s_3	s_4
Plans				
d_1	251.44	225.53	196.41	163.63
d_2	182.28	132.39	75.64	16.23
d_3	266.52	242.51	214.34	183.35
d_4	206.61	160.2	105.87	51.51

Step 3: Based on the fact that the first unit of volume saved is clearly of importance, yet the millionth might not be as important. Hence, a weighed sum method to is used to modify the data in Table IV in order to have more representative data that fit the need of the city. Weighted factors are set up in Table V.

Reduced rainfall flow	Qty (L/s)	Weight
Need	80.000	1.000
Possible future use	120.000	0.500
Not necessary		0.100

In this way, from Table IV, the first 80 L/s are worth their exact weight; Then those values between 80L/s and 120L/s, while being nice to save this volume is not relevant for the

current situation. Thus, a half weight is given, it becomes $80 + (\text{value}-80) * 0.5$; After 120L/s, there should never be any need for these volumes. Thus, it becomes $80 + 40 * 0.5 + (\text{value}-120) * 0.1$. Table VI presents the weighted results:

States of nature	s_1	s_2	s_3	s_4
Plans				
d_1	113.144	110.553	107.641	104.363
d_2	106.228	101.239	75.64	16.23
d_3	114.652	112.251	109.434	206.335
d_4	108.661	104.02	92.93	51.51

Step 4: Generate Table VII through dividing the total cost of each plan by the weighted reduced incoming flow values in Table VI. Those values in Table VII are the cost per weighted liter per second for each alternative plan under each state of nature which is the desired outcomes of the DMUSU.

$\$/(\text{L/s})$	s_1	s_2	s_3	s_4
d_1	16658.00	17048.41	17509.62	18059.59
d_2	4119.50	4322.50	5785.38	26962.79
d_3	36004.32	36774.44	37721.07	38820.40
d_4	24671.41	25772.17	28846.19	52044.66

C. Optimal Plan Selection Using Five criteria of DMUSU and NE

In this section, five criteria of DMUSU and NE are applied to the decision matrix formalized in Table VII in order to find the optimal plan.

1. Laplace's Principle of Insufficient Reason

As a reminder, according to Laplace's criterion, when the probabilities of conditions are not known, the probabilities of states of nature are accepted as equal. Thus, the expectation of each decision is computed through the average $(a_{i1} + a_{i2} + a_{i3} + a_{i4})/4$. The decision chosen is the smallest average. Hence, Plan 2 is the optimal decision for the city based on Laplace's criterion. See Table VIII.

$\$/(\text{L/s})$	s_1	s_2	s_3	s_4	Laplace average
d_1	16658.00	17048.41	17509.62	18059.59	17318.91
d_2	4119.50	4322.50	5785.38	26962.79	10297.54 **
d_3	36004.32	36774.44	37721.07	38820.40	37330.06
d_4	24671.41	25772.17	28846.19	52044.66	32833.61

2. Wald's Criterion

The Wald's criterion is an approach best summarizes as a pessimistic decision maker. Instead of maximin, minimax is applied since the idea is to minimize the cost. Hence, Plan 1 is the optimal decision for the city based on Wald's criterion. See Table IX.

TABLE IX

OPTIMAL PLAN (MARKED WITH **) BY WALD'S MINIMAX CRITERION

\$(L/s)	s1	s2	s3	s4	Maximum cost for each row
d_1	16658.00	17048.41	17509.62	18059.59	18059.59**
d_2	4119.50	4322.50	5785.38	26962.79	26962.79
d_3	36004.32	36774.44	37721.07	38820.40	38820.40
d_4	24671.41	25772.17	28846.19	52044.66	52044.66

3. Savage's Minimax Regret Criterion

Savage's regret criterion minimizes the probable regrets for decision maker. For the cost matrix, regret is calculated by $r_{ij} = a_{ij} - \min_{k=1, \dots, m} a_{kj}$ for all i, j , the regret matrix of this problem is presented in Table X. The optimal plan is Plan 2 according to this criterion.

4. The Pessimism- Optimism Index Criterion of Hurwicz

With the Hurwicz's criterion, the decision maker's attitude is between pessimistic and optimistic and measured by one optimistic coefficient $0 < \alpha < 1$. For the cost matrix, in each

row, a_i denote the smallest component and A_i the largest, then Hurwicz's measurement H_i is defined as:

$$H_i = \alpha a_i + (1 - \alpha)A_i \text{ where } i = 1, \dots, m.$$

the optimal plan is with $\min_i H_i$. Hence, Plan 1 is the optimal choice if $\alpha \leq 0.4152$ and Plan 2 is optimal if $\alpha > 0.4152$. See Table XI.

TABLE X
OPTIMAL PLAN (MARKED WITH **) BY SAVAGE'S MINIMAX REGRET CRITERION

\$(L/s)	s ₁	s ₂	s ₃	s ₄	Maximum regret for each row
d_1	12538.50	12725.91	11724.24	0	12725.91
d_2	0	0	0	8903.20	8903.20**
d_3	31884.82	32451.93	31935.69	20760.81	32451.93
d_4	20551.92	21449.66	23060.81	33985.15	33985.15

TABLE I
OPTIMAL PLAN (MARKED WITH **) BY HURWICZ'S CRITERION

	s ₁	s ₂	s ₃	s ₄	Hurwicz's measurement H_i
d_1	16658.00	17048.41	17509.62	18059.59	$18059.59 - 1401.59\alpha$ if $\alpha \leq 0.4152$
d_2	4119.50	4322.50	5785.38	26962.79	$26962.79 - 22843.29\alpha$ if $\alpha > 0.4152$
d_3	36004.32	36774.44	37721.07	38820.40	$38820.4 - 2816.08\alpha$
d_4	24671.41	25772.17	28846.19	52044.66	$52044.67 - 27373.25\alpha$

5. Starr's Domain Criterion

Starr's domain criterion computes the volume of the set D_i for each decision and chooses the decision with the highest volume, in this way, it actually selects the decision that is most likely to have a higher expected payoff value than all the others. In this example, apply Starr's criterion to a modified matrix which is cost matrix times minus one, the dimension of the decision matrix is 4×4 , the simulation with random sampling of points in the FPS is implemented to approximate the volume. The optimal plan chosen by this criterion is Plan 2, see Table XII.

TABLE XII

OPTIMAL PLAN (MARKED WITH **) BY STARR'S DOMAIN CRITERION

\$(L/s)	s ₁	s ₂	s ₃	s ₄	Domain for each decision
d_1	-16658.00	-17048.41	-17509.62	-18059.59	0.0368
d_2	-4119.50	-4322.50	-5785.38	-26962.79	0.4632**
d_3	-36004.32	-36774.44	-37721.07	-38820.40	0.0000
d_4	-24671.41	-25772.17	-28846.19	-52044.66	0.0000

6. Nash Equilibrium

Consider the city as player 1 and nature as player 2, DMUSU problem becomes a two-player game. The representation of the game is a matrix which shows players, strategies, and payoffs, while in this example only cost matrix is given. Hence, when apply NE in this example, consider a new matrix which is cost matrix times minus one. This new matrix indicates how much player 1 loses when taking each strategy. NE chooses Plan 1 with 100% probability. See Table

XIII.

TABLE XIII
OPTIMAL PLAN (MARKED WITH **) BY NE

\$(L/s)	s ₁	s ₂	s ₃	s ₄	NE
d_1	-16658.00	-17048.41	-17509.62	-18059.59	100%**
d_2	-4119.50	-4322.50	-5785.38	-26962.79	0
d_3	-36004.32	-36774.44	-37721.07	-38820.40	0
d_4	-24671.41	-25772.17	-28846.19	-52044.66	0

D. Discussion and Analysis

This section summarizes all the results by the different criteria in Table XIV.

TABLE XIV
RESULTS FOR ALL CRITERIA

Criterion	The suggested optimal plan
Laplace's principle of insufficient reason	P2
Wald's criterion	P1
Savage's Minimax regret criterion	P2
Hurwicz's criterion	P1, if $\alpha \leq 0.4152$ P2, if $\alpha > 0.4152$
Starr's Domain criterion	P2
Nash Equilibrium	P1

Table XIV shows that P2 is an optimal choice according to Laplace's, Savage's and Hurwicz if $\alpha > 0.4152$ and Starr's, while Wald's, NE and Hurwicz if $\alpha \leq 0.4152$ chooses P1. It is worth noting that alternative 2 is the main alternative selected, however most civil engineers intuitively rooted for alternative 3 from a purely city planning perspective. On the other hand, the fact that NE points toward P1 is a strong

argument for this alternative. As a reminder, a NE is a strategy such that regardless of the choice of one's opponent, there is no incentive to change one's strategy. In the other words, regardless of the state of nature, NE says that P1 is the best choice. This is a strong recommendation. The main drawback of the NE is the fact that it can recommend a mixed strategy (several alternatives with different probability). Decision maker can hardly cope with such a recommendation. However, in this specific case, the fact that it is 100% behind Plan 1 (i.e. a pure strategy) is reassuring for decision maker.

VI. CONCLUSION

DMUSU is a situation where decision maker needs to make decision without any information about the probabilities of the various states of nature. This paper presented a decision making process under strict uncertainty in sewer network planning. Laplace's principle of insufficient reason, Wald's criterion, Savage's Minimax regret criterion, Hurwicz's criterion and Starr's Domain criterion is introduced and compared. Furthermore, DMUSU problem is considered as a two-player game, and NE is used as well to find the optimal decision. While different criteria recommend different alternative, the fact that the NE is 100% behind alternative 1 is a strong argument to choose this one. While alternative 2 is the main alternative recommended, it is interesting to note that alternative 3 is not selected by any criteria, however most civil engineers intuitively rooted for alternative 3 from a purely city planning point of view.

As a future work, it is important to compare this approach on more projects to evaluate if a trend is emerging. Also, from a pragmatic point of view, we recommend to adapt the current decision process to include the comparison of these 5 criteria (and NE) to give a better depth to the decision. The next step is clearly to form a portfolio of decision policy and evaluate the robustness of such an approach compared to the individual criterion or the current decision process of the city.

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