Study on Robot Trajectory Planning by Robot End-Effector Using Dual Curvature Theory of the Ruled Surface

Y. S. Oh, P. Abhishesh, B. S. Ryuh

Abstract**—**This paper presents the method of trajectory planning by the robot end-effector which accounts for more accurate and smooth differential geometry of the ruled surface generated by tool line fixed with end-effector based on the methods of curvature theory of ruled surface and the dual curvature theory, and focuses on the underlying relation to unite them for enhancing the efficiency for trajectory planning. Robot motion can be represented as motion properties of the ruled surface generated by trajectory of the Tool Center Point (TCP). The linear and angular properties of the six degree-of-freedom motion of end-effector are computed using the explicit formulas and functions from curvature theory and dual curvature theory. This paper explains the complete dualization of ruled surface and shows that the linear and angular motion applied using the method of dual curvature theory is more accurate and less complex.

Keywords**—**Dual curvature theory, robot end effector, ruled surface, TCP, tool center point.

I. Introduction

THE robot follows a specific path in order to perform a task. The path of the robot is determined as per the task to be performed, and the path planning of the robot is one of the most important researches in the robotics field. If precise path control is required for surfaces defined as free-form surfaces or analytical surfaces, or if path is to be traced, the number of data (points) necessary for the robot is selected, and the position and orientation (azimuth) values of the robot are obtained by the interpolation method.

Now, the accuracy of the path is proportional to the number of data points. Thus, the interpolation control of the robot path currently used has a limitation based on the approximation method.

The robot path is a continuous representation of position and orientation in three-dimensional space. Among them, there are discrete transformation methods such as quadratic transformation matrix, Quaternian, and Euler angles [1]. These methods have high disadvantages that they cannot provide continuity because they have high redundancy.

This paper focuses on the fact that the trajectory drawn by moving the robot end effector forms a ruled surface. A ruled surface is a trajectory that is drawn as a linear component moves in space. By applying the curved surface curl theory to the robot path, it is possible to obtain properties (speed, acceleration, angular velocity, angular velocity, etc.) of the robot motion by a mathematical method rather than an interpolation method. This provides a method for precise control of the robot when the curved surface to be traced is an analytical surface or a free curved surface. The advantage is that it directly controls the robot by obtaining the robot motion coefficients directly on the curved surface instead of by the interpolation method.

McCarthy and Roth [2] in Ruled Surface Curvature Theory have dealt with the theoretical study of linear trajectories in kinematics, Ryuh and Pennock [3]-[5] applied the curved surface curvature theory to DoCarmo's [6] applied analytical geometry theory, for the robot end-effector motion study, Veldcamp [7] detailed the dual curves for linear curvature theory, and Kirson [8] has generalized the dual formulas for obtaining the higher-order path curvature theory of the ruled surface.

In this paper, we use differential geometry [9] and apply the dual curvature theory to the ruled surfaces drawn by the end effector of the robot. Especially, by applying the dual curvature theory, the 1st and 2nd derivatives of the robot end effector are represented by the expression including the position and the posture, thereby reducing the calculation amount and enhancing the intuitiveness of the expression.

II. Robot Path Representation Using Ruled Surface

The path of the robot can be represented by the tool point and the end effector angle. In Fig. 1, the tool coordinate system is represented by three orthogonal unit position vectors $\mathbf{O}, \mathbf{A}, \mathbf{N}$ (boldface is displayed in the vector sense) attached to the tool point. In addition, the path drawn by the unit vector $\mathbf{O}$ in the tool coordinate system is called "Directrix", and a ruled surface is generated according to the directrix.

In Fig. 1, the vector $\mathbf{O}$ indicates the ruling direction, which is the direction of the end-effector. In order to express space, six degrees of freedom, such as position three degrees of freedom and three degrees of freedom of the bearing, are required, but they can be expressed by five variables. For this purpose, spin angle ($\boldsymbol{\eta}$) is defined as shown in Fig. 1. This unit normal vector is represented by an angle from $\mathbf{S_n}$ to $\mathbf{N}$.

The ruled surface is defined as:

This research is supported by Automotive New Technology Research Center.

Abhishesh. P. is with the Department of Mechanical System Engineering, Chonbuk National University, Jeonju, South Korea (e-mail: abhishesh.pal6@gmail.com).

B. S. Ryuh is with the Department of Mechanical System Engineering, Chonbuk National University, Jeonju, South Korea (phone: +82-10-5628-2480, e-mail: ryuhbs@jbnu.ac.kr).

$$\mathbf{X}(\psi, \nu) = \boldsymbol{\alpha}(\psi) + \nu\mathbf{r}(\psi) \tag{1}$$

$\boldsymbol{\nu}$ is an arbitrary real number, $\boldsymbol{\psi}$ is an independent variable, $\boldsymbol{\alpha}(\boldsymbol{\psi})$ is constant, and $\mathbf{O}$ is a directrix, $\mathbf{r}(\boldsymbol{\psi})$ is the ruling constant and corresponds to the direction of $\mathbf{O}$. The motion of the robot end effector can be completely represented by the ruled surface and the spin angle, and it can be used for the robot path planning by describing the ruled surface as an analytical function. Even if the ruled surface cannot be represented as a complete function, it can be expressed using a separate curve generation technique [5].

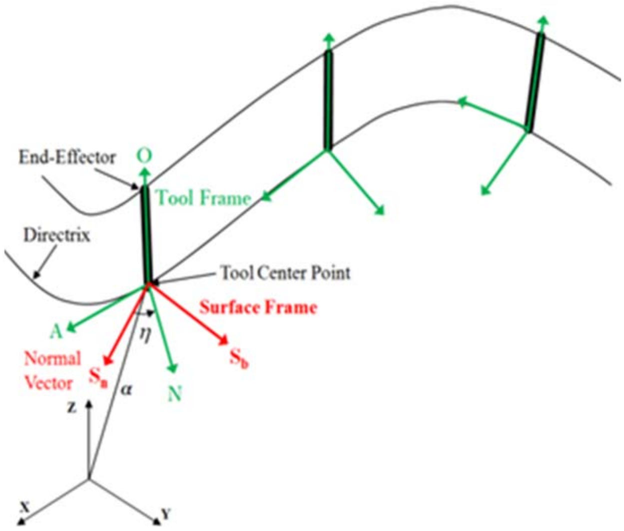

Fig. 1 Ruled Surface and spin angle

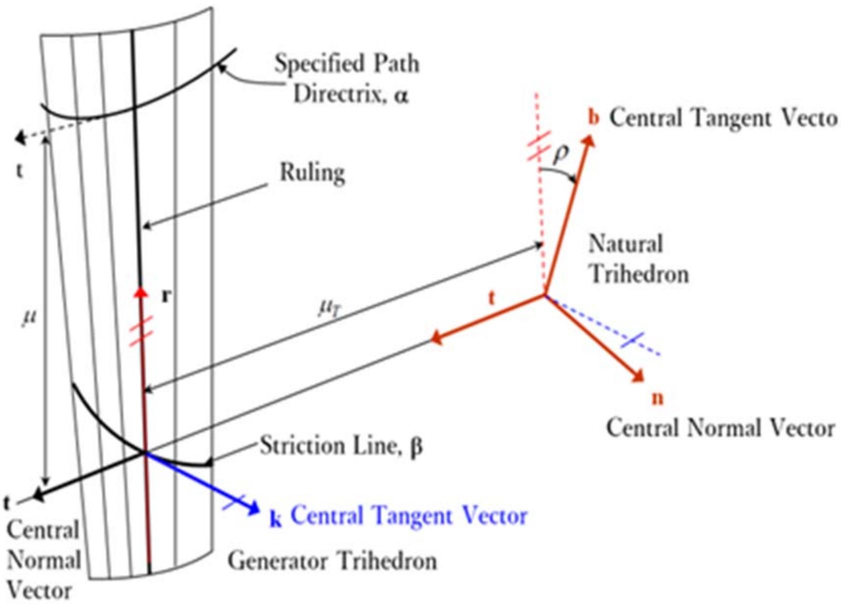

Fig. 2 Frames of reference

III. Curvature Theory of Ruled Surfaces

Robot path control requires information of the position (position, velocity, acceleration) and posture (angle, angular velocity, and angular velocity) of the end effector, which can be obtained through the curved surface curvature theory. The ruled surface, as shown in Fig. 2, has a coordinate system consisting of Generator trihedron and the unique Natural trihedron. Generator trihedron consists of unit orthogonal vectors $\mathbf{r}, \mathbf{t}, \mathbf{k}$ called Generator vector, Central normal vector, and Central tangent vector, respectively. It is expressed as:

$$\mathbf{r}, \mathbf{t} = \frac{\mathrm{dr}}{\mathrm{d}\psi} \Big/ \left|\frac{\mathrm{dr}}{\mathrm{d}\psi}\right|, \mathbf{k} = \mathbf{r} \times \mathbf{t} \tag{2}$$

$\boldsymbol{\psi}$ has been used as an independent variable for the ruled surface in (1), considering its usefulness for future calculations. Instead of using new normalized parameter, the arc length of the ruling $\mathbf{r}(\boldsymbol{\psi})$ is used. This arclength parameter, $\mathbf{s}(\boldsymbol{\psi})$ is defined as:

$$\mathrm{s}(\psi) = \int_0^{\psi} \left|\frac{\mathrm{dr}(\psi)}{\mathrm{d}\psi}\right| \mathrm{d}\psi \tag{3}$$

Here, the integral term $\left|\frac{\mathrm{d}\alpha(\psi)}{\mathrm{d}(\psi)}\right|$ is referred to as the angular velocity of the ruling $\mathbf{r}(\psi)$, and is denoted by $\boldsymbol{\delta}$. If $\boldsymbol{\delta} \neq \mathbf{0}$ in (3), which gives $\mathbf{r}(\psi(\mathrm{s})) = \mathbf{r}(\mathrm{s})$ and $\psi(\mathrm{s})$ can easily be inversely transformed. Using the normalization technique, (2) $\mathbf{t}$ can be expressed as:

$$\mathbf{t} = \mathbf{r}' \tag{4}$$

where " ′ " denotes the derivative of the arc length $s(\psi)$.The origin of the Generator trihedron is the Striction point, and this trajectory is called the Striction line $\boldsymbol{\beta}$ which is defined as:

$$\boldsymbol{\beta}(\psi) = \boldsymbol{\alpha}(\psi) - \boldsymbol{\mu}(\psi)\mathbf{r}(\psi) \tag{5}$$

Here, the coefficient $\boldsymbol{\mu}(s)$ is a Striction point and the position of the tool point is as:

$$\boldsymbol{\mu}(\psi) = \frac{\boldsymbol{\alpha}'(\psi)\cdot\mathbf{r}'(\psi)}{|\mathbf{r}'(\psi)|^2} \tag{6}$$

The first angular momentum coefficient of the Generator trihedron, we can get by (8).

$$\frac{d}{ds}\begin{bmatrix}\mathbf{r}\\ \mathbf{t}\\ \mathbf{k}\end{bmatrix} = \mathbf{u_r} \times \begin{bmatrix}\mathbf{r}\\ \mathbf{t}\\ \mathbf{k}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -1 & 0 & \gamma\\ 0 & -\gamma & 0\end{bmatrix}\begin{bmatrix}\mathbf{r}\\ \mathbf{t}\\ \mathbf{k}\end{bmatrix} \tag{7}$$

$\mathbf{u_r}$ is called the Darboux vector of the Generator trihedron, which represents the primary angular motion of the ruled surface. (7) is known as skew symmetric matrix which can also be expressed as:

$$\mathbf{u_r} = \gamma\mathbf{r} + \mathbf{k} \tag{8}$$

γ is a geodesic curvature and is defined as a curved line that is constrained on the curved surface by a point mass and is not subjected to force. It is defined as:

$$\gamma = \cot\rho = (\mathbf{r}\times\mathbf{r}')\cdot\mathbf{r}'' \tag{9}$$

First derivative of Striction point of the ruled surface, $\boldsymbol{\beta}'$, can be expressed as:

$$\boldsymbol{\beta}' = \boldsymbol{\Gamma}\mathbf{r} + \boldsymbol{\Delta}\mathbf{k} \tag{10}$$

In (10), $\boldsymbol{\Gamma}$ and $\boldsymbol{\Delta}$ can be defined as:

$$\boldsymbol{\Gamma} = \boldsymbol{\alpha}'\cdot\mathbf{r} - \boldsymbol{\mu}', \boldsymbol{\Delta} = \boldsymbol{\alpha}'\cdot(\mathbf{r}\times\mathbf{r}') \tag{11}$$

$\gamma, \boldsymbol{\Gamma}, \boldsymbol{\Delta}$ are expressed in (9) and (11) and are called Curvature functions of ruled surface [2].

IV. Introduction of Dual Coordinate System

If one can identify both the position and the orientation components of the robot end effector, then they will be able to interpret the robot work more intuitively. This section introduces dual curves. The dual curves are divided into real and dual parts. The real part shows the kinetic component of the rotation, and the dual part shows the kinetic component of the length.

For the introduction of the dual coordinate system, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are arbitrary vectors. Consider the case where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ vectors do not exist in one plane in a certain space, then an arbitrary vector $\mathbf{v} = \boldsymbol{\alpha}\mathbf{a} + \boldsymbol{\beta}\mathbf{b} + \gamma\mathbf{c}$ is shown in Fig. 3.

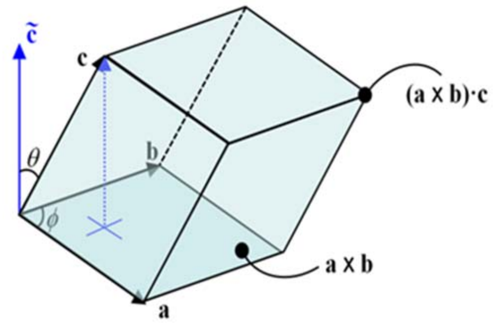

Fig. 3 Dual coordinate systems

If we multiply $a \times b$ on both the side, γ can be found easily as shown in (12). In the same way, α, β can also be found.

$$\gamma = \frac{\mathbf{a}\times\mathbf{b}}{(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}}\cdot\mathbf{v} \tag{12}$$

Here, the denominator is the same size and the dual coordinate axis, $\tilde{\boldsymbol{c}}$ is as:

$$\tilde{\boldsymbol{c}} = \frac{\mathbf{a}\times\mathbf{b}}{(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}} \tag{13}$$

$$\tilde{\boldsymbol{c}} = \frac{1}{|c|cos\theta}\bar{\tilde{\boldsymbol{c}}}, \text{where, } \bar{\tilde{\boldsymbol{c}}} \text{ is unit vector} \tag{14}$$

Therefore, $\alpha = \tilde{\mathbf{a}}\cdot\mathbf{v}, \beta = \tilde{\mathbf{b}}\cdot\mathbf{v}, \gamma = \tilde{\mathbf{c}}\cdot\mathbf{x}$, the $\mathbf{v}$ be written as shown in:

$$\mathbf{v} = \tilde{\mathbf{a}}\cdot\mathbf{v}\mathbf{a} + \tilde{\mathbf{b}}\cdot\mathbf{v}\mathbf{b} + \tilde{\mathbf{c}}\cdot\mathbf{v}\mathbf{c} \tag{15}$$

As shown in the above equation, three dual coordinate axes $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, and $\tilde{\mathbf{c}}$, together form a dual coordinate system, which is defined by two other external vector for the volume created by three vectors **a**, **b**, and **c**.

As stated above that the space vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ do not lie on the same plane hence the condition $(\mathbf{a}\times\mathbf{b})\cdot\ \mathbf{c} \neq \mathbf{0}$ does not exist. And we can say that, the vector $\tilde{\mathbf{a}}\cdot\mathbf{a} = \tilde{\mathbf{b}}\cdot\mathbf{b} = \tilde{\mathbf{c}}\cdot\mathbf{c} = 1$ fulfills the mentioned condition.

If the vector $\mathbf{a}, \mathbf{b}, \mathbf{c}$ is defined as a unit Cartesian coordinate system $(\mathbf{a}\times\mathbf{b})\cdot\ \mathbf{c} = 1(\theta = 0°, \cos\theta = 1)$, then the dual coordinate system becomes $\tilde{\mathbf{a}} = \mathbf{b}\times\mathbf{c}, \tilde{\mathbf{b}} = \mathbf{c}\times\mathbf{a}, \tilde{\mathbf{c}} = \mathbf{a}\times\mathbf{b}$. The dual coordinate axes representing the dual parts are represented as cross product of the other two vectors. The characteristics of the dual part are similar to the characteristics of a screw according to the calculation.

V. Dual Curvature Theory of Ruled Surfaces

This section shows the differential surface geometry of a ruled surface using a dual vector in a three-dimensional dual curve. This result is represented by a set of dual functions that characterize the ruled surface.

The dual vector theory consists of the Plücker vector $\mathbf{r}$ of the line $\mathbf{X}$ and $\boldsymbol{\alpha}\times\mathbf{r}$ ($\mathbf{r}$ is the direction of the line $\mathbf{X}$ and $\boldsymbol{\alpha}$ is the arbitrary point on the line $\mathbf{X}$) as a single dual vector

($\hat{\mathbf{r}} = \mathbf{r} + \epsilon\boldsymbol{\alpha} \times \mathbf{r}$). Symbol ϵ is a dual unit. The ϵ operation used is same as the operation on real numbers, except for the property of $\epsilon^2 = 0$. All vector algebra operations can be used to manipulate dual vectors [10], [11], which is advantageous since it reduces the amount of computation by allowing simple vector operations to represent lines in space.

Equation (16) representing the ruled surface can be represented by a dual vector function as:

$$\hat{\mathbf{r}}(\psi) = \mathbf{r}(\psi) + \epsilon\boldsymbol{\alpha}(\psi) \times \mathbf{r}(\psi) \quad (16)$$

The generator trihedron is defined as three dual-unit orthogonal vectors such as a dual generator vector $\hat{\mathbf{r}}$, a dual central normal vector $\hat{\mathbf{t}}$, and a dual central tangent vector $\hat{\mathbf{k}}$, which can be expressed as:

$$\hat{\mathbf{r}}, \hat{\mathbf{t}} = \frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}\psi} \Big/ \left|\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}\psi}\right|, \hat{\mathbf{k}} = \hat{\mathbf{r}} \times \hat{\mathbf{t}} \quad (17)$$

The dual arc length $\mathrm{s}(\psi)$ of the ruled surface $\hat{\mathbf{r}}(\psi)$ can be defined as:

$$\hat{s}(\psi) = \int_0^{\psi} \left|\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}\psi}\right| \mathrm{d}\psi \quad (18)$$

The integral term $|(\mathrm{d}\hat{\mathbf{r}})/\mathrm{d}\psi|$ of (18) is referred to as the dual velocity $\mathbf{r}\hat{}(\psi)$, which can be expressed as:

$$\hat{\delta} = \left|\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}\psi}\right| = \left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\psi}\right| [1 + \epsilon\boldsymbol{\alpha}' \cdot (\mathbf{r} \times \mathbf{r}')] \quad (19)$$

Since the dual term Δ on the right-hand side of (19) is the same as that defined in (11), it can be written as:

$$\hat{\delta} = \delta[1 + \epsilon\Delta] \quad (20)$$

In addition, the following relation holds between s and $\hat{\mathrm{s}}$ defined in (3), (24).

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}} = \frac{1}{1+\epsilon\Delta}\frac{\mathrm{d}}{\mathrm{d}s} \quad (21)$$

If $\hat{\delta} \neq 0$, then $\hat{\mathbf{r}}(\psi(s)) = \hat{\mathbf{r}}(s)$ can be used in (22). In addition, using the normalization technique, the dual central normal vector $\hat{\mathbf{t}}$ of (17) can be expressed as:

$$\hat{\mathbf{t}} = \mathrm{d}\hat{\mathbf{r}}/\mathrm{d}\hat{s} \quad (22)$$

The first dual differential value of the ruled surface generator trihedron can be obtained as:

$$\frac{\mathrm{d}}{\mathrm{d}\hat{s}}\begin{bmatrix}\hat{\mathbf{r}}\\ \hat{\mathbf{t}}\\ \hat{\mathbf{k}}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -1 & 0 & \hat{\gamma}\\ 0 & -\hat{\gamma} & 0\end{bmatrix}\begin{bmatrix}\hat{\mathbf{r}}\\ \hat{\mathbf{t}}\\ \hat{\mathbf{k}}\end{bmatrix} = \hat{\mathbf{u}}_{\mathrm{r}} \times \begin{bmatrix}\hat{\mathbf{r}}\\ \hat{\mathbf{t}}\\ \hat{\mathbf{k}}\end{bmatrix} \quad (23)$$

The dual vector is defined as the Dual Darboux vector, $\hat{\mathbf{u}}_{\mathrm{r}}$, of the Generator trihedron and can be expressed as:

$$\hat{\mathbf{u}}_{\mathrm{r}} = \hat{\gamma}\hat{\mathbf{r}} + \hat{\mathbf{k}} \quad (24)$$

$\hat{\gamma}$ is called dual geodesic curvature and is defined as:

$$\hat{\gamma} = \frac{\mathrm{d}\hat{\mathbf{t}}}{\mathrm{d}\hat{s}} \cdot \hat{\mathbf{k}} \quad (25)$$

VI. Robot Motion with Dual Curves

In this section, the information for robot path control, i.e., the linear motion coefficient of the tool point and the kinematic coefficients of the tool coordinate system are applied in the form of a dual curve. The reference coordinate system for studying the end effector motion of the robot is shown in Fig. 4. In Fig. 4, the tool frame has an orientation vector, $\mathbf{O}$ an approach vector, $\mathbf{A}$ and a normal vector, $\mathbf{N}$, and their dual unit vectors are represented by a dual orientation vector, a dual approach vector, and a dual normal vector $[\hat{\mathbf{O}}, \hat{\mathbf{A}}, \hat{\mathbf{N}}]^{\mathrm{T}}$.

Surface frame $[\hat{\mathbf{O}}, \hat{\mathbf{S}}_{\mathbf{n}}, \hat{\mathbf{S}}_{\mathbf{b}}]^{\mathrm{T}}$ has dual orientation vector, $\hat{\mathbf{O}}$ dual surface normal vector, $\hat{\mathbf{S}}_{\mathbf{n}}$, dual binormal vector, $\hat{\mathbf{S}}_{\mathbf{b}}$, etc. The dual surface normal vector $\hat{\mathbf{S}}_{\mathbf{n}}$ can be obtained as:

$$\hat{\mathrm{S}}_{\mathrm{n}} = \left.\frac{\frac{\partial\hat{\mathrm{X}}}{\partial\hat{\mathrm{v}}} \times \frac{\partial\hat{\mathrm{X}}}{\partial\hat{\mathrm{s}}}}{\left|\frac{\partial\hat{\mathrm{X}}}{\partial\hat{\mathrm{v}}} + \frac{\partial\hat{\mathrm{X}}}{\partial\hat{\mathrm{s}}}\right|}\right|_{\hat{\mathrm{v}}=0} = \frac{-\hat{\Delta}\hat{\mathbf{t}} + \hat{\mu}\hat{\mathbf{k}}}{\sqrt{\hat{\Delta}^2 + \hat{\mu}^2}} \quad (26)$$

$\hat{\mathbf{S}}_{\mathbf{b}}$ can be obtained by cross product of the dual orientation vector, $\hat{\mathbf{O}}$,and the dual surface normal vector, $\hat{\mathbf{S}}_{\mathbf{n}}$

$$\hat{\mathbf{S}}_{\mathbf{b}} = \hat{\mathbf{O}} \times \hat{\mathbf{S}}_{\mathbf{n}} = \frac{-\hat{\mu}\hat{\mathbf{t}} - \hat{\Delta}\hat{\mathbf{k}}}{\sqrt{\hat{\Delta}^2 + \hat{\mu}^2}} \quad (27)$$

The relationship between the tool coordinate system and the Generator trihedron coordinate system is expressed as:

$$\begin{bmatrix}\hat{\mathbf{O}}\\ \hat{\mathbf{A}}\\ \hat{\mathbf{N}}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\hat{\eta} & \sin\hat{\eta}\\ 0 & -\sin\hat{\eta} & \cos\hat{\eta}\end{bmatrix}\begin{bmatrix}\hat{\mathrm{O}}\\ \hat{\mathrm{S}}_{\mathrm{n}}\\ \hat{\mathrm{S}}_{\mathrm{b}}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\hat{\Sigma} & \sin\hat{\Sigma}\\ 0 & -\sin\hat{\Sigma} & \cos\hat{\Sigma}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{r}}\\ \hat{\mathbf{t}}\\ \hat{\mathbf{k}}\end{bmatrix} \quad (28)$$

In (28), $\hat{\Sigma} = \hat{\eta} + \hat{\sigma}$ and $\sin\hat{\sigma} = \frac{\hat{\mu}}{\sqrt{\hat{\Delta}^2 + \hat{\mu}^2}}$. If (28) is differentiated, it can be summarized as:

$$\begin{bmatrix}\widehat{\mathbf{O}'}\\ \hat{\mathbf{A}}'\\ \hat{\mathbf{N}}'\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -\cos\hat{\Sigma} & -\hat{\Omega}\sin\hat{\Sigma} & \hat{\Omega}\cos\hat{\Sigma}\\ \sin\hat{\Sigma} & -\hat{\Omega}\cos\hat{\Sigma} & -\hat{\Omega}\sin\hat{\Sigma}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{r}}\\ \hat{\mathbf{t}}\\ \hat{\mathbf{k}}\end{bmatrix} \quad (29)$$

In (29), $\hat{\Omega} = \hat{\Sigma}' + \hat{\gamma}$. If we denote $[\hat{\mathbf{r}}, \quad \hat{\mathbf{t}}, \quad \hat{\mathbf{k}}]^{\mathrm{T}}$ on the right side of (29) as $[\hat{\mathbf{O}}, \quad \hat{\mathbf{A}}, \quad \hat{\mathbf{N}}]^{\mathrm{T}}$, then the symmetric determinant is expressed as:

$$\begin{bmatrix}\widehat{\mathbf{O}'}\\ \hat{\mathbf{A}}'\\ \hat{\mathbf{N}}'\end{bmatrix} = \begin{bmatrix}0 & \cos\hat{\Sigma} & -\sin\hat{\Sigma}\\ -\cos\hat{\Sigma} & 0 & \hat{\Omega}\\ \sin\hat{\Sigma} & \hat{\Omega} & 0\end{bmatrix}\begin{bmatrix}\hat{\mathbf{O}}\\ \hat{\mathbf{A}}\\ \hat{\mathbf{N}}\end{bmatrix} = \hat{\mathrm{u}}_0 \times \begin{bmatrix}\hat{\mathbf{O}}\\ \hat{\mathbf{A}}\\ \hat{\mathbf{N}}\end{bmatrix} \quad (30)$$

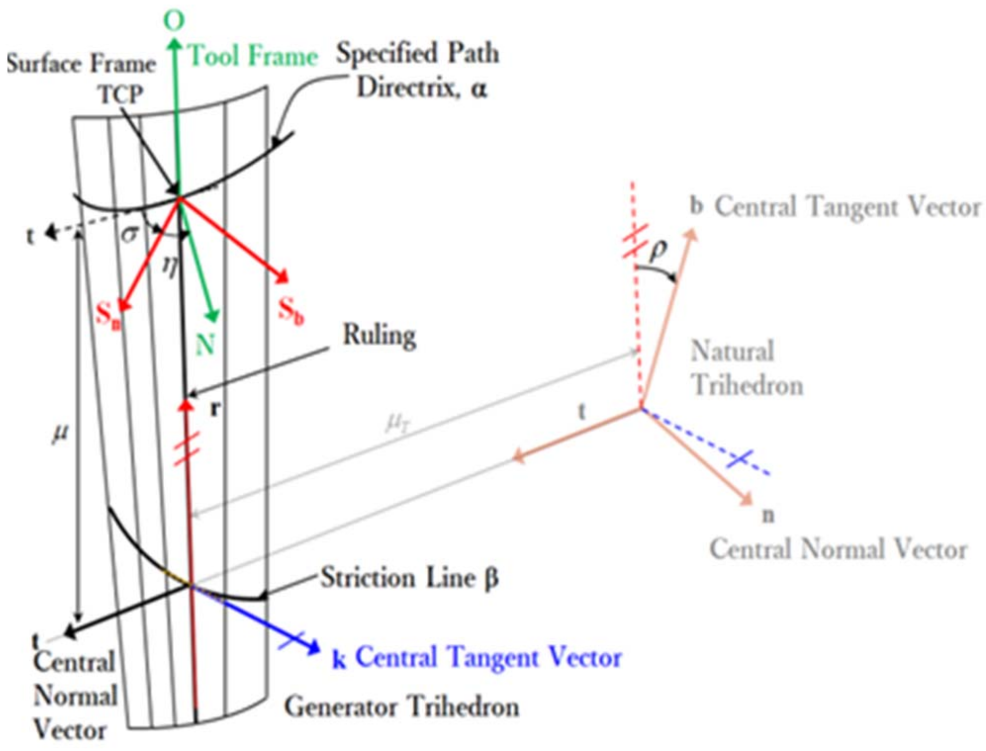

Fig. 4 Relation between surface and tool frame

$\hat{u}_0$ is obtained by the following equation:

$$\hat{u}_0 = \hat{\Omega}\hat{O} + \sin\hat{\Sigma}\,\hat{A} + \cos\hat{\Sigma}\,\hat{N} \tag{31}$$

If $[\hat{O},\ \hat{A},\ \hat{N}]$ is expressed as $[\hat{\mathbf{r}},\ \hat{\mathbf{t}},\ \hat{\mathbf{k}}]$ in (31), then:

$$\hat{u}_0 = \hat{\Omega}\hat{\mathbf{r}} + \hat{\mathbf{k}} \tag{32}$$

In (31), the dual instantaneous rotation vector of the dual tool coordinate system can be considered as the dual velocity vector of the robot end effector [12]. Therefore, the dual velocity of the robot end effector can be found on the norm of (31).

$$|\hat{\mathbf{u}}_0| = \sqrt{\hat{\mathbf{u}}_0 \cdot \hat{\mathbf{u}}_0} = \sqrt{(\hat{\Omega}\hat{\mathbf{r}} + \hat{\mathbf{k}}) \cdot (\hat{\Omega}\hat{\mathbf{r}} + \hat{\mathbf{k}})} \tag{33}$$

$$|\hat{\mathbf{u}}_0| = \sqrt{\hat{\Omega}^2 + 1} \tag{34}$$

The dual vector of $\hat{\mathbf{u}}_0$ can be expressed as $\hat{\mathbf{u}}_0 = \mathbf{u}_0 + \epsilon\tilde{\mathbf{u}}_0$. Thus, one can obtain (35) by substituting $\hat{\Omega} = \Omega + \epsilon\tilde{\Omega}$ and Taylor expansion for partitioning into real and dual parts:

$$\hat{\mathbf{u}}_0 = \sqrt{\hat{\Omega}^2 + 1} + \epsilon\frac{\Omega\tilde{\Omega}}{\sqrt{\Omega^2+1}} \tag{35}$$

Equation (35) is divided into $\mathbf{u}_0 = \sqrt{\Omega^2 + 1}$, and $\tilde{\mathbf{u}}_0 = \frac{\Omega\tilde{\Omega}}{\sqrt{\Omega^2+1}}$, the former represents the angular velocity of the robot end effector, and the latter represents the linear velocity.

In order to obtain the second-order differential value, (31) is differentiated and it becomes $\hat{\mathbf{u}}_0' = \hat{\Omega}'\hat{\mathbf{r}} + \hat{\Sigma}'\hat{\mathbf{t}}$. The dual acceleration vector equation is:

$$\hat{\mathbf{a}}_0 = |\hat{\mathbf{u}}_0'| = \sqrt{\hat{\mathbf{u}}_0' \cdot \hat{\mathbf{u}}_0'} \tag{36}$$

$$\hat{\mathbf{a}}_0 = \sqrt{\hat{\Omega}'^2 + \hat{\Sigma}'^2} \tag{37}$$

Here, $\hat{\mathbf{a}}_0$ can be expressed as a dual vector as $\hat{\mathbf{a}}_0 = \mathbf{a}_0 + \epsilon\tilde{\mathbf{a}}_0$, $\hat{\Omega} = \Omega' + \epsilon\tilde{\Omega}'$, $\hat{\Sigma} = \Sigma' + \epsilon\tilde{\Sigma}'$ and by introducing Taylor expansion, we divide into real part and dual part, and the following expression can be obtained:

$$\hat{\mathbf{a}}_0 = \sqrt{\Omega'^2 + \Sigma'^2} + \epsilon\frac{\Omega'\tilde{\Omega}' + \Sigma'\tilde{\Sigma}'}{\sqrt{\Omega'^2+\Sigma'^2}} \tag{38}$$

From (38) we get $\mathbf{a}_0 = \sqrt{\Omega'^2 + \Sigma'^2}$, $\tilde{\mathbf{a}}_0 = \frac{\Omega'\tilde{\Omega}' + \Sigma'\tilde{\Sigma}'}{\sqrt{\Omega'^2+\Sigma'^2}}$, the former represents the angular acceleration of the robot end effector, and the latter represents the acceleration.

Therefore, we apply the dual curvature theory to the ruled surface drawn by the end effector of the robot to obtain the motion components of the first and second derivatives, $\hat{\mathbf{u}}_0$, $\hat{\mathbf{a}}_0$ respectively. It can be seen that $\hat{\mathbf{u}}_0$ contains the motion of angle and position with respect to the first derivative, and $\hat{\mathbf{a}}_0$ contains the motion characteristic with angle and position as the second derivative.

VII. CONCLUSION

In this paper, we have explained the method of describing the trajectory of the robot's end effector using the differential geometry and the method of representing it as a three-dimensional dual vector using the dual curvature theory. Since the differential geometry of the curved surface is used unlike the interpolation method, which is mainly used for the robot path control, the 1st and 2nd differential motion coefficients (linear velocity, acceleration, angular velocity, angular velocity, etc.) can be obtained easily. The dual curvature theory proposed in this paper is divided into a real part and a dual part, and each motion component $\hat{\mathbf{u}}_0$, $\hat{\mathbf{a}}_0$ is obtained for the 1st and 2nd derivative values. $\hat{\mathbf{u}}_0$ contains the angular velocity and the linear velocity, and $\hat{a}_0$ contains the angular velocity and acceleration. This is advantageous because the motion expression for the robot's path becomes simpler and intuitive as compared with the existing curvature theory, and the calculation amount can also be reduced.

In the future, we plan to continue the research to compare the two values by performing the simulations using the curved surface theory of ruled surface and the dual curvature theory, and this can be widely used for robots that require precise position control afterwards.

REFERENCES

[1] Litvin, F. L. and Gao, X. C., "Analytical Representation of Trajectory of Manipulators," in *Trends and Developments in Mechanisms, Machines, and Robotics, The 1988 ASME Design Technology Conferences-20th Bie nnial Mechanisms Conference,* Kissimmee, Florida., 1988, Sept. 25-28, DE-Vol. 15-3, pp. 481-485.

[2] McCathy, J. M. and Roth, B., "The Curvature Theory of Line Trajectorie s in Spatial Kinematics," *ASME Journal of Mechanism Design,* vol. 103, No.4, 1981.

[3] Ryuh, B. S. and Pennock, G. R., "Accurate Motion of a Robot End-Effec tor using the Curvature Theory of a ruled surfaces," *Journal of Mechanis ms, Transmissions, and Automation in Design, Trans. ASME,* Dec. 1988, vol. 110, No.4, pp. 383-388.

[4] Ryuh, B. S., "Robot Trajectory planning using the curvature theory of ru led surfaces," *Doctoral dissertation,* 1989, Purdue University, West Lafa yette, Indiana, pp. 143.

[5] Ryuh, B. S. and Pennock, G. R., "Trajectory planning using the Furguso n curve model and curvature theory of a ruled surface," *Journal of Mech anical Design, Transactions of ASME,* 1990, Sep. Vol. 112, No. 4, pp. 3 77-383.

[6] DoCamo, M. P., "Differential Geometry of Curves and Surfaces," *Prenti ce-Hall,* New Jersey, 1976, pp. 503.

[7] Veldkamp, G. R., "On the Use of Dual Numbers, Vectors and Matrices i n Instantaneous Spatial Kinematics," *Mechanism and Machine Theory,*1 976, Vol. 11, No. 2, pp. 141-158.

[8] Kirson, Y., "High Order Curvature Theory in Space Kinematics," *Docto ral dissertation,* University of California, Berkeley, 1975, pp.140.

[9] Guggerhemier, H., "Differential Geometry," *Dover Publications,* 1977, pp.-378.

[10] Cumali Ekici, Yasin Unluturk, Mustafa Dede, Ryuh, B. S., "On Motion of Robot End-Effector Using the Curvature Theory of Timelike Ruled S urfaces with Timelike Rulings," *Mathematical Problems in Engineering,* Vol. 2008, pp.12.

[11] Kotelnikov, A. P., "Screw Calculus and Some Applications to Geometry and Mechanics," *Annals of Imperial University of Kazan,* 1895.

[12] Yücesan, A., Ayyıldız, N., Çöken, A. C., "On rectifying dual space curv es," *Revista Mathemática Computense*, 2007, Vol. 20(2), pp. 497-506.