

# Nuclear Fuel Safety Threshold Determined by Logistic Regression Plus Uncertainty

D. S. Gomes, A. T. Silva

**Abstract**—Analysis of the uncertainty quantification related to nuclear safety margins applied to the nuclear reactor is an important concept to prevent future radioactive accidents. The nuclear fuel performance code may involve the tolerance level determined by traditional deterministic models producing acceptable results at burn cycles under 62 GWd/MTU. The behavior of nuclear fuel can simulate applying a series of material properties under irradiation and physics models to calculate the safety limits. In this study, theoretical predictions of nuclear fuel failure under transient conditions investigate extended radiation cycles at 75 GWd/MTU, considering the behavior of fuel rods in light-water reactors under reactivity accident conditions. The fuel pellet can melt due to the quick increase of reactivity during a transient. Large power excursions in the reactor are the subject of interest bringing to a treatment that is known as the Fuchs-Hansen model. The point kinetic neutron equations show similar characteristics of non-linear differential equations. In this investigation, the multivariate logistic regression is employed to a probabilistic forecast of fuel failure. A comparison of computational simulation and experimental results was acceptable. The experiments carried out use the pre-irradiated fuels rods subjected to a rapid energy pulse which exhibits the same behavior during a nuclear accident. The propagation of uncertainty utilizes the Wilk's formulation. The variables chosen as essential to failure prediction were the fuel burnup, the applied peak power, the pulse width, the oxidation layer thickness, and the cladding type.

**Keywords**—Logistic regression, reactivity-initiated accident, safety margins, uncertainty propagation.

## I. INTRODUCTION

THIS study verifies the safety thresholds based on a few parameters capable of predicting a possible failure in nuclear fuels, under a reactivity accident conditions. Logistic regression approach can compute the fuel failure probability in burn cycles up 70 GWd/MTU. Historically, the first nuclear reactor accident occurred at the National Research Experimental (NRX) reactor at Chalk River, Canada, on December 12, 1952 [1]. The NRX event involved a partial core meltdown of the reactor. Public opinion on nuclear energy suffered further long-term setbacks as a result of the catastrophe such as the Chernobyl incident on April 26, 1986 (Ukraine), and the Fukushima disaster in 2011 (Japan). Two facts involving the most severe type of accident occurred in the late 1950s and early 1960s, i.e. the Windscale fire of October 10, 1957 (northwest England). The partial core

meltdown of the SL-1 reactor located at Idaho Falls (US) occurred on January 3, 1961. These cases are examples of reactivity-initiated accidents (RIAs). The alert was the Fukushima nuclear disaster on March 11, 2011. The consequences of the Chernobyl event clearly exceeded those of the Fukushima hazard [2]. Safety-related research efforts have been focused on Accident-tolerant Fuel (ATF) program, and on developing an alternative to replace the zirconium alloys currently used in fuel cladding. The investigated fuels, in the ATF program had the focus in materials as iron alloys and silicon carbide (SiC). The preferred options are the metallic alloys formed of iron-chromium-aluminum (FeCrAl) based on their excellent oxidation resistance at high temperatures up to 1475 °C [3]. Evaluating objectives for ATF encloses higher power ratings, extended irradiation cycles, and low cost. Therefore, it must improve the safety margins and reliability. Following the Fukushima-Daiichi intensified safety precautions. The research was directed towards a fuel with enhanced thermal conductivity, higher density, and cladding with high-performance [4].

### A. Best-Estimate Models

The quantification of uncertainty made it necessary to avoid accident risk. The licensing rules used in nuclear units may improve the security margins. Since 2009, the rules planned by International Atomic Energy Agency (IAEA) endorse the best estimate (BE) methods. The IAEA proposed the BEPU (Best Estimate Plus Uncertainty) [5]. In 2011, NEA (Nuclear Energy Agency) introduced the BEMUSE (Best Estimate Methods-Incertitude and Sensitivity Evaluation) [6]. The new directions are assuming novel computational tools, to the uncertainty quantification (UQ) and sensitivity analysis (SA). The thermal-hydraulic (TH) safety codes suggested to licensing show uncertainty models. The TH systems show better results for Large Break Loss Of Coolant-Accident (LBLOCA). Mechanical models produce inaccurate responses which were the key to use high order statistic with non-parametric methods. The new methodologies include UQ as BESUAM (Best-Estimate Plus Statistical Uncertainty Analysis Method). The BESUAM approach is a bootstrap method started in 2010. The Bayesian approach calculates through Wilks' formula the minimum number of full simulation to reach an accuracy of 95-percentile. The peak cladding temperature (PCT) reaches the 95% confidence level (95/95). The physical models create new techniques to measure uncertainties as PREMIUM (Post-BEMUSE Reflood Model Input Uncertainty Methods) [7]. The TH codes together with PREMIUM framework can predict the PCT response based on sensitivity analysis. The PREMIUM base was the

D. S. Gomes is with Institute of Energy and Nuclear Research (IPEN/CNEN-SP), Nuclear Engineering Center (CEN), SP, Brazil (phone: 55-11-3133-9447, fax: 55 11 3133 9018, e-mail: dsomes@ipen.br).

A. T. Silva is with Institute of Energy and Nuclear Research (IPEN/CNEN-SP), Nuclear Engineering Center (CEN), SP, Brazil (phone: 55-11-3133-9459, fax: 55 11 3133 9018, e-mail: teixeira@ipen.br).

simulation of reflooding phase utilized in LBLOCA scenarios. The uncertainty evolution also is applied to normal operation and transient conditions.

## II. REACTIVITY INITIATED ACCIDENT

### A. Theoretical Background

In the last decade, international RIA test programs establish at least two safety correlations among the available data on nuclear fuel performance. In 2001, Halden Project derived a relationship based on tests carried out at CABRI reactor [8]. Nam et al. proposed, in the Korean Atomic Energy Institute (KAERI), a statistical model incorporating a simple enthalpy correlation to predict failure limits [9]. The examination of the fuel failure is the starting point according to results derived from RIAs accidents' benchmarks. In the correlations determined by Vitanza [8] and Nam et al. [9], the extensions of the burnup cycle were below of 62 GWd/MTU. Since the mid-1990s, high-burnup programs have been dedicated to investigating the RIAs consequences, conducted in France and Japan. In general, it determined that cladding rupture might appear on the case that exhibits loss of ductility and toughness. The test of type RIAs growth on a global scale totaling over thousand experiments carried out, although the facilities employed did not count with any standardization.. The first units began operating in 1958, located in Idaho was Transient Reactor Test Facility (TREAT). The Sandia National Laboratories build a fast-pulsed unit, as the Annular Core Research Reactor (ACRR) facility.

In Japan, the tests utilized The Nuclear Safety Research Reactor (NSSR), supported by the Japan Atomic Energy Agency (JAEA). In France, the studies began in the sodium loop test carried out at CABRI reactor. In Russia at Semipalatinsk, tests were performed using the Impulse Graphite Reactor (IGR, Kazakhstan). One of the nuclear facilities used for tests of insertion of reactivity in Russia was Fast Impulse Graphite Reactor (BIGR) [10]. In France, research programs were focus mainly on the mechanical effects and penalties to nuclear fuels under RIAs conditions, in 2007. The PATRICIA was French program focused in thermal-hydraulic response and PROMETRA aimed efforts in the loss of ductility and hydride concentration. The PATRICIA investigated heat transfers between the cladding and the coolant during RIA [11]. The PROMETRA series were the mechanical tests performed under conditions of the plane strain deformation practiced in the transverse direction of the cladding. The research program investigated the effect of hydrogen diffusion in material and oxide spallation [12].

### B. Failure Mechanism

The observed relationships developed about RIA failure thresholds show some physical restraints for immediate application. In fuel performance tests, there exist several dependences integrated as the coolant temperature and pressure. However, the RIAs were series performed using water coolant system and long burns cycles, creating benchmarks with restrict conditions. The data analysis can

predict the physical limits of fuel rods in Light-Water Reactors (LWRs) [13]. The stress analysis of the fuel rod calculates the elastic responses, besides of the plastic deformations developed, in the normal and off-normal operations. In the gap, closure had the solid contact reinforcing the pellet-cladding mechanical interaction (PCMI). Usually, the gap closes for burnup values beyond 40 GWd/MTU, with material contact. The PCMI constitutes one of the primary causes of fuel collapse during a rapid power increase. Under transient events, the gap closes rapidly with hard contact between the pellet outer surface and the cladding inner surface.

Two parallel modes that might define the fuel rod failure, due to the enthalpy inserted, are conclusions formalized from the experimental RIAs tests. The loss of cladding properties is an alteration that depends on burnup. If the preservation of the cladding ductility was at high temperatures, failure must happen during the Departure from Nucleating Boiling (DNB) occurrence. The fast temperature increases are resulting from the boiling heat transfer and two-phase flow. In extended cycles, the cladding has the ductility reduced, and the mechanical stress combined with the chemical interaction accelerates the fuel degradation. Hydrogen uptake from 200 to 800 ppm is standard for fuel burn values of approximately 60 GWd/MTU [14]. Under harsh circumstances, a small energy increase in the fuel can lead to failure. Thus, the corrosion of the cladding coupled with hydride deposition can produce the loss of the mechanical properties, and this is considered as the initial point to occur a failure in the cladding. However, it is shown that no cladding crack produced the oxide layer spalling up to 80- $\mu$ m thickness. If the oxide layer formed is below of the limit, then it preserves the sufficient ductility to avoid the cladding failure.

### C. Analysis of Fuel Rod Failure

The classification of the rod failure threshold is a function of the minimum amount of energy deposited in the fuel. The database produced over the years has more than 2000 experiments performed using pulsed reactors. Under RIA conditions, we look for security margins given as a function of fuel parameters [15]. Cladding corrosion at high temperatures produces a fast degradation of the material properties. The oxidation drives the hydrogen uptake. During prolonged burn cycles, gaseous emissions increase, typically (Kr-85) and (Xe-133). The fission gas release produces the fuel swelling and contributes to pellet gap closure. The damages caused by irradiation may accelerate the rupture of cladding because of the hydride absorption. In Japan, the NSRR program carried out tests using average peak fuel enthalpy of 150 cal/g [16]. The possible failure correlation is a function of oxidation kinetics coupled with hydride uptake at high temperatures. However, if the internal rod pressure exceeds the coolant pressure by more than 1 MPa, also failure can occur below of 150 cal/g. At intermediate and high burnup regime, the limit defined to radial average peak enthalpy combined with PCMI. The performance code uses the integrity model based on critical strain energy density (CSED). The burnup causes the lost mechanical properties, which is a function of hydrogen

content and corrosion calculated by CSED. The importance of oxide thickness limit of 80  $\mu\text{m}$  was deemed necessary to maintain sufficient ductility to prevent failure.

#### D. Fuchs Hansen Model

The tests of reactivity insertion have the behavior defined by nonlinear systems. The modeling describes the dynamic nuclear fission analysis using the Point-Kinetics Reactor Equation (PKRE) [17]. The enthalpy pulses have a narrow width, because of the effect of the fast energy insertion. Note that, if any reactor enters the critical state, this can lead to rod failure. The Fuchs-Hansen (FH) model is an adiabatic solution to the PKRE. The FH model can solve the system composed by integro-differential equation (IDE) that used six precursor-delayed neutrons. There are different manners to solve the PKRE as low-order Taylor's series expansions of neutron density (ND), or an estimating (ND) using the Lagrange interpolation polynomials [18]. The response must calculate the effects of the peak fuel enthalpy inserted by pulse reactors. The Germany research reactor TRIGA at Johannes Gutenberg University Mainz, the pulse energy can reach values of 250 MWs produce reactivity insertion of approximately 2\$, with the pulse width, was about 30 ms [19]. These values depend on the used test conditions. The reactor power  $P(t)$  and pulse width defined by the full-width half maximum (FWHM) functions respecting the reactivity insertion are expressed in (1), the reactor power density as time function using (2).

$$\tau_{FWHM} = \frac{3.5255 \Lambda}{\Delta\rho - \beta} \quad (1)$$

$$P(t) = P_{\max} \operatorname{sech}^2\left(\frac{(\Delta\rho - \beta)(t - t_{\max})}{2\Lambda}\right) \quad (2)$$

where  $P_{\max}$  represents the maximum power;  $t_{\max}$  is the pulse width (time),  $\beta$  is the fraction of delayed neutrons (0.006502). The step-like reactivity insertion is given by  $\Delta\rho$ , and  $\Lambda$  is the useful neutron lifetime of approximately  $10^{-7}$  s [20]. The modeling regarding to enthalpy pulse, adopt the FWHM approach. The events developed may represent the deep loss of performance during the fast transient. The effects combined represent the poor thermal and mechanical responses of the zirconium alloys. The energy quickly inserted during the transient is one of the key factors for understanding the failure mechanism.

#### E. The RIA Acceptance Criteria

The U.S. NRC (Nuclear Regulatory Commission) over the years establish the basis for safety operations. In 1974, this authority defined the rules for the emergency core cooling systems named (10CFR50.46). In the same epoch, the RIA tests reported concerning total energy deposition on fuel. The USNRC regulations define the security margins regarding the concept of fuel enthalpy inserted. Despite the security thresholds early, they were conservative and obtained from test performed with lower burnup. The original, cladding failure limit relates to a radial average enthalpy peak of 280

cal/g at any axial node. However, melting in unirradiated  $\text{UO}_2$  begins at 267 cal/g. Another threshold was of 230 cal/g [21]. In the French tests executed at Cadarache, a small number of failures occurring at limits of 170 cal/g and 140 cal/g using irradiated fuels were reported. The mechanical contact might establish a low threshold of 140 cal/g. [22], [23].

The interim acceptance criteria recommended a limit for fuel enthalpy greater than 170 cal/g radially for Boiling Water Reactor (BWRs). At low burnup with an internal rod pressure at or below system, pressure accepts the limit of 150 cal/g. Regarding safety limits, the majority of analyses accept a safety margin for the peak fuel enthalpy based on low and intermediate burnup cycle extension. The insertion energy limit is a function of the fuel rod degradation level. The revisions of these criteria depend on the magnitude of the burn cycles and alloys that will be proposed by ATF program. A partial assumption is that the fuel failure threshold is a function of the average rod burnup.

#### F. Logistic Regression Model

The method adopted work with binary values is given as function of the dependent variables. In general, the regression consists of a parametric model, as the probit analysis, log-linear, and logistic regression (LR), so called of the logit model [24]. Safety nuclear is a case of distribution free; it may employ non-parametric models that are similar with the nuclear problems. There are other approaches as discriminant analysis also classified as non-parametric models; The LRs may estimate the probability of a given response for two values, varying between zero and one. The LRs measures the relationship between a categorical variable having a binary value [25]. A group of independent variables is strategic to fuel failure response. The selected predictors must indicate the most sensitivities from the physical properties. Therefore, predictors include the burnup level, oxidation, the deposited enthalpy peak, and the pulse width. In the first step, numerical coefficients for multivariate regression can be found, based on an LR that predicting the fuel performance. The fuel behavior predicts the safety limits employed LR, based on experimental data. The objective consists of improving fuel management under long burn cycles. In the case of a multinomial regression, the model  $p$  is the probability of the occurrence of fuel failure. The failure probabilities were calculated by using (3) or (4):

$$\log\left(\frac{p}{1-p}\right) = b + b_1x_1 + \dots + b_nx_n \quad (3)$$

$$p = \frac{\exp(b_0 + b_1x_1 + \dots + b_nx_n)}{1 + \exp(b_0 + b_1x_1 + \dots + b_nx_n)} \quad (4)$$

The LRs model have a few similarities with linear regression methods. The LR attributes to each regressor, coefficients  $b_n$ , which measures the independent variable contribution to deviations in the dependent parameters. The model does not need to determine the precise numerical value

of a subordinate variable from knowledge of the relevant free variables. The logistic fitting can obtain the likelihood of failure based on few parameters.

### III. EXPERIMENTAL PROGRAMS

Determining fuel failure thresholds is a technological challenge researched to an extended burnup of the nuclear fuel rods. During the 1980s, the expansion of burn cycles to 40 GWd/MTU necessitated global efforts to ensure safe operation occurred. In the 1990s, the experimental programs started to high-burnup cycles verifying that fuel failure could occur at 65-70 cal/g, in burn cycles around 60 GWd/MTU. The actual USNRC limit is of 62 GWd/MTU. Over the last three decades, extensive testing of high-burnup fuel rods was performed. The experiments initially conducted in the United States and continued in France, Japan, and Russia during the 1990s.

#### A. Special Excursion Reactor Test

A total of 10-rod tests were conducted as part of the Special Power Excursion Reactor Test (SPERT) at Scoville near Idaho. The fuel rods were BWR-type fuel rods manufactured by General Electric. The tests were conducted practically with fresh fuel. In Table I, list the data from the experiments carried out at SPERT using a design featuring a Capsule Driver Core (CDC).

Test	Burnup (GWd/MTU)	Oxide ( $\mu\text{m}$ )	Enthalpy (cal/g)	Pulse Width (ms)	Fail
CDC-567	3.1	0	214	18	Yes
CDC-568	3.8	0	161	24	Yes
CDC-569	4.1	0	282	14	Yes
CDC-571	4.6	0	137	31	No
CDC-684	13	0	170	20	No
CDC-685	13	0	158	23	No
CDC-703	1.1	0	163	15	No
CDC-709	1.0	0	202	13	Yes
CDC-756	32	65	143	17	Yes
CDC-859	32	65	154	16	Yes

The average burnup values were in the 1-32 GWd/MTU range, using a stagnant coolant water with a temperature of 19.85 °C and a pressure of 0.1 MPa. The fuel rods had an active length of 132 mm, and the oxide layer varied from zero to 65  $\mu\text{m}$ . The pulse width ranged from 13 to 31 ms, where the average fuel enthalpy ranged from 137 to 282 cal/g. The lowest level reported failure enthalpy was 85 cal/g.

#### B. Power Burst Facility Tests

The data obtained from a second facility using fresh fuel, namely, the Power Burst Facility (PBF). The test conducted in Idaho National Laboratory Engineering at Scoville and sponsored by the NRC. The early RIA experiments performed in programs as PBF and SPERT used unirradiated fuel rods or low burnup. In PBF had three series of RIA experiments involving 17 fuel-rod-type PWRs. The water was employed as the coolant, at 284.85 °C, with a flowing water pressure of 6.45 MPa. PBF was the first facility capable of facilitating

loss-of-coolant transient research. The used fuel rod enrichment was 4.7 to 5.8 %, and the rods were 900 mm long. The experiments utilized the pre-irradiated fuels at Saxton (a PWR reactor) limited to 6.1 GWd/MTU. The fuel rods exhibit an active length of 150 mm. The oxide layers were 5- $\mu\text{m}$  thick. The absorption average hydrogen content was approximately 40 ppm. The 801 series fuel rods, exhibited fuel fragmentation and dispersal. The 804 series showed a failure limit of 185 cal/g with PCMI. The pulse width varied from 11 to 16 ms FWHM. Table II shows data from experiments conducted at PBF [26].

TABLE II  
PBF - PWR FUEL ROD TEST PERFORMED (1978-1980)

Test	Burnup (GWd/MTU)	Oxide ( $\mu\text{m}$ )	Enthalpy (cal/g)	Pulse Width (ms)	Fail
801-1	4.6	5.0	285	13	No
801-2	4.7	5.0	285	13	No
801-3	0.0	0.0	285	13	No
801-4	0.0	0.0	285	13	No
802-1	5.2	5.0	185	16	No
802-2	5.1	5.0	185	16	Yes
802-3	4.4	5.0	32	16	No
802-4	4.5	5.0	255	16	Yes
804-1	6.1	5.0	234	11	Yes
804-3	5.5	5.0	255	11	Yes
804-4	5.0	5.0	253	11	Yes
804-5	5.5	5.0	255	11	Yes
804-6	5.1	5.0	253	11	Yes
804-7	5.9	5.0	253	11	Yes
804-8	4.7	5.0	255	11	Yes
804-9	5.7	5.0	253	11	Yes
804-10	4.4	5.0	255	11	Yes

#### C. Characteristic and Result of CABRI Test

TABLE III  
CABRI TESTS ON PWR FUEL TEST - PERFORMED (1993-2002)

Test	Burnup (GWd/MTU)	Oxide ( $\mu\text{m}$ )	Enthalpy (cal/g)	Pulse Width (ms)	Fail
NA-1	64	80	30	9.5	Yes
NA-2	33	4	199	9.5	No
NA-3	54	40	124	9.5	No
NA-4	62	80	95	76	No
NA-5	64	20	76	8.8	No
NA-6	47	40	8.8	32	No
NA-7	55	50	32	40	Yes
NA-8	60	130	40	75	Yes
NA-9	28	20	75	33	No
NA-10	63	80	33	31	Yes
NA-11	63	15	31	31	No
NA-12	65	80	63	63	No
CIP0-1	75	80	32	32	No
CIP0-2	77	20	28	28	No

In France, conducted RIA experiments were sponsored the Nuclear Safety and Protection Institute (IPSN, now IRSN) located at Cadarache. Tests were performed at CABRI reactor. The facility comprises of a pool-type nuclear reactor. The first phase of the CABRI experiments involved a sodium loop coolant with 14 trials. The sodium coolant test was performed the temperature of 280 °C. Pulse widths of 9-75 ms, FWHM,

were applied, and the resultant burnup ranged from 33 to 77 GWd/MTU. Table III shows CABRI tests [27].

The irradiated fuel showed oxidation with a layer thickness of 4-126  $\mu\text{m}$ . The CABRI sodium-loop tests involved a peak fuel enthalpy ranging from 88 to 199 cal/g, with lower failure occurring in the 22-36 cal/g range. The CABRI International Project (CIP) began in March 2000, where tests conducted on high burnup PWR fuel. In 2002, the second stage of trials investigated the rods denoted CIPO-1 and CIPO-2, using Zirlo and M5 cladding. CIP also carried out eight experiments with  $\text{UO}_2$  fuel, and four tests used a mixture oxide fuel (MOX) fuel, which was pre-irradiated from 28 to 65 GWd/MTU.

#### D. Nuclear Safety Research Reactor Experiments

In Japan, the Nuclear Safety Research Reactor (NSRR) carried out over thousand RIA tests. The series conducted with PWR fuel, high-burnup were divided into MH, GK, OI, and HBO performed in the NSRR. The failure occurred for the HBO-1 and HBO-5 rods in Table IV.

TABLE IV  
NSRR TESTS (1991-2000)

Test	BU (GWd/MTU)	Oxide ( $\mu\text{m}$ )	Enthalpy (cal/g)	Pulse Width (ms)	Fail
TS-1	26	6	55	6.0	No
TS-2	26	6	66	5.3	No
TS-3	26	6	88	4.8	No
TS-4	26	6	89	4.6	No
TS-5	26	6	98	4.4	No
FK-1	45	16	130	4.4	No
FK-2	45	19	70	6.5	No
FK-3	41	24	145	4.4	No
FK-4	46	22	140	4.3	No
FK-5	56	22	140	7.3	No
FK-6	61	25	70	4.4	Yes
FK-7	61	25	70	4.4	Yes
FK-8	61	25	62	7.3	No
FK-9	61	25	86	5.7	Yes
FK-10	61	25	80	5.1	Yes
FK-12	61	25	72	5.8	Yes

The MH and GK series mechanical designs show (14 x14) fuel rod arrays, while those for OI and HBO is (17x17) fuel rod arrays. A series of 15 PWR fuel experiments carried out exhibiting burnup values ranging from 39 to 50 GWd/MTU. The burn cycles reached 61 GWd/MTU with a cladding oxide thickness of 5-60  $\mu\text{m}$ . The pulse width ranged from 4.4 to 6.9 ms, and the applied radial peak fuel enthalpy of 37-108 cal/g. However, regulatory agencies have intentions of the irradiation cycles up to 75 GWd/MTU. The NSRR experiments are performed using fuel rod with medium radiation cycle. Under RIA conditions, the high- burnup fuel simulated in the NSRR and CABRI test show similar responses [28].

#### E. Fuel Performance Code

The USNRC recommends a series of nuclear codes to verify the safety limit defined as the licensing process. During a permanent state, calculate the standard operation response to

burn cycle limiting to 62 GWd/MTU. The simulation method uses the FRAPCON (A Computer Code for the Calculation of Steady-State, Thermal-Mechanical Behaviour of Oxide Fuel Rods for High Burnup). The FRAPCON code is set to simulate the experiments in the permanent state [29]. The enthalpy pulse can be simulated with the FRAPTRAN (The Fuel Rod Analysis Program Transient) [30]. In the initial stage of irradiation, the FRAPCON system produces a file that becomes an input for the transient stage, estimated by FRAPTRAN, replicating RIAs scenarios. The fuel codes FRAPTRAN and FRAPCON must be coupled to produce the results. Both systems work with the same library of physical properties, defined for the light-water reactor featured as a package of material properties. The library is composed of modular subroutines.

#### IV. LOGISTIC REGRESSION CORRELATIONS

The results of the various experiments described to constitute the database used to find the peak fuel enthalpy. For zirconium-based alloys, the regression determines the safety threshold as a function of burnup where  $H$  is the fuel enthalpy (cal/g),  $Bu$  is the burnup (GWd/MTU). Further, if  $q$  is the chance of collapse, we can define the probability as a function of burnup, oxide, pulse width, enthalpy, and cladding type. If a lower burnup than 75 GWd/MTU occurs, the curve fitting for enthalpy peak given as a function of burnup, if the burnup is greater than 40 GWd/MTU, can be calculated by using (5). The results are plotted in Fig. 1.

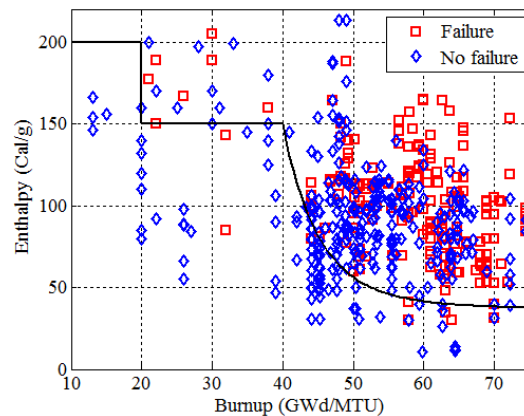


Fig. 1 Safety threshold for Zircaloy with pulse width up to 6 ms

$$H_{(Entalpy)} = 0.001669Bu^3 - 0.1929Bu^2 + 2.86.Bu^1 + 193.4 \quad (5)$$

Table V display LR parameters where  $p$  is the probability of failure;  $T$  is temperature ( $^{\circ}\text{C}$ ),  $Ox$  is the oxide thickness ( $\mu\text{m}$ ),  $W$  is the pulse width (ms),  $H$  is the enthalpy (cal/g), and  $Cd$  indicates the cladding type. For zirconium-based alloys, the probability not failing ( $q$ ) was determined using (6). The failure probability ( $p$ ), based on the LR model is determined using (7).

$$q = \exp(A + BT + CBu + DOx + EW + FH + GCd) \quad (6)$$

$$p = q/(1+q) \quad (7)$$

The failure thresholds defined throughout this study were based on  $UO_2$  and zirconium-based alloys. The cladding type is a predictor variable, where the values of 2, 4, 5, and 6 are substituted for Zircaloy-2, Zircaloy-4, M5, and Zirlo, respectively. The results obtained using the LR expressed in (6) adhere to the safety limit for the supplied parameters. Moreover, (7) indicates the probability of failure. Thus, this regression is a risk calculation. If the measured p-value is over 0.5, possible rupture of the nuclear fuel rod should be considered.

Parameters	Zirconium-based alloy
A	-7.832
B	-0.007
C	0.005
D	0.62
E	-0.001
F	0.039
G	0.405

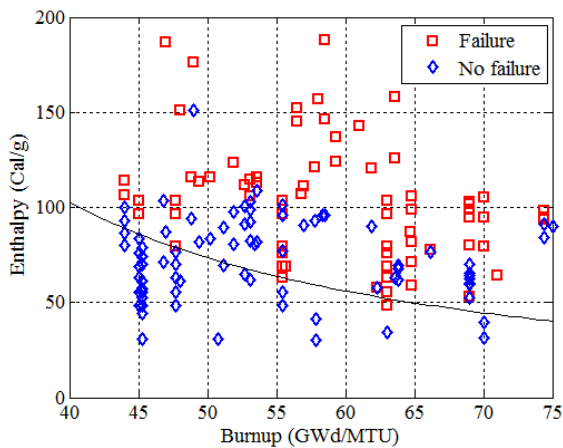


Fig. 2 Safety threshold for M5 with pulse width up to 6 ms

#### A. M5 Logistic Regression

As noted in the regression model,  $q$  indicates the chance of collapse and the regression calculation indicates possible failure, of the nuclear fuel rods. If the numerical values were, ( $q > 0.5$ ) may consider as fail cases. The fuel rods that are using the M5 alloy, the failure threshold can be calculated using (8) as burnup function using (9) and (10), the probabilistic value.

$$H_{(M5)} = 316.4 \exp(-0.021 Bu) \quad (8)$$

$$q_{(M5)} = \exp(A + BT + CBu + DOx + EW + FH) \quad (9)$$

$$P_{(M5)} = q_{(M5)} / (1 + q_{(M5)}) \quad (10)$$

where  $A = -14.127$ ,  $B = 0.010$ ,  $C = -0.124$ ,  $D = 0.318$ ,  $E = -$

0.160, and  $F = 0.157$ . The results are shown in Fig. 2 for a pulse width limited to 6 ms.

#### B. ZIRLO Logistic Regression

Similarly,  $q$  can be calculated for the ZIRLO alloy. In such cases, the pulse width must be limited to 6 ms. Fig. 3 illustrated the curve fitting response. The correlation fitting, for enthalpy peak as a function of the burn cycle, can be calculated by using (11). The probability not failing based on logistic a regression model is determined using (12) and failure using (13), if the burnup is lower than 75 GWd/MTU.

$$H_{(ZIRLO)} = 183.1 \exp(-0.0126 Bu) \quad (11)$$

$$q_{(Zirlo)} = \exp(A + BT + CBu + DOx + EW + FH) \quad (12)$$

$$P_{(Zirlo)} = q_{(Zirlo)} / (1 + q_{(Zirlo)}) \quad (13)$$

where  $A = -12.348$ ;  $B = 0.237$ ,  $C = 0.017$ ,  $D = 0.035$ ,  $E = 0.114$ , and  $F = 0.048$ . These new correlations are applicable to irradiation levels at 40–72 GWd/MTU with Zr alloy cladding having 1% niobium.

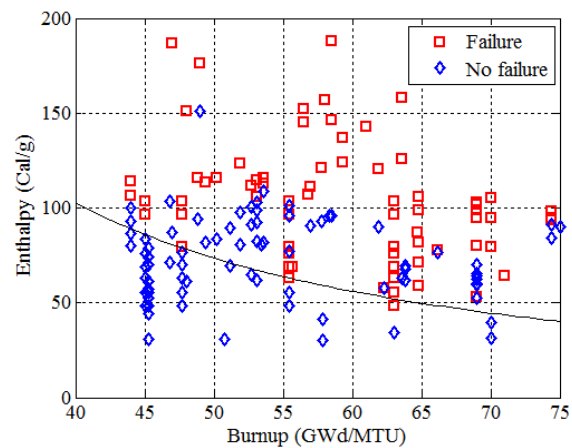


Fig. 3 Safety threshold for Zirlo with pulse width up to 6 ms

#### V. CONCLUSION

The correlations found can define an adequate safety margin, presenting an uncertainty of about 15%. We note that the extension of the irradiation cycle is the lifespan of the fuel rod. The calculated limits exhibit the possibility of failure when subjected to a radial enthalpy pulse, combined with the extension of the burn cycle, oxidation of the cladding and alloy performance. In the simulations, we observe points that are located below of the threshold established. The outlier points can occur when the pulse width exceeds 6 ms of FWHM, or the burning exceeds the proposed 75 GWd/MTU. The precision of the regression model is similar to those of other methods employed throughout this search. The simulations were conducted for both transient and steady states using the FRAPCON and FRAPTRAN software. The predictions for experimental cases have a precision,

corresponding to 85% of the total of failure cases. A comparison between the simulated results obtained using fuel performance codes or using the LR approach indicates similar uncertainties.

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