Dynamic State Estimation with Optimal PMU and Conventional Measurements for Complete Observability

M. Ravindra, R. Srinivasa Rao

Abstract—This paper presents a Generalized Binary Integer Linear Programming (GBILP) method for optimal allocation of Phasor Measurement Units (PMUs) and to generate Dynamic State Estimation (DSE) solution with complete observability. The GBILP method is formulated with Zero Injection Bus (ZIB) constraints to reduce the number of locations for placement of PMUs in the case of normal and single line contingency. The integration of PMU and conventional measurements is modeled in DSE process to estimate accurate states of the system. To estimate the dynamic behavior of the power system with proposed method, load change up to 40% considered at a bus in the power system network. The proposed DSE method is compared with traditional Weighted Least Squares (WLS) state estimation method in presence of load changes to show the impact of PMU measurements. MATLAB simulations are carried out on IEEE 14, 30, 57, and 118 bus systems to prove the validity of the proposed approach.

Keywords—Observability, phasor measurement units, PMU, state estimation, dynamic state estimation, SCADA measurements, zero injection bus.

I. INTRODUCTION

THE introduction of synchrophasors into power system is one of the recent developments in the area of state estimation. Synchrophasors allow direct measurement of phase angles associated with current and voltages which are synchronized with time signals provided by Global Position Systems (GPS) [1], [2].

State estimation plays a vital role in real-time control of the power system and it is active in providing the security to the system [3], [4]. State vectors derived from the present commercial estimator algorithms are based on conventional measurements of real and reactive power flows and voltage magnitudes derived from Supervisory Control and Data Acquisition (SCADA) data. The algorithms used in state estimators are much affected with convergence problems affecting measurement accuracy in the power system. The WLS method have been widely used in real-time control centers worldwide, but without the proper formulation of WLS method it may lead to convergence and measurement uncertainty problems. One of the major problems in state estimation is the integration of PMU and conventional measurements of the bus network. Hybrid state estimation and fusion methods are discussed in [5]-[7]. There are two ways to integrate PMU measurements with conventional measurements in SE process [8], [9]:

1) A single step state estimator, in which PMU measurements are combined with conventional measurements.
2) A two-step scheme in which the state estimate obtained from traditional SCADA measurements is improved using the second state estimator followed by PMU measurements.

Three different methods of integrating PMU measurements with conventional measurements are discussed in [10]. But the transformation of the current phasors into real and imaginary quadrant leads to propagation of uncertainty measurements [11]. An error free estimation method with integration of PMU and conventional measurements is in need. But, PMU allocation on every bus of the system is not feasible as it leads to high cost investment. The first analysis of optimal PMU placement using Integer Linear Programming (ILP) approach had been developed by Abur [12]. Optimal multistage scheduling of PMU placement using the ILP approach is implemented in [13]. Optimal allocation with Genetic Algorithm (GA) requires performing mutation, crossover, and fitness functions which take much time for computation [14], [15]. For optimal allocation of PMUs at critical buses a Binary Integer programming method is used in [16]. All optimal allocation methods so far proposed, did not show the impact of optimal allocation of PMUs in state estimation process and did not describe how far they are more accurate compared to conventional measurements in the presence of load increase for complete observability.

In power system, state estimation method should have the capability of keeping the system observable during different contingencies besides detecting and identifying gross errors in the measurement set [17], [18]. However, this capability relies on PMU locations and redundant measurement set.

In this paper, a single step dynamic state estimator is established for estimating the state of the system. A bus connectivity matrix based on zero injection constraints is modeled considering GBILP method. The problem of optimal PMU placement is considered with modeling zero injection constraints, line contingency, cost factor and redundant measurements. The proposed DSE method with the optimal allocation of PMU is analyzed by comparing with the traditional state estimation method. Test case systems, i.e., 14-bus system and 57-bus systems are considered in analyzing the performance.

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II. CONVENTIONAL STATE ESTIMATION

A. WLS Estimation

The objective of state estimation is to determine the state of the system based on quantities that are measured. The most traditionally used method is WLS method [4]. Consider the set of l measurements given by a vector Z

$$Z = h(x) + r \tag{1}$$

where Z is $l \times 1$ measurement vector, x is $n \times 1$ state vector variables to be estimated, $h(x)$ is $l \times 1$ vector of the nonlinear function relating measurement vectors and state vectors, and r is a vector of measurement errors. Covariance matrix is denoted by $R = diag\{\sigma_1^2, \sigma_2^2......\sigma_l^2\}$. The standard deviation σ_i of each measurement is calculated to reflect the estimated accuracy of each measurement from the corresponding unit used.

The WLS state estimation is based on the minimization of the weighted sum of the squared residuals.

$$J(x) = [Z - h(x)]^T R^{-1} [Z - h(x)] \tag{2}$$

The above minimization problem is solved using Newton Raphson method, an iterative process in which each equation is solved.

$$G\Delta x = H^T R^{-1} \Delta Z \tag{3}$$

$G = H^T R^{-1} H$ is defined as Gain matrix, H is the Jacobian matrix of size $l \times n$, $h(x)$ is determined at a given point x^t

$$\Delta Z = Z - h(x^t) \tag{4}$$

The solution of (3) yields the vector Δx i.e., an increment to the states. As a result, the updated state vector is obtained as

$$x^{t+1} = x^t + \Delta x \tag{5}$$

The convergence of the iterative process is attained when Δx is smaller than pre-defined tolerance value.

III. PROBLEM FORMULATION

A. Optimal Allocation of PMUs

The problem of optimal PMU allocation for complete observability on considering redundancy and cost factor is presented as

$$Min \sum_{p=1}^{n} F_p x_p \tag{6}$$

Subjected to observability constraints

$$AX \geq b \tag{7}$$

$X = [x_1, x_2,.....x_p]$ is a binary decision variable matrix and defined as

$$x_p = \begin{cases} 1 & \text{if PMU is installed at bus p} \\ 0 & \text{otherwise} \end{cases}$$

A is bus incidence matrix formed from line connectivity data and defined as

$$A = \begin{cases} 1 & \text{if p = q or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

F is cost vector written as $[1\ 1\ 1\ 1....n]$ and b is vector of observability constraints defined as $[1\ 1\ 1\ 1....n]$.

B. Modeling of Inequality Constraints

While considering buses of the power system, the inequality associated with power flow and power injections need to be analyzed [18]. Let us consider the power flow measurements on the line $p\ q$. The inequality constraint associated with P and q buses is defined as

$$Z_p + Z_q \geq 1 \tag{8}$$

Let bus 'm'connects the bus l, p and q. Suppose an injection measurement is at bus 'm' then inequality constraints is defined as

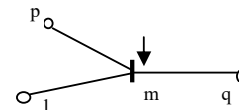$$Z_l + Z_p + Z_q + Z_m \geq 3 \tag{9}$$

Fig. 1 Buses p,q,l connected to injection bus m.

The power injection measurements and power flow measurements are associated such that inequalities are represented as

$$Z_l + Z_p + Z_q + Z_m - Z_p - Z_q \geq 3 - 1$$
$$Z_l + Z_m \geq 2 \tag{10}$$

Due to single injection I_m, the right-hand side of the equation is to be reduced by one. Thus the inequality constraints to be satisfied are

$$\begin{aligned} Z_l + Z_m &\geq 1 \\ Z_p + Z_q &\geq 1 \end{aligned} \tag{11}$$

C. Procedure for Formation of Bus Connectivity Matrix with Zero Injection Modeling for Optimal Allocation of PMUs

The bus to which neither generator nor load connected is considered as ZIB. With the modeling of ZIB constraints in the PMU placement problem, number of PMUs required for measurement can be reduced.

Step1. A vector of bus, including ZIBs and buses associated with ZIBs are represented as

$$Z_a = \begin{bmatrix} Z_b & Z_{as} \end{bmatrix}$$

Step2. Bus vector that are not associated with ZIBs are represented as a vector Z_{ns}

Step3. A new vector formed combining step (1) and (2) and written as

$$b_{in} = \begin{bmatrix} Z_{ns} & Z_a \end{bmatrix}$$

Step4. Permutation matrix P is established utilizing the vector b_{in}

Step5. Zero injection connectivity matrix Z_m with buses associated with ZIBs is established.

Step6. Zero injection constraint matrix is created as

$$Z_{con} = \begin{bmatrix} I & 0 \\ 0 & Z_m \end{bmatrix}$$

where I is Identity matrix, Z_m - matrix which represents buses associated with ZIBs.

Step7. The final connectivity matrix for optimal allocation is formulated as

$$Z_{pmu} = A * P * Z_{con}$$

A= binary connectivity matrix, P= permutation matrix, Z_{con} = zero injection constraint matrix

Step8. The constraint vector b_{con} is formed from number of constraints in the Z_{pmu} matrix to check observability.

Step9. Bus connectivity matrix Z_{pmu} and vector b_{con} are the constraints that are modeled for optimal allocation of PMUs.

The proposed objective function to minimize the number of PMUs can be formulated as

$$Min \sum_{p=1}^{n} F_p x_p \qquad (12)$$

Subjected to observability constraints

$$Z_{pmu} X \geq b_{con} \qquad (13)$$

Consider single line diagram of seven bus system shown in Fig. 2. Here, bus-2 is assumed as ZIB.
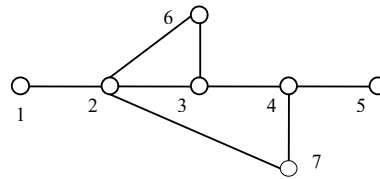

Fig. 2 Single line diagram of seven bus system

From the step 3 using the vector b_{in} permutation matrix for seven bus system can be formed as

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Zero injection constraint matrix is defined from step 6 as

$$Z_{con} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Binary connectivity matrix is represented as

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

From step 7 the final connectivity matrix obtained as

$$Z_{pmu} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 2 & 5 & 3 & 2 & 0 & 3 & 2 \end{bmatrix}$$

The constraint vector to check observability is obtained as $b_{con} = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$. Using Z_{pmu} and b_{con} constraints in minimization function, the optimal allocation of PMU obtained as $x = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ i.e., PMU is allocated at bus 4.

D. Performance of the System

To measure the performance of the optimization, we

propose Complete System Observability Redundancy Index (CSORI) which is defined as the sum of observability of each bus in the network of the power system.

$$CSORI = \sum_{p=1}^{n} \beta_p \qquad (14)$$

β_p is defined as Bus Observability Index (BOI) of pth bus.

Maximum BOI is limited to maximum connectivity (χ_i) of the bus plus one [13] i.e.

$$\beta_p \leq \chi_p + 1 \qquad (15)$$

For bus p, BOI is defined as no PMUs which can observe the given bus. Maximum redundancy of the bus can be defined as

$$Max \sum_{p=1}^{n} b_p{}^T A x_p \qquad (16)$$

Subjected to the following constraints

$$\sum_{p=1}^{n} x_p = \gamma_0 \,, \qquad (17)$$

$$AX \geq b \qquad (18)$$

where γ_0 is defined as the minimum number of PMUs obtained for complete system observability.

E. Modeling of Zero Injection Constraints with Single Line Contingency

In real-time control of power systems to enhance the system control, each bus of the system should be observed by at least two buses as it will not lead to loss of observability even with the single PMU outage. The single line contingency issue is a subset of PMU outage problems. The problem can be formulated as

$$Min \sum_{p=1}^{n} F_p x_p \qquad (19)$$

Subjected to observability constraints

$$AX \geq 2b \qquad (20)$$

F. Integration of PMU Measurements with Conventional Measurements

The state estimation problem involving measurements such as power injections and power flows is nonlinear. The formulation of the measurement function for DSE is as

$$Z_k = h_k(x) + r_k \qquad (21)$$

Z_k is measurement data, $h_k(x)$ is a nonlinear measurement function relating state vectors and r_k is a vector of residual errors. The formulated Jacobian matrix for WLS state estimation involving both conventional and PMU measurements is illustrated as

$$H_{PMU}(k) = \begin{bmatrix} \dfrac{\partial P_{in}}{\partial \theta} & \dfrac{\partial P_{in}}{\partial v} \\[2mm] \dfrac{\partial Q_{in}}{\partial \theta} & \dfrac{\partial Q_{in}}{\partial v} \\[2mm] \dfrac{\partial P_{fl}}{\partial \theta} & \dfrac{\partial P_{fl}}{\partial v} \\[2mm] \dfrac{\partial Q_{fl}}{\partial \theta} & \dfrac{\partial Q_{fl}}{\partial v} \\[2mm] 0 & \dfrac{\partial v_{meas}}{\partial v} \\[2mm] \dfrac{\partial \theta_{meas}}{\partial \theta} & 0 \\[2mm] \dfrac{\partial I_{rpmu}}{\partial \theta} & \dfrac{\partial I_{rpmu}}{\partial v} \\[2mm] \dfrac{\partial I_{ipmu}}{\partial \theta} & \dfrac{\partial I_{ipmu}}{\partial v} \end{bmatrix} \qquad (22)$$

where P_{in} and Q_{in} are active and reactive power injections, P_{fl} and Q_{fl} are active and reactive power flows, v_{meas} and θ_{meas} are Voltage magnitude and phase angles calculated from SCADA data, I_{rpmu} and I_{ipmu} are real and imaginary current magnitudes measured through PMU. The weights R_{kk} in the form of diagonal matrix are derived from the variance of the individual measurements as

$$\text{Standard deviation } \sigma = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(x_k - \hat{u})^2} \qquad (23)$$

Variance of the measurement is σ^2 where N is the total number of measurements, x_k is individual k^{th} measurement, and $\hat{u}$ is mean of the measurements. $R_{kk} = 1/\sigma^2$ are the weights of the measurements represented in the diagonal matrix. These weights are used in proposed DSE to obtain the accurate estimated value of the measurement.

G. Proposed DSE Algorithm

The proposed DSE can be formulated with iterative processes as

$$x^{t+1}(k+1) = x^t(k) + \Delta x \qquad (24)$$

$$\Delta x = (H_{pmu}{}^T R_{kk}{}^{-1} H_{pmu})^{-1} H_{pmu}{}^T R_{kk}{}^{-1}(Z - h(x^t)) \quad (25)$$

where x^t is a vector of state variables at t iteration. H_{pmu} is Jacobian matrix of PMU and conventional measurements, R_{ii} is measurement error covariance matrix, Z is measurement vector of PMU and conventional measurements, h is a vector of nonlinear measurement functions associated with state vectors. The DSE algorithm is shown in Fig. 3.
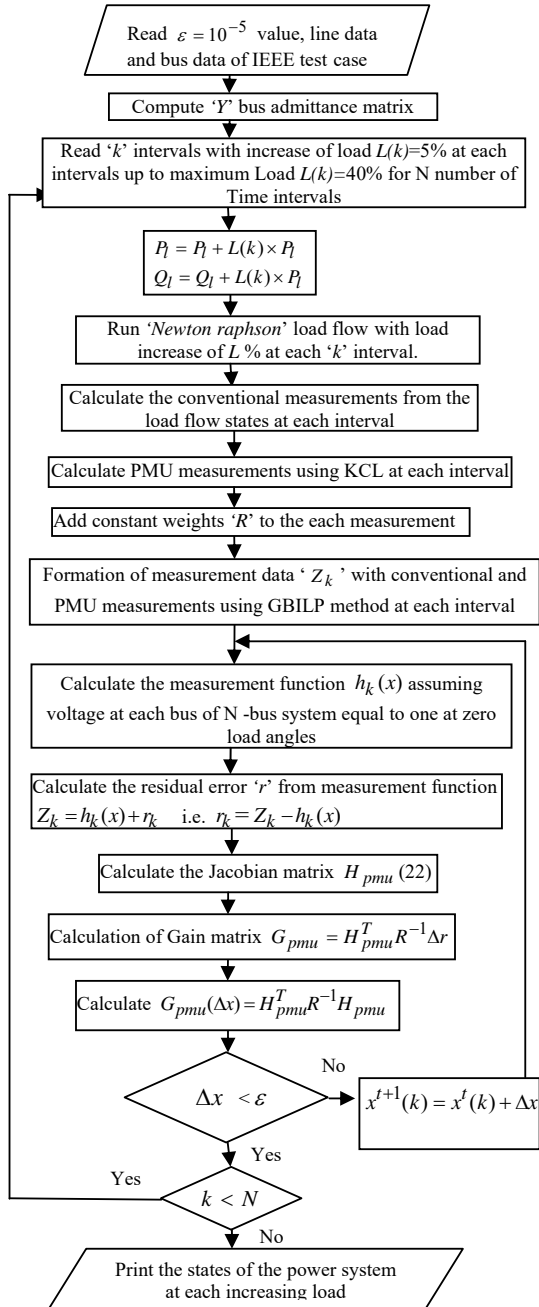

Fig. 3 DSE with Integration of Synchrophasors

The state variables, voltage and phase angle (V, θ)

generated by the NR load flow method are taken as actual states for comparing with the state estimation results.

To accurately estimate states of the system with integrated data (PMU and conventional measurements), the standard deviation of the measurements which are considered as weight of the measurements are included in state estimation formulation. Finally, the realistic measurement data (perturbed), measurement noise and nominal network parameters are inputs to WLS state estimator which results in output states (V_{est}, θ_{est}) that are compared with NR states to obtain state estimation error.

In state estimation procedure, to know the dynamic behavior of state estimation and performance accuracy of PMU measurements, we incremented load from 5% to 40% considering ten time intervals.

H. Performance of the Proposed DSE with Absolute Root Mean Square Error (ARMSE) Index

The performance and accuracy of the state estimation process are measured by the proposed ARMSE index.

$$\text{ARMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (x_{est}(k) - x_{meas}(k))^2} \qquad (26)$$

where x_{est} is state estimated by the DSE method, x_{meas} is the true state of the system obtained from the measurement and N is represented as a total number of buses in the system. As the scale of measurement is different for the two states, ARMSE is calculated separately.

The computational work is done under MATLAB/ Programming environment. The input constraints are modeled to generate a new binary connectivity matrix in GBILP framework. MATLAB programming is carried out using Intel(R) core(TM) i3 processor at 2.20GHz with 4GB of RAM.

IV. RESULTS AND ANALYSIS

A. Optimal Allocation of PMUs with ZIB Constraint Modeling

The ZIB locations for IEEE bus test case systems are shown in Table I.

TABLE I
ZIB LOCATIONS

IEEE test systems	No of ZIB buses	ZIB bus locations
14 bus	1	7
30 bus	6	6,9,22,25,27,28
57 bus	15	4,7,11,21,22,24,26,34,36,37,39.40,45,46,48
118 bus	10	5,9,30,37,38,63,64,68,71,81

Comparison of PMU locations with and without ZIB modeling in the ideal case for IEEE 14, 30, 57, 118 buses is shown in Table II. From Table II, it is observed that ZIB constraint modeling decreases the number of PMU locations in the IEEE bus network.

PMU locations, with and without ZIB modeling, for the single line contingency are shown in Table III. The number of PMUs required for measurement decreases with zero injection modeling. CSORI is a performance indicator of the quality of optimization. Comparison of CSORI with and without ZIB modeling for the single line contingency and no line contingency is shown in Table IV. The PMU placement with maximum CSORI is chosen for final placement with a particular number of allocations. The bus with maximum BOI is considered in optimization subjected to observability constraints of the system.

TABLE II
COMPARISON OF PMU LOCATIONS WITH AND WITHOUT ZIB MODELING

IEEE Test systems	Without ZIB Modeling		With ZIB Modeling	
	No. of PMUs	PMU locations	No. of PMUs	PMU locations
14bus	4	2,6,7,9	3	2,6,9
30 bus	10	1,7,9,10,12,18,24,25,27,28	7	1,7,10,12,19,24,27
57 bus	17	1,4,6,13,19,22,25,27,29,32,36,39,41,45,47,51,54	13	1,6,9,14,20,25,27,32,37,38,50,53,56
118 bus	32	3,7,9,11,12,17,21,25,28,34,37,41,45,53,56,62,63,68,70,71,76,79,85,86,89,92,96,100,105110,105,114	27	1,5,12,15,17,21,25,28,34,40,45,49,53,56,62,64,70,77,80,85,87,90,94,101,105,110,114

TABLE III
COMPARISON OF PMU LOCATIONS WITH AND WITHOUT ZIB MODELING FOR SINGLE LINE CONTINGENCY

IEEE Test systems	Without ZIB Modeling for Single Line Contingency		With ZIB Modeling for Single Line Contingency	
	No. of PMUs	PMU locations	No. of PMUs	PMU locations
14bus	9	2,4,5,6,7,8,9,11,13	7	2,3,5,6,9,11,13
30 bus	21	1,3,5,7,8,9,10,11,12,13,15,17,19,20,22,24,25,26,2829,30	16	2,3,4,7,10,12,13,15,17,19,20,21,24,25,27,29
57 bus	33	1,3,4,6,9,12,15,19,20,22,24,26,28,29,30,31,32,33,35,36,38,39,41,43,45,46,47,50,51,53,54,5,57	29	1,2,4,6,9,12,15,19,20,22,24,25,28,29,30,32,33,35,36,38,41,45,48,49,50,51,53,54,56
118 bus	68	2,3,5,7,9,10,11,12,15,17,19,21,22,24,25,26,27,29,31,32,34,36,37,40,42,44,45,46,49,52,53,56,57,58,59,62,64,65,67,68,70,71,73,75,77,79,80,84,85,86,87.89,91,92,94,96,100,102,105,107,109,110,111,112,115,116,117,118	61	1,2,5,6,11,12,15,17,18,20,21,23,27,29,30,31,32,35,36,37,41,,42,43,44,47,48,49,52,53,56,57,58,59,62,64,66,67,70,75,77,79,80,84,85,86,87.89,91,92,94,96,100,102,105,107,101,110,111,115,117,118

TABLE IV
CSORI FOR NO LINE CONTINGENCY AND SINGLE LINE CONTINGENCY

IEEE Test systems	No Line Contingency		Single Line Contingency	
	CSORI with Zero Injection Modeling	CSORI without Zero Injection Modeling	CSORI with Zero Injection Modeling	CSORI without Zero Injection Modeling
14 bus	15	19	30	39
30 bus	31	43	64	74
57 bus	57	67	117	127
118 bus	144	157	272	299

B. DSE with Optimal Allocation of PMUs

From the line data and bus data of the SCADA systems, true state variables are determined to perform NR load flow method. Traditional State estimation (WLS) is performed with the conventional measurements that are determined from the load flow analysis. In DSE process, a binary connectivity matrix is designed in GBILP method for optimal allocation of PMUs in the bus network. Integration of Conventional and PMU measurements in DSE process results accurate state variables. The convergence tolerance value of DSE algorithm is set to 10^{-5}. Weights for each measurement are formed from their respective variances in the case of traditional state estimation. In DSE, the weights with minor deviations are considered to obtain accurate measurements.

Standard deviations for power injections power flows, and voltage phasors are defined to 0.0001, 0.0064 and 0.0001. For current flows, it is set to 0.01.

Case 1:14-Bus System

In 14-bus system considered, bus-1 and 2 are generator buses and bus-3, 6 and 8 are connected to compensating devices. The total measurements utilized in 14-bus system for traditional state estimation and proposed DSE are

1. Conventional measurements:
- Power injections: 1, 2, 3, 4, 7, 8, 10, 11, 12, 14}
- Power flows: {(1,2),(3,2),(3,4),(4,2),(4,7),(4,9),(5,2),(5,4),(5,6),(6,13),(7,9),(11,6),(12,13)}
2. PMU measurements:
- Voltage phasors: {2,6,9}
- Current flows: {(2,1),(2,3),(2,4),(2,5),(6,5),(6,11),(6,12),(6,13),(9,4),(9,7),(9,10),(9,14),}

Single line diagram of 14-bus system with optimal PMU allocation is shown in Fig. 4.
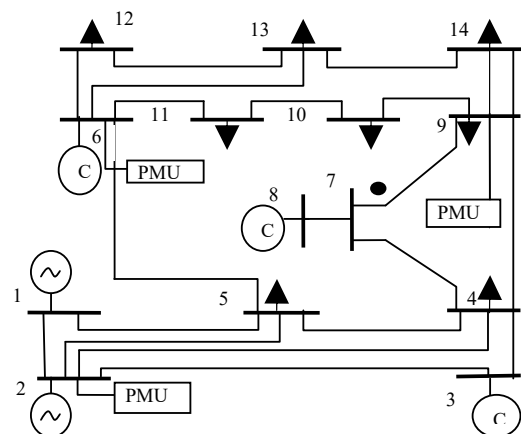

Fig. 4 Single line diagram of 14-bus system with PMU Allocation

To know the accuracy of the DSE, we have increased the load by 5% from the first interval to 40% at 8-interval. To estimate the voltage and phase angle error with load change,

we considered load increases up to 8 intervals and base load for 9 and 10 intervals. The MATLAB simulations results show the difference of error between traditional state estimation and DSE with optimal PMU allocation. Comparison of voltage magnitude error at 14-bus system considering with and without PMU is shown in Fig. 5. The voltage magnitude error is calculated as

$$V(k)_{err} = V(k)_{est} - V(k)_{meas} \qquad (27)$$

DSE with PMU gives error free measurements which are approximately equal to zero. Similarly, comparison of phase angle error plot with load change at bus-14 in 14-bus system is shown in Fig. 6. The phase angle error is calculated as

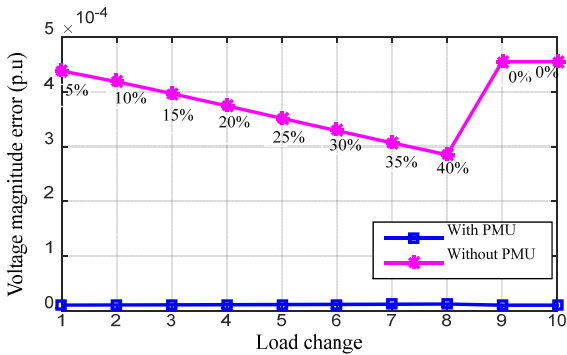$$\theta(k)_{err} = \theta(k)_{est} - \theta(k)_{meas} \qquad (28)$$

Fig. 5 Voltage magnitude error at 14 -bus considering with PMU and without PMU with increasing of load 5% at each interval
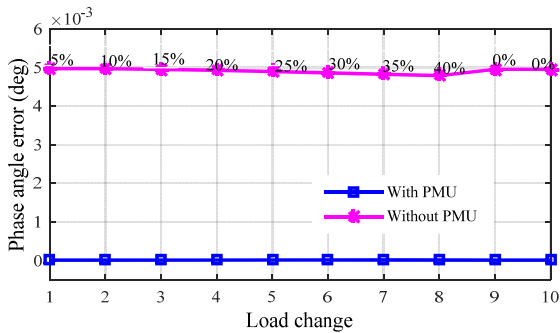

Fig. 6 Phase angle error at bus-9 considering with PMU and without PMU with increasing of load 5% at each interval

In the test case systems with compensating devices, the error decreases with the increase of load. The DSE with optimal PMU measurements provided the best accurate measurements showing the voltage and phase angle errors approximately equal to zero.

Case 2: 57-Bus System

In this power system bus-1, 2, 3, 6, 8, 9 and 12 are generator buses and no compensating devices connected. The total measurements utilized in 57- bus systems for traditional

state estimation and proposed DSE are
1. Conventional measurements:
▪ Power injections: {1,2,3,4,5,6,7,8,9,14,15,16,20,25,32,33, 37,40,50,53,56}
• Power flows: {(1,2),(2,3),(3,4),(3,15),(4,5),(4,18),(4,9),(5, 6),(6,7),(6,9),(7,29),(9,10),(10,12),(10,51),(11,13),(11,41), (11,4),(12,17),(13,49),(14,46),(15,45),(18,1),(19,20),(21, 22),(22,23),(23,24),(24,2),(24,25),(27,28),(28,29),(29,52), (30,3),(34,33),(34,35),(35,36),(36,40),(37,38),(37,39),(38, 48),(39,57),(41,42),(41,43),(44,45),(46,47),(47,48),(48, 49),(50,51),(52,53),(54, 55),(57,56)}
2. PMU measurements:
• Voltage phasors: {1,6,9,14,20,25,27,32,37,38,50,53,56} Current flows: {(1,2),(1,15),(1,16),(1,17),(6,4),(6,5),(6,7), (6,8),(9,10),(9,11),(9,12),(9,13),(9,55),(14,13),(14,15),(14, 46)(20,19),(20,21),(25,24),(25,30),(27,26),(27,28),(32,31) (32,33),(32,34),(37,36),(37,38),(37,39),(38,22),(38,37), (38,44),(38,48),(38,49),(50,49),(50,51),(53,52),(53,54), (56,57),(56,41),(56, 42), (56, 40)}
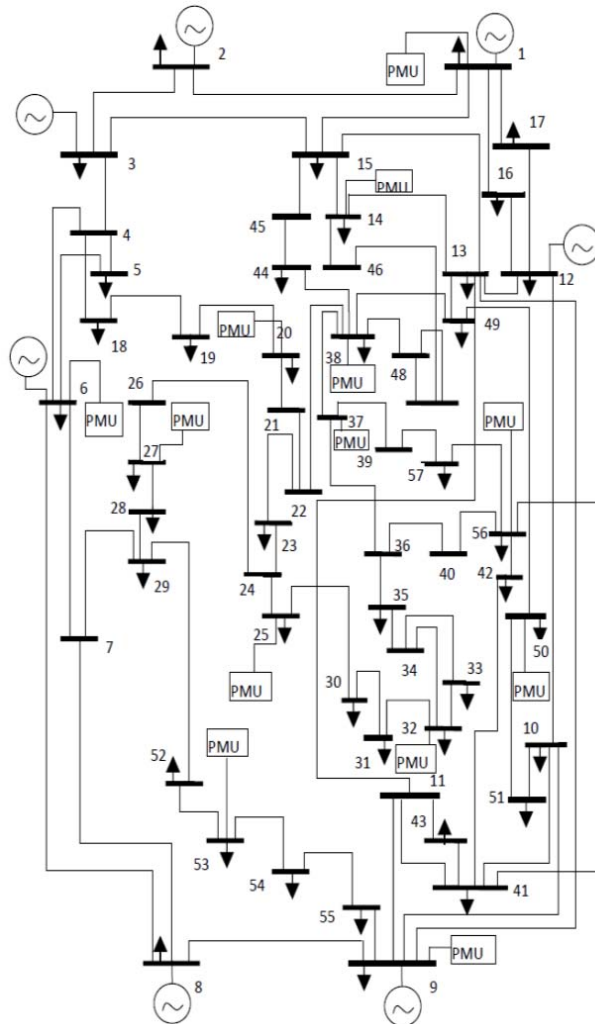

Fig. 7 Single line diagram of 57-bus system with optimal placement of PMUs

The single line diagram of 57-bus system with optimal PMU allocation is shown in Fig. 7. Fig. 8 and 9 show the voltage and phase angle error plot of bus-31 with load change. To estimate the voltage and phase angle error with load change in 57-bus system, we considered load increases up to 8 intervals and base load for 9 and 10 intervals. Fig. 9 shows the accuracy of PMU measurements compared to conventional measurements. From Fig. 9, it can be observed that with the load change, the error obtained with PMU is very less compared to conventional measurements.
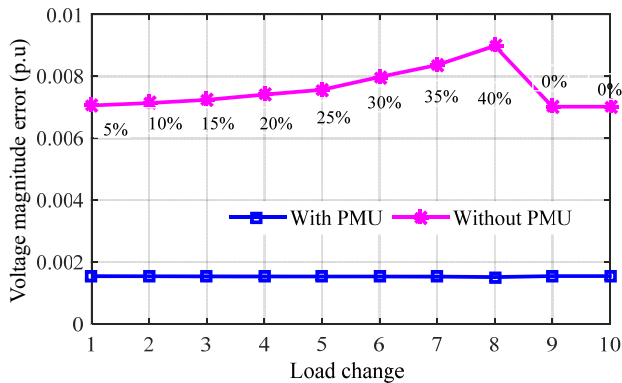

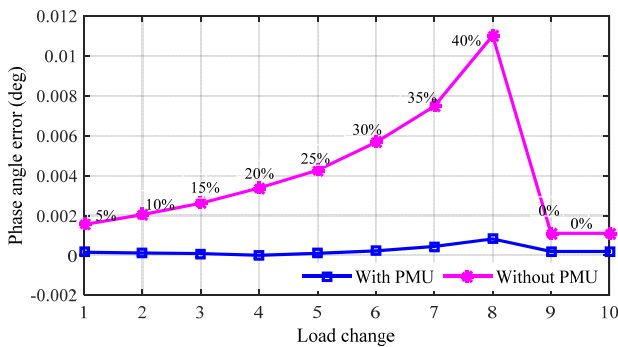Fig. 8 Voltage magnitude error at bus-31 with load change considering with PMU and without PMU for 57 bus system

Fig. 9 Phase angle error of bus-31 with load change considering with PMU and without PMU for 57- bus system

C. Performance Analysis and Discussion

TABLE VI
ARMSE INDEX FOR 14 AND 57 BUS SYSTEMS

IEEE Test System	Traditional State Estimation		Proposed DSE method	
	V	θ	V	θ
14 bus	6×10^{-4}	47×10^{-4}	1.5319×10^{-5}	8.2962×10^{-6}
57 bus	7×10^{-3}	11×10^{-4}	5.1905×10^{-4}	25093×10^{-5}

The performance of the state estimation algorithm with PMU allocation is effective for accurate measurement of states in the power network. The threshold value of state estimation accuracy is of order 10^{-6}. Performance Index ARMSE (26) of the state estimation is shown in Table VI. From the results, it is observed that the accuracy of state estimation remains below a threshold value.

From Table VI, it can be observed that the error obtained by integration of PMU measurements is very less compared to the traditional state estimation method (WLS method). The Comparison of Proposed GBILP approach with other methods is shown in Table VII.

TABLE VII
COMPARISON OF PROPOSED GBILP METHOD WITH OTHER METHODS FOR MINIMUM PMU ALLOCATIONS IN THE BUS SYSTEM

Optimization Methods	14-Bus System	30-Bus System	57-Bus System	118 -Bus System
ILP [13]	3	-	14	-
Unified Approach BILP [19]	4	7	13	29
GILP [20]	4	10	17	-
Integer Quadratic [21]	4	10	17	32
Binary Search [22]	3	7	-	-
Proposed Generalized BILP	3	7	13	27

From Table VII, it is observed that the application of ZIB modeling with GBILP approach reduces locations of PMUs compared to the optimal methods proposed in [19]-[22].

V. CONCLUSION

A DSE method is formulated with optimal PMU and conventional measurements to estimate measurement errors in the system with complete observability. A binary connectivity matrix is designed with modeling of ZIB constraints with GBILP method for optimal allocation of PMUs. The GBILP method with ZIB constraint modeling is able to minimize the number of locations for placement of PMUs in the network. The performance of the GBILP method is determined with CSORI in case of single line and no line contingency in the power system. The measurement impact of PMUs in DSE with an increase of load at each interval can be observed. The performance of the estimation is measured with proposed Absolute Root Mean Square Error. The error obtained with proposed DSE is very small compared to traditional state estimation. DSE with optimal PMUs gives many accurate results.

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