

Role of ICT and Wage Inequality in Organization

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Abstract—This study deals with wage inequality in organization and shows the relationship between ICT and wage in organization. To do so, we incorporate ICT's factors in organization into our model. ICT's factors are efficiencies of Enterprise Resource Planning (ERP), Computer Assisted Design/Computer Assisted Manufacturing (CAD/CAM), and NETWORK. The improvement of ICT's factors decrease the learning cost to solve problem pertaining to the hierarchy in organization. The improvement of NETWORK increases the wage inequality within workers and decreases within managers and entrepreneurs. The improvements of CAD/CAM and ERP increases the wage inequality within all agent, and partially increase it between the agents in hierarchy.

Keywords—Endogenous economic growth, ICT, inequality, capital accumulation, technology.

I. INTRODUCTION

AS [13] pointed out, inequality has widened around the world. Reference [13] mentions that inequality has stemmed from the causes of two types. One is that the rate of return for capital (r) is greater than the economic growth rate (g). Another is the wage (income) inequality in organization. And [12] mentions that technological progress led to higher wage differentials, so that advances in information and communication technologies in particular have been more beneficial for workers with higher skills. Taking into consideration the above, this study deals with the cause of wage inequality of the latter type in [13], and incorporates ICT factors in organization into our model.

Recently ICT has strong influence to the various aspects. For example, it is said that the evolving form of new innovation, Industry 4.0, has succeeded to steam engine, electrical power and automation. As it enhances the competitiveness of the manufacturing industry in Germany, the Berlin aims to retain the factories within the country with the new innovation. Efficiency is enhanced by the fusion with ICT and manufacturing technology. Furthermore, we illustrate that by connection with internet between manufacturing devices and factories production management and orders are fully automated. In addition to this, rate of operation of manufacturing devices is able to automatically controlled by internet with correspondence to situation of shipment.

Many literatures have theoretically mentioned that IT/ICT brings about the impacts to economies [3], [10], [11], [7], [14]. However, almost all of these precedent researches analyze the impacts of IT/ICT from macroeconomic point of view, but not from microeconomic point of view, such as from firms and industries.

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Among others, [2] analyzes the role of ICT in organization (firm), and how to influence the firm organization by ICT. They present a simplified version of [4], and introduce hierarchy in firm organization. Regarding the hierarchy, there are head quarter and factory, and they stratify the firm organization into three type agents, central managers at corporate head quarters, local managers and workers at site. There central managers have decision making as to non-production, local managers at factories have decision making as to production, and workers engage in production at factories. The costs for acquiring knowledge and communication are reduced by ICT. Problems in each layer are solved by acquiring knowledge and communication. According to [2], the technologies have at least two distinct components. First, through the spread of cheap storage and processing of data, information stored in database is becoming cheaper to access. Second, through the spread of cheap wired (IP-based) and wireless communications, agents find it easier to communicate with each other (e.g. e-mail and mobile devices).

The first one includes two types of technology. The first type is Enterprise Resource Planning (ERP) relating to non-production decisions. ERP system increases dramatically the availability of information to decision makers in the company, that is they reduce the cost of acquiring information to solve a problem. The second one is Computer Assisted Design/Computer Assisted Manufacturing (CAD/CAM) relating to production decisions. Acquiring knowledge ERP systems are able to make decisions about production and its investment and employment. Falls in acquiring knowledge costs enlarge the span of control for decision making of local managers and production workers at plant site, and prompt organization to decentralized decision making.

The second one is the communication technology to centralize decision making. A key technology innovation affecting communication is growth of network and connectedness. Communication costs are able to be reduced through NETWORK, and falls in communication costs lead to more decision making at head quarters and proceed to centralization. Reference [2] indicates that ICT/IT influence corporate organization. However it takes the wage as exogenous, so that they do not refer to the wage inequality.

Although [5] does not use the term ICT, it is possible that the progress of ICT reduces communication cost, and with two layers of organization and agents to be differ in knowledge, they analyze how to influence wages of workers, self-employments and managers through falls of communication cost. As a result of numerical calculation, its falls decreases the shares of self-employment and manager, increases the share of worker and decreases the wage inequality within workers. In addition, it widens the wage

inequality between workers and managers. In [6], they deal with multiple layers in organization and analyze the wage inequality.

As empirical studies, there are many literatures, such as [1] and [8]. Reference [8] analyzes the critical factor for making use of ICT in Ireland with probit method, and indicates that the services of computer utilization and online are related to firm scale, human capital, management, technology, clerks, and export intensity. Reference [9] shows that with using data for United States from 1970 to 2010, the progress of ICT causes the income inequality. More concretely, it indicates that the proportion of the upper decile of income in that country increases in accordance with the progress of ICT, and its tendency started from about the middle of 1980. Furthermore it shows that the intrinsic characteristic in ICT (skill biased technology) is in favor of upper decile of income. Reference [1] shows that with using data of Italian industry from 1995 to 1997 the productivity of firms with many replacement of investment is higher than the one with less replacement by regression analysis, and ICT investment has 8 times as productivity as non-ICT investment. This indicates that ICT investment much influences firms on various aspects.

To clarify the wage inequality in the framework of [2], we incorporate the cost function, which is based on [6], into the model of [2], and we analyze wage inequality in the changes of situation for information and communication technologies. In addition to this, we consider the externality for agents behavior in organization (for example X-Efficiency) and analyze wage inequality between agents.

II. THE MODEL

This model is a partial equilibrium one and based on [2] and [6]. In these studies, agents of heterogeneous ability learn to solve problems, choose an occupation, and a team to join. Agents supply some constant time, which may be used in production or helping others solve problem. Regarding organization, we consider the two cases, two layers and three layers in organization. At first, we consider the model where firm or economy consists of two layers (Case 1: layer 0 (l_0), layer 1 (l_1)) or three layers (Case 2: layer 0 (l_0), layer 1 (l_1), layer 2 (l_2)). Layer 0 (l_0) comprises of workers, such as factory workers, layer 1 (l_1) managers of decision makers for production such as plant manager, layer 2 (l_2) entrepreneurs of decision makers for non-production.

A. Production and Knowledge

Based on the concept of [6], we specified the production and knowledge as follows. Production requires labor and knowledge. Agents are production workers, managers and entrepreneurs. They spend time in production and non-production, and solve the problems they confront in order to produce. Problems are ranked by the likelihood that they will be confronted, so that problem z is associated with a continuous density $f(z) \in [0, \bar{z}]$, and c.d.f. $F(z)$, where $f'(z) < 0$. \bar{z} is the maximum value of problem. Solving problems requires knowledge. We defined the proportion of problems a worker can solve as $q = F(\tilde{z})$. Then $\tilde{z} = z(q)$,

where $z(\cdot) = F^{-1}(\cdot)$, and so $z' > 0$, $z'' > 0$. Thus $z(q)$ denotes the knowledge required to solve a proportion of q of problems.

B. Cost, Information Technology and Externality

Agents differ in their cognitive ability so that higher ability agents incur lower earning costs. We assume that the distribution of ability in the population can be described by a continuous density function, $\alpha \sim \phi(\alpha)$, with support in $[0, 1]$. Especially, we define ability so that the cost of learning to solve an interval of problems of length 1 is given by

$$c_0(\alpha; t_0, \beta_0) = t_0 - \beta_0 \alpha \quad \text{for } \alpha \in A_{0M} \quad (1)$$

$$c_1(\alpha; t_1, \beta_1) = t_1 - \beta_1 \alpha \quad \text{for } \alpha \in A_{1M} \quad (2)$$

$$c_2(\alpha; t_2, \beta_2) = t_2 - \beta_2 \alpha \quad \text{for } \alpha \in A_{2E} \quad (3)$$

Cost functions are piecewise continuous ones. A decrease in t_i ($i = 0, 1, 2$) represents an improvement in information technology that decreases the cost of learning (e.g. a technology that decreases the cost of accessing knowledge, such as cheaper database storage and research). The cost of information technology t_0 is related to CAD, t_1 to CAM, and t_2 to ERP. An increase in β_i ($i = 0, 1, 2$) presents an improvement of external effect with ability in the corresponding layer (e.g. X-Efficiency: efficiency being produced by agents in their layer and for working together by using ICT apparatus and software).

C. Communication and Organization

Agents can communicate their knowledge to others, and thus help them solve problems. Thus, agents form organizations where several individuals combine their time and knowledge to produce together. These organizations take the form of knowledge hierarchies. On layer level (l_0) of these teams is set of equally knowledgeable production workers, who learn the most routine problems and spend all of their time in production, and generate one problem each. Above them are layers of managers and entrepreneurs. Workers draw a problem per their own time (some constant time). Managers and entrepreneurs do not engage in production, and thus do not draw problems. If workers can solve it, they produce; otherwise, they ask for help to the managers in the layer immediately above them (l_1), in which these managers incur a communication cost of h units of time ($0 < h < \bar{h}$, \bar{h} denotes some constant). If these managers know how to solve the problem, they solve it; otherwise, they pass it on the layer immediately above them (l_2), in which these entrepreneurs incur a communication cost of h units of time like managers. Higher layer has smaller number of agents than the previous one, since only a fraction of problems are passed on. The communication cost h is related to NETWORK.

Consider an organization with n_0 production workers with knowledge $q_0 = F(z_0)$; and n_1 problem solving managers in layer l_1 , with knowledge q_1 . Workers in production draw one problem each, and solve in expectation a fraction q_0 of them. Hence, they pass on a fraction, $1 - q_0$, of all problems. Managers in layer 1 are thus asked to solve $n_0(1 - q_0)$

problems, which they can address in $n_0(1-q_0)h$ units of time. Since all agents have same units of time available, the number of managers in layer 1 (l_1) is $hn_0(1-q_0) = n_1$. Managers in layer 1 can only solve a fraction q_1 of problems, and pass up to the next layer (l_2) $n_0(1-q_1)$ problems. Thus, the number of entrepreneurs in layer 2 (l_2) is $hn_0(1-q_1) = n_2$, and entrepreneurs can only solve a fraction q_2 of problems, that is, a problem is solved with probability q_2 . Therefore, expected total output y produced the organization is given as below.

$$y = q_2 n_0 \quad (4)$$

D. Firm's Problem

In this section, we assume that a hierarchy is integrated in a firm, and mention the profit maximization problem for a firm.

Profits of a hierarchy are given by production minus labor costs, since we normalize the price of output to unity. Thus, the problem of a hierarchy of two (three) layers that faces a wage schedule, $w(\alpha)$, is to choose the ability α , knowledge q , and number of agents in each layer of the team n . Let L denote the number of layer ($L=1,2,3$). Profits are given by output minus wages, $w(\alpha)$, and learning costs, $n_l c_l z(q_l)$ ($l = 0, 1, 2$). Here we just mention the firm's problem for $L = 3$. The expected profits of hierarchy are

$$\begin{aligned} \Pi(L=3) &= \max_{[q_l, n_l, \alpha_l]_{l=0}^2} q_2 n_0 - n_2 (c_2(\alpha_2; t_2) z(q_2) \\ &+ w(\alpha_2)) - n_1 (c_1(\alpha_1; t_1) z(q_1) + w(\alpha_1)) \\ &- n_0 (c_0(\alpha_0; t_0) z(q_0) + w(\alpha_0)) \end{aligned} \quad (5)$$

subject to time constraints for the different layers of managers and entrepreneurs,

$$hn_0(1-q_1) = n_2 \quad (6)$$

$$hn_0(1-q_0) = n_1 \quad (7)$$

Then using (5)-(7), we obtain the first order conditions (f.o.c) for profit maximization problem as:

$$\begin{aligned} \frac{\partial \Pi}{\partial q_1} &= h(c_2(\alpha_2; t_2) z(q_2) + w(\alpha_2)) \\ &- h((1-q_0)c_1(\alpha_1; t_1) z'(q_1)) = 0 \quad (foc_{q_1}) \end{aligned} \quad (8)$$

$$\frac{\partial \Pi}{\partial q_2} = 1 - h(1-q_1)c_2(\alpha_2; t_2) z'(q_2) = 0, \quad (foc_{q_2}) \quad (9)$$

$$\frac{\partial \Pi}{\partial \alpha_i} = -c'_i(\alpha_i; t_i) z(q_i) - w'(\alpha_i) = 0 \quad (i = 0, 1, 2) \quad (10)$$

III. EQUILIBRIUM

In this section, we analyze an equilibrium in this economy. An equilibrium allocation specifies the sets of agents in different occupation, the assignment of agents to supervisor, and the wage schedule that support this assignment.

Based on [6], we mention the labor market equilibrium condition briefly. In equilibrium, the supply of workers or managers for a corresponding set of abilities at each layer is equal to the demand for these workers or managers by managers or entrepreneurs at each layer. Let $n(\alpha)$ denote the total number of workers or managers hired as direct subordinates of managers or entrepreneurs with ability α in equilibrium. Let $a(\alpha)$ denote the ability of the manager

or entrepreneur assigned to an employee of ability α in equilibrium. In order for $a(\alpha)$ to be defined over the whole set of abilities, $[0, 1]$, we set $a(\alpha) = 1$ for all entrepreneurs. Let A_S denote the set of agents with subordinates and let A_M denote the set of agents who are not at the top of hierarchy. $\alpha \in A_{0M}$ agents become workers and $\alpha \in A_{1M}$ do managers. Then, labor markets clear if for every $\alpha \in A_M = A_{0M} \cup A_{1M}$,

$$\int_{0 \cap A_M}^{\alpha \cap A_M} \phi(\alpha') d\alpha' = \int_{a(0) \cap A_S}^{a(\alpha) \cap A_S} \frac{n(\alpha')}{n(a(\alpha'))} \phi(\alpha') d\alpha' \quad (11)$$

where $A_S \equiv [0, 1] \setminus A_{0M}$. The left-hand side is the supply of employees in the interval $[0, \alpha]$. The right-hand side is the demand for employees by manager and entrepreneurs in the interval $[a(0), a(\alpha)]$: managers and entrepreneurs of ability α hire $n(\alpha)$ employees, and there are $n(a(\alpha))$ of them. The definition of equilibrium in this setup is then given by Definition 1.

Definition 1: A competitive equilibrium is

- for $\alpha \in A_{1M}$ agents become managers of layer 1 (l_1), workers of layer 0 (l_0) for $\alpha \in A_{0M}$, and entrepreneurs of layer 2 (l_2) for $\alpha \in A_{2E}$,
- a wage function, $w(\alpha) : [0, 1] \rightarrow R_+$,
- an assignment function, $a(\alpha) : [0, 1] \rightarrow A_S$ and $a(\alpha) = 1$ for $\alpha \in A_{2E}$.
- a knowledge function $q(\alpha) : [0, 1] \rightarrow [0, 1]^1$ and
- a total number of direct subordinates of agents with ability α , $n(\alpha) : A_S \rightarrow R_+$, that (i) Firms choose the skill of their employees, knowledge, and their number, to maximize (5).
(ii) Firms make zero profits.
(iii) Labor markets clear, that is, (11) is satisfied for $\alpha \in A_M$.

A. Assignment

On layers in organization high ability managers hire high ability agents so as to be shield from solving easy and common problems. Hiring better workers allows managers to specialize in solving only the harder problems that lower layer agents cannot solve. Driving the labor market condition (11) with Leibniz's Rule, we obtain

$$\frac{\partial a(\alpha)}{\partial \alpha} = \frac{n(a(\alpha))}{n(\alpha)} \frac{\phi(\alpha)}{\phi(a(\alpha))} \quad \text{for } \alpha \in A_M \quad (12)$$

Following [6], we obtain the assignment function as below.

$$\frac{\partial a(\alpha)}{\partial \alpha} = \frac{1 - q(\alpha)}{1 - q(a^{-1}(\alpha))} \frac{\phi(\alpha)}{\phi(a(\alpha))} \quad \text{for } \alpha \in A_M \setminus A_{0M}, \quad (13)$$

or

$$\frac{\partial a(\alpha)}{\partial \alpha} = h(1 - q(\alpha)) \frac{\phi(\alpha)}{\phi(a(\alpha))} \quad \text{for } \alpha \in A_{0M} \quad (14)$$

¹ Although theoretically it is acceptable that $q(\alpha) : [0, 1] \rightarrow [0, 1]$, $q(\alpha) = 1$ means that $z \rightarrow \infty$. Therefore it is practical that $q(\alpha) : [0, 1] \rightarrow [0, b]$, b is constant and $0 < b < 1$.

Equations (13 and 14) are a collection of ordinary different equations that determine the functions $a(\alpha)$ given some initial values.

B. Procedure of Existence of Assignment and Wages at Equilibrium

In this section, we show the procedure of existence of assignment. An equilibrium can be constructed as follows.

- 1) Set $L = 2$ (Case 1:workers and managers) or $L = 3$ (Case 2:workers, managers and entrepreneurs).
- 2) In Case 1, we fix $\alpha_{01}(\alpha_{10})$ by using (14), through reiteration to satisfying the labor market (11).
- 3) In Case 2, we fix $0 < \alpha_{01}(\alpha_{10}) < \alpha_{11}(\alpha_{20})$.
- 4) Set the initial value of $w(0)$ ².
- 5) In Case 2, based on the fixed value of $\alpha_{01}(\alpha_{10})$, we fixed the final value of $\alpha_{11}(\alpha_{20})$ by using (13), through reiteration to satisfy the labor market (11).
- 6) After the fixed values $\alpha_{01}(\alpha_{10})$ and $\alpha_{11}(\alpha_{20})$, we obtain the wages for each layer by using (1)-(3) and (10) ³.

C. Theoretical Results

With the above results, we perform the statics analysis in equilibrium. In accordance with [2], firstly we show the statics results of two layers model (Case 1) which comprises workers and managers. Afterwards we deal with three layers model (Case 2). In Appendix the calculation for statics analysis for two and three layers is shown.

The results of statics analysis for Case 1 are as follows.

$$\text{sign} \begin{pmatrix} \frac{\partial q_0}{\partial h} \\ \frac{\partial q_1}{\partial h} \end{pmatrix} = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (15)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_0}{\partial t_0} \\ \frac{\partial q_1}{\partial t_0} \end{pmatrix} = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix} \quad (16)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_0}{\partial t_1} \\ \frac{\partial q_1}{\partial t_1} \end{pmatrix} = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (17)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_0}{\partial \beta_0} \\ \frac{\partial q_1}{\partial \beta_0} \end{pmatrix} = \begin{pmatrix} > 0 \\ > 0 \end{pmatrix} \quad (18)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_0}{\partial \beta_1} \\ \frac{\partial q_1}{\partial \beta_1} \end{pmatrix} = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (19)$$

²In Simulation of this paper we set $w(0) = 1$.

³For the thresholds, such as $\alpha_{01}(\alpha_{10})$ and $\alpha_{11}(\alpha_{20})$, we assume that firms make a decision to choose workers or managers, (managers or entrepreneurs) at thresholds, taking into consideration their backgrounds such as curriculum vitae, personality and school achievements an so on, in addition to their abilities.

Regarding the span of control $S_M = \frac{n_0}{n_1} = \frac{1}{h(1-q_0)}$, we obtain the following results of sign.

$$\frac{\partial S_M}{\partial h} = < 0 \text{ or } > 0 \quad (20)$$

$$\frac{\partial S_M}{\partial t_0} = < 0 \quad (21)$$

$$\frac{\partial S_M}{\partial t_1} = < 0 \text{ or } > 0 \quad (22)$$

$$\frac{\partial S_M}{\partial \beta_0} = > 0 \quad (23)$$

$$\frac{\partial S_M}{\partial \beta_1} = < 0 \text{ or } > 0 \quad (24)$$

From the above results, as to communication cost h and acquiring knowledge cost of managers t_1 , we obtain the ambiguous outcome in comparison with [2]. Regarding acquiring cost of workers t_0 , we have same outcome for workers as [2]. As to the span of control S_M , we have almost the same result as [2]. Then we obtain the following proposition.

Proposition 1: The improvement of information technology for workers t_0 increases the knowledge for workers and managers. That is, q_0 and q_1 are decreasing in t_0 . The improvement of communication technology h and information technology for managers t_1 lead to the ambiguous outcomes for knowledge in layer 0 and 1. The improvement of externality for workers β_0 increases the knowledge for both workers and managers. However the improvement of externality for managers β_1 is ambiguous to knowledge. The changes of communication technology h , information technology for managers t_1 , and externality for managers β_1 have the ambiguous outcomes for S_M . The improvement of information technology for workers t_0 increases the span of control S_M . The deterioration of externality for worker β_0 increases the span of control for managers S_M .

Next we show the results of statics analysis for three layers (Case 2). However we just mention the results of the relationship between managers and entrepreneurs. The results are as follows.

$$\text{sign} \begin{pmatrix} \frac{\partial q_1}{\partial h} \\ \frac{\partial q_2}{\partial h} \end{pmatrix} = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix} \quad (25)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_1}{\partial t_1} \\ \frac{\partial q_2}{\partial t_1} \end{pmatrix} = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix} \quad (26)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_1}{\partial t_2} \\ \frac{\partial q_2}{\partial t_2} \end{pmatrix} = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (27)$$

$$\text{sign} \begin{pmatrix} \frac{\partial q_1}{\partial \beta_1} \\ \frac{\partial q_2}{\partial \beta_1} \end{pmatrix} = \begin{pmatrix} > 0 \\ > 0 \end{pmatrix} \quad (28)$$

$$\text{sign} \left(\frac{\partial q_1}{\partial \beta_2} \right) = \begin{pmatrix} < 0 & \text{or} > 0 \\ < 0 & \text{or} > 0 \end{pmatrix} \quad (29)$$

Regarding the span of control $S_M = \frac{n_0}{n_1} = \frac{1}{h(1-q_0)}$ and $S_E = \frac{1-q_0}{1-q_1}$ for managers and entrepreneurs, we obtain the following results of sign.

$$\frac{\partial S_M}{\partial h} = < 0 \quad (30)$$

$$\frac{\partial S_E}{\partial h} = < 0 \quad (31)$$

$$\frac{\partial S_M}{\partial t_1} = 0 \quad (32)$$

$$\frac{\partial S_E}{\partial t_1} = < 0 \quad (33)$$

$$\frac{\partial S_M}{\partial t_2} = 0 \quad (34)$$

$$\frac{\partial S_E}{\partial t_2} = < 0 \text{ or } > 0 \quad (35)$$

$$\frac{\partial S_M}{\partial \beta_1} = 0 \quad (36)$$

$$\frac{\partial S_E}{\partial \beta_1} = > 0 \quad (37)$$

$$\frac{\partial S_M}{\partial \beta_2} = 0 \quad (38)$$

$$\frac{\partial S_E}{\partial \beta_2} = < 0 \text{ or } > 0 \quad (39)$$

Regarding the costs for communication h and information technologies for managers t_1 , we obtain the same outcome as [6]. Then we obtain the following proposition.

Proposition 2: The improvement of communication technology h and information technology for managers t_1 increases the knowledge of entrepreneurs and managers. That is, q_1 and q_2 are decreasing in h and t_1 . The improvement of the externality for managers β_1 increases the knowledge of entrepreneurs and managers, q_1 and q_2 . The improvement information technology t_2 and externality for entrepreneurs β_2 leads to ambiguous to knowledge. The improvement of communication technology h increases the both span of control. And improvement of information technology for managers t_1 increases the span of control for entrepreneurs S_E , but the improvement of externality for managers β_1 increases S_E . The improvements of information technology and the externality for entrepreneurs, t_2 and β_2 , are ambiguous to S_E .

IV. SIMULATION: EFFECT OF ICT ON WAGE AND ORGANIZATION

With the results mentioned above, we study examples with an exponential density of problems, $f(z) = \lambda e^{-\lambda z}$ and uniform distribution of workers ability, $\alpha \sim U[0, 1]$. Moreover, in all exercises, we use $\lambda = 2$ which is cited from [6]. The software to be used is MATLAB. Also we set the initial value of wage to 1.

A. The Data: Values of Parameters

The parameters of Baseline and Modified Simulations for each case to be used are shown on Tables IA and IB.

B. The Procedure of Knowledge at Threshold for Wage

In the Modified Simulations, regarding the threshold for wages, such as the threshold between workers and managers or managers and entrepreneurs, we follow the procedure as mentioned below except a parameter of communication h . Basic concept of this procedure is that wage level reflects the corresponding ability or knowledge. When wage is less than the initial wage level (w_0), wage is set at $w_0 = 1$.

1) Case 1

- As to information technology $t_0(t_1)$ we take the knowledge α which satisfies the condition: The minimum(maximum) wage for managers(workers) in Baseline Simulation is equal to the wage of workers(managers) in Modified Simulation.
- As to externality $\beta_0(\beta_1)$ we take the knowledge α which satisfies the condition: The maximum(minimum) wage for workers(managers) in Baseline Simulation is equal to the wage of managers(workers) in Modified Simulation.

TABLE IA
PARAMETERS OF BASELINE AND MODIFIED SIMULATIONS

Case	h	t_0	t_1	t_2
Case 1(Baseline)@	2.7	0.6	0.4	-
Case 1(Mod.1)	2.5	0.6	0.4	-
Case 1(Mod.2)	2.7	0.5	0.4	-
Case 1(Mod.3)	2.7	0.6	0.3	-
Case 1(Mod.4)	2.7	0.6	0.4	-
Case 1(Mod.5)	2.7	0.6	0.4	-
Case 2(Baseline)@	4.66	0.6	0.4	0.3
Case 2(Mod.1)	4.5	0.6	0.4	0.3
Case 2(Mod.2)	4.66	0.5	0.4	0.3
Case 2(Mod.3)	4.66	0.6	0.5	0.3
Case 2(Mod.4)	4.66	0.6	0.4	0.2
Case 2(Mod.5)	4.66	0.6	0.4	0.3
Case 2(Mod.6)	4.66	0.6	0.4	0.3
Case 2(Mod.7)	4.66	0.6	0.4	0.3

TABLE IB
PARAMETERS OF BASELINE AND MODIFIED SIMULATIONS

Case	β_0	β_1	β_2
Case 1(Baseline)	2.0	2.5	-
Case 1(Mod.1)	2.0	2.5	-
Case 1(Mod.2)	2.0	2.5	-
Case 1(Mod.3)	2.0	2.5	-
Case 1(Mod.4)	1.9	2.5	-
Case 1(Mod.5)	2.0	2.4	-
Case 2(Baseline)	2.0	2.5	3.0
Case 2(Mod.1)	2.0	2.5	3.0
Case 2(Mod.2)	2.0	2.5	3.0
Case 2(Mod.3)	2.0	2.5	3.0
Case 2(Mod.4)	2.0	2.5	3.0
Case 2(Mod.5)	1.9	2.5	3.0
Case 2(Mod.6)	2.0	2.4	3.0
Case 2(Mod.7)	2.0	2.5	2.9

2) Case 2

- As to t_0 we follow the procedure of t_0 in Case 1.
- As to t_1 we take the knowledge $\alpha_{01} = \alpha_{10}(\alpha_{11} = \alpha_{20})$ which satisfies the condition: The minimum wage for managers(entrepreneurs) in Baseline Simulation is equal to the wage of managers(entrepreneurs) in Modified Simulation.

- c) As to t_2 we take the knowledge $\alpha_{11} = \alpha_{20}$ which satisfies the condition: The minimum wage for entrepreneurs in Baseline Simulation is equal to the wage of entrepreneurs in Modified Simulation.
- d) As to β_0 we follow the procedure of β_0 in Case 1.
- e) As to β_1 we take the knowledge $\alpha_{01} = \alpha_{10}(\alpha_{11} = \alpha_{20})$ which satisfies the condition: The minimum(maximum) wage for managers in Baseline Simulation is equal to the wage of managers(entrepreneurs) in Modified Simulation.
- f) As to β_2 we take the knowledge $\alpha_{11} = \alpha_{20}$ which satisfies the condition: The minimum wage for entrepreneurs in Baseline Simulation is equal to the wage of entrepreneurs in Modified Simulation.

By using the parameters shown on Tables IA and I, we simulate the wages for both cases.

For both cases, with the Baseline Simulations, we compare the Modified Simulations where the parameters values are changed from the Baseline Simulation in equilibrium.

In the Tables IIA, IIIB, we summarize the results for Baseline and Modified Simulations for both Cases.

Tables IIA and B show the wage inequality within the agents in the same layer. The values are the ratio of the wage for agents with the highest ability relative to the one for agents with the lowest ability. The values at lower row (the value with parenthesis) are ratio of the value of Modified Simulation relative to the one of Baseline Simulation.

From Tables IIA and IIB, we obtain the following results.

Result 1 (for both Cases):

- 1) In Baseline Simulation, the inequality within managers is higher (the highest) than the one within workers (of them all).
- 2) The improvement of communication (decrease of h) increases the inequality within worker in comparison with the one within managers or entrepreneurs.

Result 2 (for Case 1):

- 1) Improvement of information technology for workers (decrease of t_0) increases the inequality within workers and decreases the one within managers.
- 2) Improvement of information technology for manager (decrease of t_1) decreases the inequality within workers, and increases the one within managers.
- 3) Deterioration of externality of worker (decrease of β_0) decreases the inequality within workers and increases the one within managers.
- 4) Deterioration of externality of managers (decrease of β_1) increases the inequality within workers and decreases the one within managers.

TABLE IIA
WAGE INEQUALITY RATIO WITHIN LAYER (CASE 1)

Employee	Workers	Managers
Baseline Simulation	1.23	1.46
Communication (h):	1.56	1.09
Decrease of h	(1.27)	(0.75)
Information Techn.(t_0):	1.39	1.29
Decrease of t_0	(1.13)	(0.88)
Information Tech.(t_1):	1.10	1.69
Decrease of t_1	(0.89)	(1.10)
Externality(β_0):	1.10	1.66
Decrease of β_0	(0.89)	(1.14)
Externality(β_1):	1.26	1.40
Decrease of β_1	(1.02)	(0.96)

TABLE IIB
WAGE INEQUALITY RATIO WITHIN LAYER (CASE 2)

Employee	Workers	Managers	Entrepre.
Baseline Simulation	1.22	1.34	1.10
Communication(h):	1.33	1.23	1.07
Decrease of h	(1.09)	(0.92)	(0.97)
Information Tech.(t_0):	1.39	1.31	1.10
Decrease of t_0	(1.14)	(0.98)	(1.00)
Information Tech.(t_1):	1.24	1.35	1.07
Increase of t_1	(1.02)	(1.01)	(0.97)
Information Tech.(t_2):	1.22	1.32	1.12
Decrease of t_2	(1.00)	(0.99)	(1.02)
Externality(β_0):	1.11	1.52	1.10
Decrease of β_0	(0.91)	(1.13)	(1.00)
Externality(β_1):	1.24	1.16	1.26
Decrease of β_1	(1.02)	(0.87)	(1.15)
Externality(β_2):	1.22	1.40	1.05
Decrease of β_2	(1.00)	(1.04)	(0.95)

Result 3 (for Case 2):

- 1) Improvement of information technology for workers (decrease of t_0) increases the inequality within workers and decreases the one within managers. However, it has no effect to the inequality within entrepreneurs.
- 2) Deterioration of information technology for manager (increase of t_1) increases the inequality within workers and managers, and decreases the one within entrepreneurs.
- 3) Improvement of information technology for entrepreneur (decrease of t_2) increases the inequality within entrepreneurs, and decreases the one within managers. However, it has no effect to the inequality within workers.
- 4) Deterioration of externality of worker (decrease of β_0) decreases the inequality within workers and increases the one within managers. However, it has no effect to the inequality within entrepreneurs.
- 5) Deterioration of externality of managers (decrease of β_1) decreases the inequality within managers and increases the ones within workers and entrepreneurs.
- 6) Deterioration of externality for entrepreneur (increase of β_2) decreases the inequality within entrepreneurs and increases the one within managers. However, it has no effect to the inequality within workers.

Tables IIIA and B show the wage inequality ratios between the different layers. For Baseline Simulations of the both cases, the values without parenthesis are the ratios for the lowest ability and the ones with parenthesis are the ratios for the highest ability in different layer. For Modified Simulation, the values of the first row are the ratios for the lowest

ability (values without parenthesis) and the ones for the highest ability (values with the parenthesis). Then in the second row, we indicate the wage inequality ratios between Modified and Baseline Simulation at lowest ability (values without parenthesis) and at the highest ability (values with the parenthesis).

TABLE IIIA
WAGE INEQUALITY RATIO BETWEEN LAYERS (CASE 1)

Upper Layer/Lower Layer	M/W
Baseline Simulation	1.40(1.67)
Communication(h):	1.87(1.41)
Decrease of h	1.34(0.84)
Information Techn.(t_0):	1.58 (1.47)
Decrease of t_0	1.23(0.88)
Information Tech.(t_1):	1.22(1.87)
Decrease of t_1	0.87(1.12)
Externality(β_0):	1.23(1.85)
Decrease of β_0	0.88(1.11)
Externality(β_1):	1.40(1.56)
Decrease of β_1	1.00(0.93)

TABLE IIIB
WAGE INEQUALITY RATIO BETWEEN LAYERS (CASE 2)

Upper Layer/Lower Layer	M/W	E/M
Baseline Simulation	1.39(1.53)	1.55(1.27)
Communication(h):	1.55(1.44)	1.43(1.23)
Decrease of h	1.12(0.94)	0.92(0.97)
Information Tech.(t_0):	1.42(1.34)	1.51(1.27)
Decrease of t_0	1.02(0.88)	0.97(1.00)
Information Tech.(t_1):	1.39(1.51)	1.60(1.51)
Increase of t_1	1.00(0.99)	1.03(1.19)
Information Tech.(t_2):	1.39(1.51)	1.53(1.29)
Decrease of t_2	1.00(0.99)	0.99(1.02)
Externality(β_0):	1.23(1.68)	1.75(1.27)
Decrease of β_0	0.88(1.10)	1.13(1.00)
Externality(β_1):	1.40(1.31)	1.35(1.46)
Decrease of β_1	1.01(0.86)	0.87(1.15)
Externality(β_2):	1.39 (1.59)	1.54 (1.15)
Decrease of β_2	1.00(1.04)	0.99(0.91)

From Table III we obtain the following results:

Result 4 (for Case 1):

- 1) For Baseline Simulation the inequality at the highest ability is greater than the one at the lowest ability.
- 2) As to the improvement of communication (decrease of h), the inequality at the lowest ability is greater than the one at the highest ability. The inequality at lowest ability increases and the one at the highest ability decreases.
- 3) As to the improvement of information technology for worker (decrease of t_0), the inequality at the lowest ability is greater than the one at the highest ability. The inequality at the lowest ability increases and the one at the highest ability decreases.
- 4) As to the improvement of information technology for manager (decrease of t_1), the inequality at the highest ability is greater than the one at the lowest ability. The inequality at the lowest ability decreases and the one at the highest ability increases.
- 5) As to the deterioration of externality for worker (decrease of β_0), the inequality at the highest ability is greater than the one at the lowest ability. The inequality at the lowest ability decreases and the one at the highest ability increases.
- 6) As to the deterioration of externality for manager (decrease of β_1), the inequality at the highest ability is

greater than the one at the lowest ability. The inequality at the lowest ability has no change and the one at the highest ability decreases.

Result 5 (for Case 2):

- 1) For Baseline Simulation the inequality at the lowest ability between entrepreneur and managers is the greatest in all the ones between the different layers. The inequality at the highest ability between entrepreneur and manager is the smallest in all the ones between the different layers.
- 2) As to the improvement of communication (decrease of h), the inequality at the lowest ability between managers and workers is the greatest in all the ones between the different layers. The inequality at the lowest ability between managers and workers increases and the other ones decrease.
- 3) As to the improvement of information technology for worker (decrease of t_0), the inequality at the lowest ability between entrepreneurs and managers is the greatest in all the ones between the different layers. The inequality at lowest ability between managers and workers increases and the other ones decrease or not change.
- 4) As to the deterioration of information technology for manager (increase of t_1), the inequality at the lowest ability between entrepreneurs and managers is the greatest in all the ones between the different layers. The inequalities between entrepreneurs and managers increase and the one at the highest ability between managers and workers decrease and the one at the lowest ability between managers and workers has no change.
- 5) As to the improvement of information technology for entrepreneur (decrease of t_2), the inequality at the lowest ability between entrepreneurs and managers is the greatest in all the ones between the different layers. The inequality at the highest ability between entrepreneurs and managers increases and the ones at the highest ability between managers and worker and at the lowest ability between entrepreneurs and managers decrease, and one at the lowest ability between managers and workers has no change.
- 6) As to the improvement of externality for worker (increase of β_0) the inequality at the lowest ability between entrepreneurs and managers is the greatest in all the ones between the different layers. The inequalities at the highest ability between managers and workers and at the lowest ability between entrepreneurs and managers increase. Then the inequality at the lowest ability between managers and worker decreases and the one at the highest ability between entrepreneurs and managers has no change.
- 7) As to the deterioration of externality for manager (increase of β_1) the inequality at the highest ability between entrepreneurs and managers is the greatest in all the ones between the different layers. The inequalities at the lowest ability between manager and worker and at the highest ability between entrepreneurs and managers

increase. Then the inequalities at the highest ability between managers and worker and at the lowest ability between entrepreneurs and managers decrease.

- 8) As to the deterioration of externality for entrepreneur (decrease of β_2), the inequality at the highest ability between manager and worker is the greatest in all the ones between the different layers. The inequality at the highest ability between managers and workers increases and the ones between entrepreneurs and managers decrease. The inequality at the lowest ability between managers and workers has no change.

V. CONCLUSION

In this study we develop our model based on [2] and [6]. Our final goal is to clarify the role of ICT for wage inequality.

Regarding the theoretical results, the improvement of information technology (CAD/CAM) increases the knowledge of workers and managers, and the span of controls for managers (Case 1) and the one for entrepreneurs (Case 2) increases. The improvement of communication (NETWORK) increases the knowledge and span of control for managers (Case 1 and 2) and for entrepreneurs (Case 2). The improvement of externality increases the knowledge for worker (Case 1) and for managers (Case 2).

Regarding the simulation for wages, the improvement of communication (NETWORK) increases the inequality within workers and decreases the inequality within managers and entrepreneurs. The improvement of information technology (CAD/CAM) increases the inequality within workers (Case 1 and Case 2) and within managers (Case 2), and the improvement of information technology (ERP) increases the inequality within entrepreneurs (Case 2). The improvement of externality at each layer increases the corresponding inequality within the agents (Case 1 and Case 2). Furthermore the improvements of communication (NETWORK) and information technology (CAD) increase the inequality at the lowest ability between managers and workers (Case 1 and Case 2). The improvement of information technology (CAM) increases the inequality at the highest ability between managers and workers (Case 1 and Case 2). The improvement of information technology (ERP) increases the inequality at the highest ability between entrepreneurs and managers. The improvement of externality increases the inequalities at the highest ability between managers and workers (Case 1) and between entrepreneurs and managers (Case 2).

From the above results, ICT is strongly related to the inequality in organization, since improvement of NETWORK/CAD/CAM/ERP increases the inequality within the agents at large. Then the improvement of externality at each layer increases the corresponding inequality within the agents. As a further research, the most influential factor among ICT's factor is determined through the simulations, and an empirical analysis using actual data like [2] have to be performed.

APPENDIX

A. Two Layers: Workers and Managers

The profit maximization problem is specified as:

$$\begin{aligned} \Pi(1) = & \max_{[q_1, n_1, \alpha_1]_{l=0}^1} q_1 n_0 \\ & - n_1 (c_1(\alpha_1; t_1) z(q_1) + w(\alpha_1)) \\ & - n_0 (c_0(\alpha_0; t_0) z(q_0) + w(\alpha_0)), \end{aligned} \quad (40)$$

subject to time constraints for the different layer of managers,

$$h n_0 (1 - q_0) = n_1. \quad (41)$$

We mention the first order conditions as:

$$\begin{aligned} \frac{\partial \Pi}{\partial q_0} = & h n_0 (c_1(\alpha_1, t_1) z(q_1) + w(\alpha_1)) \\ & - n_0 c_0(\alpha_0; t_0) z'(q_0) = 0, \quad (foc_{q_0}) \end{aligned} \quad (42)$$

$$\begin{aligned} \frac{\partial \Pi}{\partial q_1} = & n_0 - h n_0 (1 - q_0) c_1(\alpha_1; t_1) z'(q_1) \\ = & 0, \quad (foc_{q_1}) \end{aligned} \quad (43)$$

$$\frac{\partial \Pi}{\partial \alpha_i} = -c_i(\alpha_i; t_i) z(q_i) - w'(\alpha_i) = 0. \quad (i = 0, 1) \quad (44)$$

Using the first order conditions, the elements of Hessian are given as:

$$\frac{\partial foc_{q_0}}{\partial q_0} = -n_0 c_0(\alpha_0; t_0) z''(q_0) < 0, \quad (45)$$

$$\frac{\partial foc_{q_0}}{\partial q_1} = h n_0 c_1(\alpha_1; t_1) z'(q_1) > 0, \quad (46)$$

$$\frac{\partial foc_{q_1}}{\partial q_1} = -h n_0 (1 - q_0) c_1(\alpha_1; t_1) z''(q_1) < 0, \quad (47)$$

$$\frac{\partial foc_{q_1}}{\partial q_0} = h n_0 c_1(\alpha_1; t_1) z'(q_1) > 0. \quad (48)$$

Then we obtain the Hessian as:

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}, \quad (49)$$

where

$$\begin{aligned} H_{11} &= -n_0 c_0(\alpha_0; t_0) z''(q_0), \\ H_{12} &= h n_0 c_1(\alpha_1; t_1) z'(q_1), \\ H_{21} &= h n_0 c_1(\alpha_1; t_1) z'(q_1), \\ H_{22} &= -h n_0 (1 - q_0) c_1(\alpha_1; t_1) z''(q_1). \end{aligned}$$

Since we are maximizing profit, the sign of the determinant of the Hessian has to be positive ($\det|H| > 0$). Letting the vector $foc = (foc_{q_0}, foc_{q_1})$, we obtain:

$$\frac{\partial foc}{\partial h} = \begin{pmatrix} n_0 (c_1(\alpha_1, t_1) z(q_1) + w(\alpha_1)) \\ -n_0 (1 - q_0) c_1(\alpha_1, t_1) z'(q_1) \end{pmatrix}, \quad (50)$$

$$\frac{\partial foc}{\partial t_0} = \begin{pmatrix} -n_0 \frac{\partial c_0(\alpha_0, t_0)}{\partial t_0} z'(q_0) \\ 0 \end{pmatrix}, \quad (51)$$

$$\frac{\partial foc}{\partial t_1} = \begin{pmatrix} h n_0 \frac{\partial c_1(\alpha_1, t_1)}{\partial t_1} z(q_1) \\ -h n_0 (1 - q_0) \frac{\partial c_1(\alpha_1, t_1)}{\partial t_1} z'(q_1) \end{pmatrix}, \quad (52)$$

$$\frac{\partial foc}{\partial \beta_0} = \begin{pmatrix} -n_0 \frac{\partial c_0(\alpha_0, t_0)}{\partial \beta_1} z'(q_0) \\ 0 \end{pmatrix}, \quad (53)$$

$$\frac{\partial foc}{\partial \beta_1} = \begin{pmatrix} h n_0 \frac{\partial c_1(\alpha_1, t_1)}{\partial \beta_1} z(q_1) \\ -h n_0 (1 - q_0) \frac{\partial c_1(\alpha_1, t_1)}{\partial \beta_1} z'(q_1) \end{pmatrix}. \quad (54)$$

Let the vectors vars=(q_0, q_1). Then for each parameters, $\frac{\partial vars}{\partial t} = H^{-1} \frac{\partial foc}{\partial t}$ gives the following results.

$$\text{sign} \left(\frac{\partial q_0}{\partial h} \right) = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix}. \quad (55)$$

$$\text{sign} \left(\frac{\partial q_0}{\partial t_0} \right) = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix}, \quad (56)$$

$$\text{sign} \left(\frac{\partial q_0}{\partial t_1} \right) = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix}, \quad (57)$$

$$\text{sign} \left(\frac{\partial q_0}{\partial \beta_0} \right) = \begin{pmatrix} > 0 \\ > 0 \end{pmatrix}, \quad (58)$$

$$\text{sign} \left(\frac{\partial q_0}{\partial \beta_1} \right) = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix}. \quad (59)$$

Regarding the span of control $S_M = \frac{n_0}{n_1} = \frac{1}{h(1-q_0)}$, we obtain the following results of sign.

$$\frac{\partial S_M}{\partial h} = < 0 \text{ or } > 0, \quad (60)$$

$$\frac{\partial S_M}{\partial t_0} = < 0, \quad (61)$$

$$\frac{\partial S_M}{\partial t_1} = < 0 \text{ or } > 0, \quad (62)$$

$$\frac{\partial S_M}{\partial \beta_0} = > 0, \quad (63)$$

$$\frac{\partial S_M}{\partial \beta_1} = < 0 \text{ or } > 0. \quad (64)$$

B. Three Layers: Workers, Managers and Entrepreneurs

In this section we only confirm the relationship between managers and entrepreneurs.

The profit maximization problem is specified as:

$$\begin{aligned} \Pi(2) = & \max_{[q_1, n_1, \alpha_1]_{i=0}^2} q_2 n_0 - n_2 (c_2(\alpha_2; t_2) z(q_2) \\ & + w(\alpha_2)) - n_1 (c_1(\alpha_1; t_1) z(q_1) + w(\alpha_1)) \\ & - n_0 (c_0(\alpha_0; t_0) z(q_0) + w(\alpha_0)) \end{aligned} \quad (65)$$

subject to time constraints for the different layers of managers and entrepreneurs,

$$h n_0 (1 - q_1) = n_2 \quad (66)$$

$$h n_0 (1 - q_0) = n_1 \quad (67)$$

Using (65)-(67), we obtain the first order conditions as:

$$\begin{aligned} \frac{\partial \Pi}{\partial q_1} = & h(c_2(\alpha_2; t_2) z(q_2) + w(\alpha_2)) \\ & - h(1 - q_0) c_1(\alpha_1; t_1) z'(q_1) = 0 \quad (foc_{q_1}) \end{aligned} \quad (68)$$

$$\begin{aligned} \frac{\partial \Pi}{\partial q_2} = & 1 - h(1 - q_1) c_2(\alpha_2; t_2) z'(q_2) \\ = & 0, \quad (foc_{q_2}) \end{aligned} \quad (69)$$

$$\frac{\partial \Pi}{\partial \alpha_i} = -c_i(\alpha_i; t_i) z(q_i) - w'(\alpha_i) = 0. (i = 0, 1, 2) \quad (70)$$

Using the first order conditions, the elements of Hessian are given as:

$$\frac{\partial foc_{q_1}}{\partial q_1} = -h(1 - q_0) c_1(\alpha_1; t_1) z''(q_1) < 0 \quad (71)$$

$$\frac{\partial foc_{q_2}}{\partial q_2} = -h(1 - q_1) c_2(\alpha_2; t_2) z''(q_2) < 0 \quad (72)$$

$$\frac{\partial foc_{q_1}}{\partial q_2} = h c_2(\alpha_2; t_2) z'(q_2) > 0 \quad (73)$$

$$\frac{\partial foc_{q_2}}{\partial q_1} = h c_2(\alpha_2; t_2) z'(q_2) > 0 \quad (74)$$

Then we obtain the Hessian as:

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \quad (75)$$

$$H_{11} = -h(1 - q_0) c_1(\alpha_1; t_1) z''(q_1),$$

$$H_{12} = h c_2(\alpha_2; t_2) z'(q_2),$$

$$H_{21} = h c_2(\alpha_2; t_2) z'(q_2),$$

$$H_{22} = -h(1 - q_1) c_2(\alpha_2; t_2) z''(q_2).$$

Since we are maximizing profit, the sign of the determinant of the Hessian has to be positive ($\det[H] > 0$). Letting the vector foc=(foc_{q_1}, foc_{q_2}), we obtain:

$$\frac{\partial foc}{\partial h} = \begin{pmatrix} c_2(\alpha_2; t_2) z(q_2) + w(\alpha_2) \\ -(1 - q_0) c_1(\alpha_1; t_1) z'(q_1) \\ -(1 - q_1) c_2(\alpha_2; t_2) z'(q_2) \end{pmatrix} \quad (76)$$

$$\frac{\partial foc}{\partial t_1} = \begin{pmatrix} -h(1 - q_0) \frac{\partial c_1(\alpha_1; t_1)}{\partial t_1} z'(q_1) \\ 0 \end{pmatrix} \quad (77)$$

$$\frac{\partial foc}{\partial t_2} = \begin{pmatrix} h \frac{\partial c_2(\alpha_2; t_2)}{\partial t_2} z(q_2) \\ -h(1 - q_1) \frac{\partial c_2(\alpha_2; t_2)}{\partial t_2} z'(q_2) \end{pmatrix} \quad (78)$$

$$\frac{\partial foc}{\partial \beta_1} = \begin{pmatrix} -h(1 - q_0) \frac{\partial c_1(\alpha_1; t_1)}{\partial \beta_1} z'(q_1) \\ 0 \end{pmatrix} \quad (79)$$

$$\frac{\partial foc}{\partial \beta_2} = \begin{pmatrix} h \frac{\partial c_2(\alpha_2; t_2)}{\partial \beta_2} z(q_2) \\ -h(1 - q_1) \frac{\partial c_2(\alpha_2; t_2)}{\partial \beta_2} z'(q_2) \end{pmatrix} \quad (80)$$

Let the vectors vars=(q_1, q_2). Then for each parameters,

$\frac{\partial vars}{\partial t} = H^{-1} \frac{\partial foc}{\partial t}$ gives the following results.

$$\text{sign} \left(\frac{\partial q_1}{\partial h} \right) = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix} \quad (81)$$

$$\text{sign} \left(\frac{\partial t_1}{\partial q_2} \right) = \begin{pmatrix} < 0 \\ < 0 \end{pmatrix} \quad (82)$$

$$\text{sign} \left(\frac{\partial q_1}{\partial t_2} \right) = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (83)$$

$$\text{sign} \left(\frac{\partial \beta_1}{\partial q_2} \right) = \begin{pmatrix} > 0 \\ > 0 \end{pmatrix} \quad (84)$$

$$\text{sign} \left(\frac{\partial q_1}{\partial \beta_2} \right) = \begin{pmatrix} < 0 \text{ or } > 0 \\ < 0 \text{ or } > 0 \end{pmatrix} \quad (85)$$

Regarding the span of control $S_M = \frac{n_0}{n_1} = \frac{1}{h(1-q_0)}$ and $S_E = \frac{1-q_0}{1-q_1}$ for managers and entrepreneurs, we obtain the following results of sign.

$$\frac{\partial S_M}{\partial h} = < 0 \quad (86)$$

$$\frac{\partial S_E}{\partial h} = < 0 \quad (87)$$

$$\frac{\partial S_M}{\partial t_1} = 0 \quad (88)$$

$$\frac{\partial S_E}{\partial t_1} = < 0 \quad (89)$$

$$\frac{\partial S_M}{\partial t_2} = 0 \quad (90)$$

$$\frac{\partial S_E}{\partial t_2} = < 0 \text{ or } > 0 \quad (91)$$

$$\frac{\partial S_M}{\partial \beta_1} = 0 \quad (92)$$

$$\frac{\partial S_E}{\partial \beta_1} = > 0 \quad (93)$$

$$\frac{\partial S_M}{\partial \beta_2} = 0 \quad (94)$$

$$\frac{\partial S_E}{\partial \beta_2} = < 0 \text{ or } > 0 \quad (95)$$

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