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Heat and Mass Transfer of Triple Diffusive Convection in a Rotating Couple Stress Liquid Using Ginzburg-Landau Model

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***Abstract*—**A nonlinear study of triple diffusive convection in a rotating couple stress liquid has been analysed. It is performed to study the effect of heat and mass transfer by deriving Ginzburg-Landau equation. Heat and mass transfer are quantified in terms of Nusselt number and Sherwood numbers, which are obtained as a function of thermal and solute Rayleigh numbers. The obtained Ginzburg-Landau equation is Bernoulli equation, and it has been elucidated numerically by using Mathematica. The effects of couple stress parameter, solute Rayleigh numbers, and Taylor number on the onset of convection and heat and mass transfer have been examined. It is found that the effects of couple stress parameter and Taylor number are to stabilize the system and to increase the heat and mass transfer.

***Keywords*—**Couple stress liquid, Ginzburg-Landau model, rotation, triple diffusive convection.

I. Introduction

IN the classical Bénard problem, the instability is determined by the difference in density produced by the variation in temperature amongst the upper and lower surfaces confining the liquid. This kind of problem in literature is referred as single component convection. When the instability in a liquid is caused by two opposing density components like temperature and solute or two different diffusivity solutes, then it is termed as double diffusive convection or two-component convection. The dissimilarity between single and two-component convection is that, in two-component convection, the convection can arise even if the system is hydrostatically stable with two different diffusing components. The behaviour in the two-component convection is further remarkable than that of the single component condition as new instability may arise which does not exist in the classical Bénard problem.

When the instability in a liquid is caused by three different diffusivities then the mathematical and physical situation becomes progressively better-off. Such problems in literature are termed as triple diffusive convection or three component convection. Pearlstein et al. [1] obtained very fascinating results in triply diffusive convection. The outcomes of Pearlstein et al. are noteworthy. They determined that, for triple diffusive convection, linear uncertainty can arise in discrete units of Rayleigh number since for few parameters the neutral curve has its oscillatory curve lying lower than that of the standard boundless stationary convection. The problems of triple diffusive convection have also been studied by Lopez et al. [2], Sumithra [3] and Rionero [4] and recently by Sameena and Pranesh [5].

An important class of natural convection is the rotating system. The Coriolis Effect is a deflection of moving elements if it is observed in a gyrating situation. The deflection is towards left of the motion of element in case of clockwise rotation and the deflection is towards right in case of anti-clockwise rotation. Coriolis force is a result of inertia and it is not assigned to a distinguishable body, like in the case of electromagnetic forces. We can see the Coriolis effect in oceanography, meteorology, astrophysics, geophysics and even in earth's rotation. In natural convection, the influence of rotation is to stabilise the system (see [6]).

In the classification of non-Newtonian liquids, couple stress has unique features like polar effect and also the capacity to possess enormous viscosity. Couple stress liquid has directed to the latest expansion of numerous concepts of liquid with microstructure which was developed by Stokes [7]. The effects of couple stresses in liquid have no microstructure; therefore, the kinematic energy of spin density and angular momentum are not considered and are thereby determined completely by the velocity field. In view of this, couple stresses in liquid result in the equation similar to Navier-Stokes equation. There are many applications of couple stress liquid, and one of such applications can be perceived in the mechanism of lubrication of synovial joints. Many authors have considered couple stress liquid in single and double diffusive convection like Siddeshwar and Pranesh [8] and Shivakumara et al. [9].

As mentioned by Pranesh and Sameena [10], the linear analysis provides only the stability condition of the problem and does not help examining the rate of heat and mass transfer. In view of this, we have done nonlinear study of the problem using Ginzburg-Landau model (see [11]). A good understanding of nonlinear convection is justified in many practically important liquids to assess the amount of heat and mass transfer, finite amplitude cellular motion, and other related topics.

The main benefit of using Ginzburg Landau model is that it provides nonlinear solution in the form of a series whose convergence is guaranteed. To the best of our knowledge from the literature review done, none of the authors have studied triple diffusive convection using Ginzburg-Landau model and

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hence there is a motivation to study triple diffusive convection in a rotating couple stress liquid using Ginzburg-Landau model.

The problem under investigation also has many applications like the material processing, space crafts, underground spreading of chemical pollutants, petroleum reservoirs and waste and fertilizer migration in saturated soil and solidification of alloys. Therefore, the main objective of this paper is to study the effect of triple diffusive convection in a rotating couple stress liquid.

II. Mathematical Formulation

Consider a layer of couple stress liquid confined between two infinite horizontal surfaces separated by a distance d apart and rotating at a constant angular velocity Ω_0 in the vertical direction. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upward. Lower surface is maintained at higher temperature $T_0 + \Delta T$ and higher solute concentration $S_{i0} + \Delta S_i$, where i = 1,2 and upper surface is maintained at temperature T_0 and solute concentration S_{i0} (see Fig. 1).

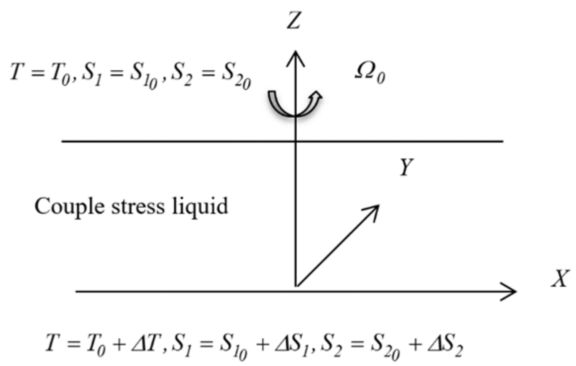

Fig. 1 Physical configuration

The basic governing equations are:

$$\nabla . \vec{q} = 0 \tag{1}$$

$$\rho_o \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} . \nabla) \vec{q} + 2 \vec{\Omega} \times \vec{q} \right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{q} - \mu' \nabla^4 \vec{q} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\vec{q} . \nabla) T = \chi \nabla^2 T \tag{3}$$

$$\frac{\partial S_1}{\partial t} + (\vec{q} . \nabla) S_1 = \chi_{S1} \nabla^2 S_1 \tag{4}$$

$$\frac{\partial S_2}{\partial t} + (\vec{q} . \nabla) S_2 = \chi_{S2} \nabla^2 S_2 \tag{5}$$

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0) + \alpha_{S1} (S_1 - S_{10}) + \alpha_{S2} (S_2 - S_{20}) \right] \tag{6}$$

where $\vec{q}$ is the velocity, Ω is the rotation, p is the pressure, ρ_0 is the constant density, ρ is the density, $\vec{g}$ is the gravitational force, μ is the viscosity, μ' is the couple stress viscosity, T is the temperature, S_1 is the solute 1, S_2 is the solute 2, χ is the thermal diffusivity, χ_{S1} is the solute 1 diffusivity, χ_{S2} is the solute 2 diffusivity, α_t is a coefficient of thermal expansion to determine how fast the density decreases with temperature, α_{S1} is the coefficient of solute 1 expansion, and α_{S2} is the coefficient of solute 2 expansion.

Equations (1)-(6) are solved for free-free velocity, isothermal and isoconcentration boundary conditions:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S_1 = S_2 = 0 \text{ at } z = 0,1. \tag{7}$$

The basic state of the fluid is quiescent and is described by:

$$\left. \begin{aligned} &\vec{q}_b = (0,0,0), p = p_b(z), \rho = \rho_b(z), T = T_b(z), \\ &S_1 = S_{1b}(z), S_2 = S_{2b}(z), \vec{\Omega} = \Omega \hat{k} \end{aligned} \right\} \tag{8}$$

where the subscript 'b' denotes the basic state.

The stability of the basic state is analysed by introducing the following perturbation:

$$\left. \begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}', p = p_b + p', \rho = \rho_b + \rho', \\ &T = T_b + T', S_1 = S_{1b} + S_1', S_2 = S_{2b} + S_2' \end{aligned} \right\} \tag{9}$$

where, the prime indicates that the quantities are infinitesimal perturbations.

Substituting (9) into (1) – (6), eliminating the pressure by operating curl twice, introducing the stream functions $u' = \frac{\partial \psi}{\partial z}$ and $w' = -\frac{\partial \psi}{\partial x}$ and non-dimensionalizing the resulting equations, we get the following dimensionless equations:

$$\left. \begin{aligned} &\begin{bmatrix} -\nabla^4 + C\nabla^6 & -\sqrt{Ta}\frac{\partial}{\partial z} & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \sqrt{Ta}\frac{\partial}{\partial z} & -\nabla^2 + C\nabla^4 & 0 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \\ &\begin{bmatrix} \psi \\ v \\ \theta \\ \phi_{S1} \\ \phi_{S2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2\psi\right) + \frac{1}{Pr}J\left(\psi, \nabla^2\psi\right) \\ -\frac{1}{Pr}\frac{\partial v}{\partial t} + \frac{1}{Pr}J(\psi, v) \\ -\frac{\partial \theta}{\partial t} + J(\psi, \theta) \\ -\frac{\partial \phi_{S1}}{\partial t} + J(\psi, \phi_{S1}) \\ -\frac{\partial \phi_{S2}}{\partial t} + J(\psi, \phi_{S2}) \end{bmatrix} \end{aligned} \right\} \tag{10}$$

Here, nondimensionalizing parameters in the above system are $Pr = \frac{\mu}{\rho_0 \chi}$ (Prandtl number), $C = \frac{\mu'}{\mu d^2}$ (Couple stress parameter), $Ta = \frac{4\rho_0^2 \Omega_0^2 d^2}{\mu^2}$ (Taylor number), $Ra = \frac{\rho_o \alpha_t g \Delta T d^3}{\mu \chi}$ (Rayleigh number), $R_{S1} = \frac{\rho_o \alpha_{S1} g \Delta S_1 d^3}{\mu \chi}$ (solute Rayleigh number 1), $R_{S2} = \frac{\rho_o \alpha_{S2} g \Delta S_2 d^3}{\mu \chi}$ (solute Rayleigh number 2), $\tau_1 = \frac{\chi_{S1}}{\chi}$ (ratio of diffusivity of solute1 and heat diffusivity), $\tau_2 = \frac{\chi_{S2}}{\chi}$ (ratio of diffusivity of solute2 and heat diffusivity).

We now introduce asymptotic expansions as follows:

$$\left.\begin{array}{l} Ra = Ra_0 + \delta^2 Ra_2 + \delta^4 Ra_4 + ... \\ v = \delta v_1 + \delta^2 v_2 + \delta^3 v_3 + ... \\ \psi = \delta \psi_1 + \delta^2 \psi_2 + \delta^3 \psi_3 + ... \\ \theta = \delta \theta_1 + \delta^2 \theta_2 + \delta^3 \theta_3 + ... \\ \phi_{S1} = \delta \phi_{S11} + \delta^2 \phi_{S12} + \delta^3 \phi_{S13} + ... \\ \phi_{S2} = \delta \phi_{S21} + \delta^2 \phi_{S22} + \delta^3 \phi_{S23} + ... \end{array}\right\} \tag{11}$$

At the lowest mode, we have,

$$\left.\begin{array}{l} \begin{bmatrix} -\nabla^4 + C\nabla^6 & -\sqrt{Ta}\frac{\partial}{\partial z} & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \sqrt{Ta}\frac{\partial}{\partial z} & -\nabla^2 + C\nabla^4 & 0 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_1 \nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & 0 & -\tau_2 \nabla^2 \end{bmatrix} \\ \begin{bmatrix} \psi_1 \\ v_1 \\ \theta_1 \\ \phi_{S11} \\ \phi_{S21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array}\right\} \tag{12}$$

The solution to the lowest order of the system is,

$$\left.\begin{array}{l} \psi_1 = A(\tau)\sin(ax)\sin(\pi z) \\ v_1 = -\frac{\pi\sqrt{Ta}\,A(\tau)}{k^2\left(1 + Ck^2\right)}\sin(ax)\cos(\pi z) \\ \theta_1 = -\frac{a}{k^2}A(\tau)\cos(ax)\sin(\pi z) \\ \phi_{S11} = -\frac{a}{k^2 \tau_1}A(\tau)\cos(ax)\sin(\pi z) \\ \phi_{S21} = -\frac{a}{k^2 \tau_2}A(\tau)\cos(ax)\sin(\pi z) \end{array}\right\} \tag{13}$$

The Rayleigh number for triple diffusive convection in a rotating couple stress liquid is

$$Ra_0 = \frac{k^6\left(1 + Ck^2\right)}{a^2} + \frac{\pi^2 Ta}{a^2\left(1 + Ck^2\right)} + \frac{R_{S1}}{\tau_1} + \frac{R_{S2}}{\tau_2} \tag{14}$$

where $k^2 = \pi^2 + a^2$.

At the second order, we have,

$$\left.\begin{array}{l} \begin{bmatrix} -\nabla^4 + C\nabla^6 & -\sqrt{Ta}\frac{\partial}{\partial z} & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \sqrt{Ta}\frac{\partial}{\partial z} & -\nabla^2 + C\nabla^4 & 0 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_1 \nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & 0 & -\tau_2 \nabla^2 \end{bmatrix} \\ \begin{bmatrix} \psi_2 \\ v_2 \\ \theta_2 \\ \phi_{S12} \\ \phi_{S22} \end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \\ R_{24} \\ R_{25} \end{bmatrix} \end{array}\right\} \tag{15}$$

where $R_{21} = 0$, $R_{22} = 0$, $R_{23} = \frac{\partial \psi_1}{\partial x}\frac{\partial \theta_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial \theta_1}{\partial x}$, $R_{24} = \frac{\partial \psi_1}{\partial x}\frac{\partial \phi_{S11}}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial \phi_{S11}}{\partial x}$, $R_{25} = \frac{\partial \psi_1}{\partial x}\frac{\partial \phi_{S21}}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial \phi_{S21}}{\partial x}$.

The second order solutions are obtained as follows:

$$\left.\begin{array}{l} \psi_2 = 0 \\ v_2 = 0 \\ \theta_2 = -\frac{a^2}{8\pi k^2}A^2(\tau)\sin(2\pi z) \\ \phi_{S12} = -\frac{a^2}{8\pi k^2 \tau_1^2}A^2(\tau)\sin(2\pi z) \\ \phi_{S22} = -\frac{a^2}{8\pi k^2 \tau_2^2}A^2(\tau)\sin(2\pi z) \end{array}\right\} \tag{16}$$

The horizontally averaged Nusselt number Nu and Sherwood numbers Sh_1 and Sh_2 are given by

$$Nu(\tau) = 1 + \frac{a^2}{8\pi k^2}A^2(\tau) \tag{17}$$

$$Sh_1(\tau) = 1 + \frac{a^2}{8\pi k^2 \tau_1^2}A^2(\tau) \tag{18}$$

$$Sh_2(\tau) = 1 + \frac{a^2}{8\pi k^2 \tau_2^2}A^2(\tau) \tag{19}$$

At the third order, we have,

$$\left.\begin{bmatrix} -\nabla^4 + C\nabla^6 & -\sqrt{Ta}\frac{\partial}{\partial z} & Ra\frac{\partial}{\partial x} & -R_{S1}\frac{\partial}{\partial x} & -R_{S2}\frac{\partial}{\partial x} \\ \sqrt{Ta}\frac{\partial}{\partial z} & -\nabla^2 + C\nabla^4 & 0 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & -\nabla^2 & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\tau_1\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & 0 & -\tau_2\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_3 \\ v_3 \\ \theta_3 \\ \phi_{S13} \\ \phi_{S23} \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \\ R_{34} \\ R_{35} \end{bmatrix} \right\} \tag{20}$$

where

$$\begin{aligned} R_{31} &= -Ra\frac{\partial \theta_1}{\partial x} - \frac{1}{Pr}\frac{\partial}{\partial \tau}\left(\nabla^2 \psi_1\right) \\ R_{32} &= -\frac{1}{Pr}\frac{\partial v_1}{\partial x} \\ R_{33} &= -\frac{\partial \theta_1}{\partial \tau} + J\left(\psi_1, \theta_2\right) \\ R_{34} &= -\frac{\partial \phi_{S11}}{\partial \tau} + J\left(\psi_1, \phi_{S12}\right) \\ R_{35} &= -\frac{\partial \phi_{S21}}{\partial \tau} + J\left(\psi_1, \phi_{S22}\right) \end{aligned} \tag{21}$$

For the existence of the third order solution of the system, we apply the solvability condition which leads to arrive at the autonomous Ginzburg-Landau equation in the form

$$A_1 A'(\tau) - A_2 A(\tau) + A_3 A^3(\tau) = 0 \tag{22}$$

where

$$A_1 = \frac{k^2}{Pr} + \frac{\pi^2 Ta}{k^4\left(1 + Ck^2\right)^2} + \left(-Ra + \frac{R_{S1}}{\tau_1^2} + \frac{R_{S2}}{\tau_2^2}\right)\frac{a^2}{k^4}$$

$$A_2 = -\frac{a^2\, Ra}{k^2}$$

$$A_3 = \left(-Ra + \frac{R_{S1}}{\tau_1^3} + \frac{R_{S2}}{\tau_2^3}\right)\frac{a^4}{8k^4}$$

The Ginzburg-Landau equation given in (22) is Bernoulli equation and obtaining its analytical solution is difficult as it is autonomous in nature. In view of this, it has been solved numerically using Mathematica, subject to the initial condition $A(0) = a_0$, where a_0 is the chosen initial amplitude of convection.

III. Results and Discussions

In this paper, a nonlinear analysis is done to study the effect of triple diffusive convection in a rotating couple stress liquid. Autonomous Ginzburg-Landau equation is derived to study the effects of parameters of the problem on heat and mass transfer.

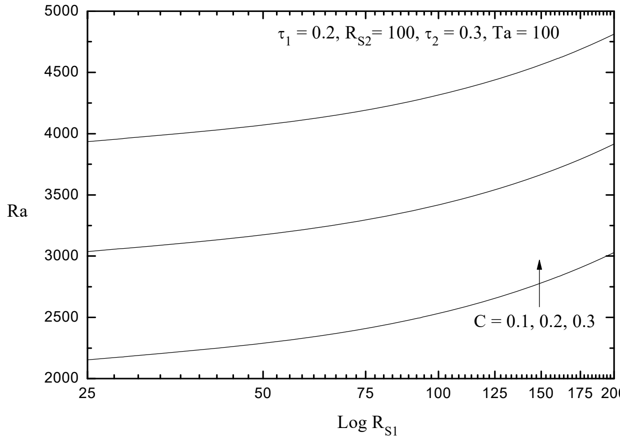

Fig. 2 Plot of solute Rayleigh number1 R_{S1} versus Rayleigh number *Ra* for different values of couple stress parameter *C*

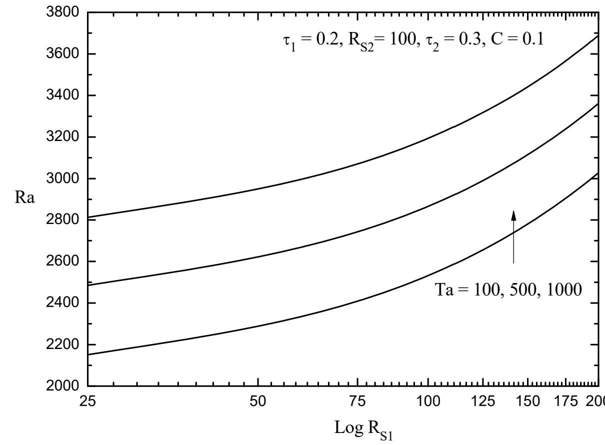

Fig. 3 Plot of solute Rayleigh number1 R_{S1} versus Rayleigh number *Ra* for different values of Taylor number *Ta*

Figs. 2-4 are the plots of Rayleigh number *Ra* versus solute Rayleigh number1 R_{S1} for different values of couple stress parameter *C*, Taylor number *Ta*, and solute Rayleigh number2 R_{S2}, respectively. From Fig. 2, we observe that as *C* increases, *Ra* also increases. This may be attributed to the fact that the presence of couple stress is to increase the viscosity of the liquid and hence higher heating is required to have the instability with an increasing value of *C*. Thus, increase in *C* delays the onset of triple diffusive convection and hence stabilizes the system. From Fig. 3, we observe that as *Ta* increases, *Ra* also increases, indicating that effect of increasing the Taylor number *Ta* is to stabilize the flow. The stabilizing effect of *Ta* is due to the fact that rotation induces vorticity into the fluid. Thus, the fluid moves in horizontal planes with higher velocity. On account of this motion, the velocity of the fluid perpendicular to the plane reduces. Thus, the onset of triple diffusive convection is delayed. From Fig. 4, we observe that as R_{S2} increases, *Ra* and R_{S1} also increase. This is because when the solutes are added from below, the concentration of solutes settles at the bottom boundary without disturbing the system, and hence, the solute Rayleigh numbers enhance the stability of the system.

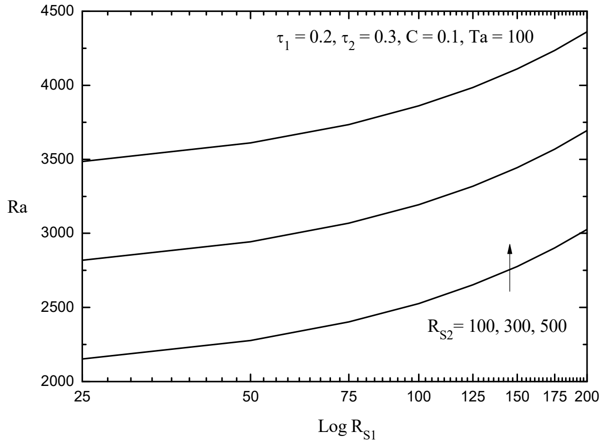

Fig. 4 Plot of solute Rayleigh number 1 R_{S1} versus Rayleigh number *Ra* for different values of solute Rayleigh number 2 R_{S2}

Figs. 5-10 are the plots of Nusselt number *Nu*, Sherwood number 1 Sh_1 and Sherwood number 2 Sh_2, respectively versus time τ for different values of *C* and *Ta*. From the figures, we observe that increase in *C* and *Ta*, increases the heat and mass transfer.

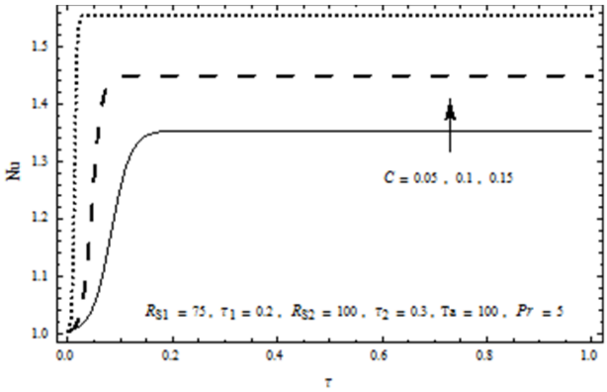

Fig. 5 Nusselt number *Nu* versus time τ for different values of *C*

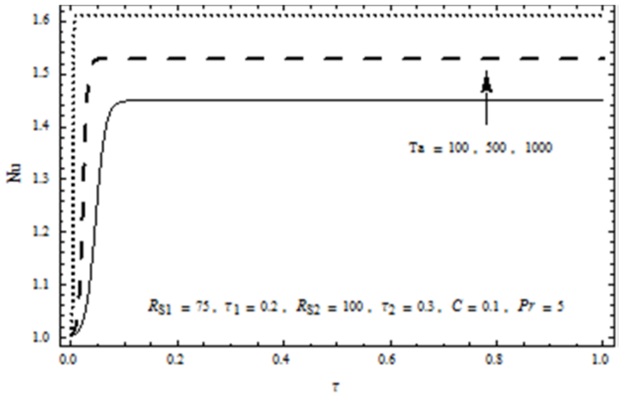

Fig. 6 Nusselt number *Nu* versus time τ for different values of *Ta*

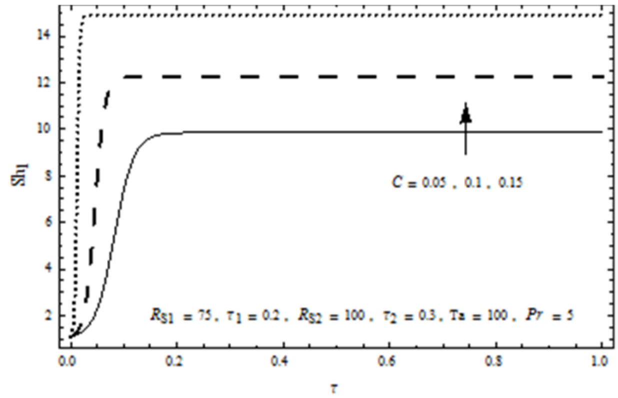

Fig. 7 Sherwood number 1 Sh_1 versus time τ for different values of *C*

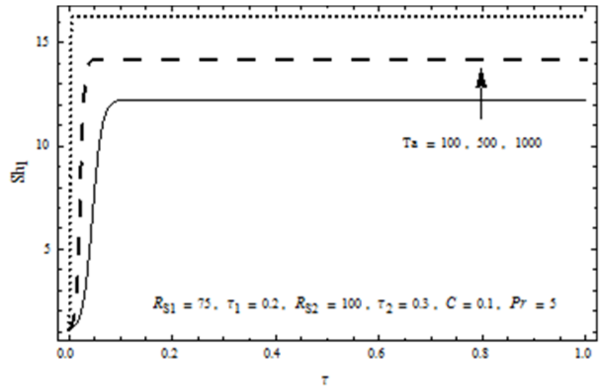

Fig. 8 Sherwood number 1 Sh_1 versus time τ for different values of *Ta*

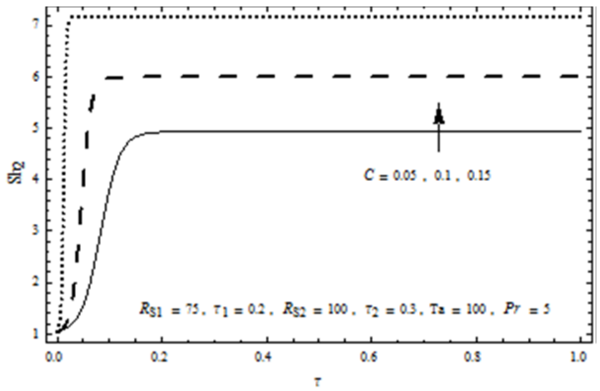

Fig. 9 Sherwood number 2 Sh_2 versus time τ for different values of C

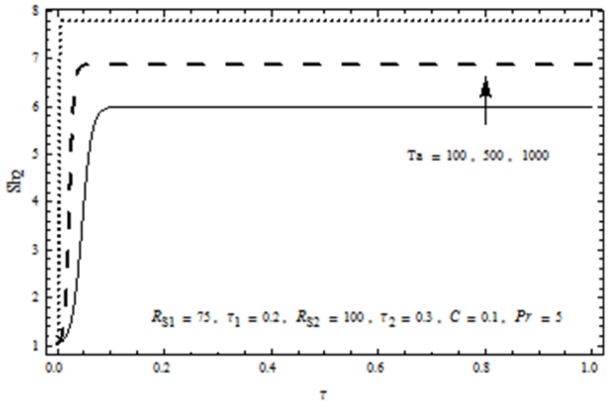

Fig. 10 Sherwood number2 Sh_2 versus time τ for different values of Ta

IV. Conclusion

We observed that it is possible to control the onset of triple diffusive convection and also to regulate the heat and mass transfer with the help of couple stress liquid and rotation.

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References

[1] A. J. Pearlstein, R. D. Harris and G. Terrones, "The onset of convective instability in a triply diffusive fluid layer," *J. Fluid Mech.*, vol. 202, pp. 443-465, 1989.
[2] R. A. Lopez, L. A. Romero and A. J. Pearlstein, "Effect of rigid boundaries on the onset of convective instability in a triply diffusive fluid layer," *Phys. Fluids A*, vol. 2, pp. 897, 1990.
[3] R. Sumithra, "Exact solution of triple diffusive Marangoni-convection in a composite layer," *Int. J. Engg. Research and Tech.*, vol. 1, no. 5, pp. 1-13, 2012.
[4] S. Rionero, "Triple diffusive convection in porous media," *Acta Mech.*, vol. 224, pp. 447–458, 2013.
[5] Sameena Tarannum and S. Pranesh, "Triple diffusive convection in Oldroyd-B liquid," *IOSR J. Math.*, vol. 12, no. 4, pp. 7-13, 2016.
[6] S. Chandrasekhar, "Hydrodynamic and hydromagnetic stability," *Oxford: Clarendon Press*, 1961.
[7] V. K. Stokes, "Couple stress in fluids," *Phys. Fluids*, pp. 1079-1715, 1966.
[8] P. G. Siddheshwar and S. Pranesh, "An analytical study of linear and non-linear convection in Boussinesq-Stokes suspensions," *Int. J. Non-Linear Mech.*, pp. 165-172, 2004.
[9] I. S. Shivakumara, S. Sureshkumar and N. Devaraju, "Effect of Non-Uniform Temperature Gradients on the Onset of Convection in a Couple-Stress Fluid-Saturated Porous Media," *J. Applied Fluid Mech.*, vol. 5, pp. 49-55, 2012.
[10] S. Pranesh and Sameena Tarannum, "Linear and weakly non-linear stability analyses of double-diffusive electro-convection in a micropolar fluid," *IOSR J. Math.*, vol. 11, no. 6(1), pp. 44-70, 2015.
[11] P. G. Siddheshwar, "A series solution for the Ginzburg-Landau Equation with a time-periodic coefficient," Appl. Math. vol. 3, pp. 542-554, 2010.