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Travel Time Model for Cylinder Type Parking System

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***Abstract*—**In this paper, we mainly analyze an automated parking system where the storage and retrieval requests are performed by a tower crane. In this parking system, the S/R crane which is located at the middle of the bottom of the cylinder parking area can rotate in both clockwise and counterclockwise and three kinds of movements can be done simultaneously. We develop some mathematical travel time models for the single command cycle under the random storage assignment using the characteristics of this system. Finally, we compare these travel models with discrete case and it is shown that these travel models display a good satisfactory performance.

***Keywords*—**Parking system, travel time model, tower crane.

I. Introduction

WITH the rapid increase in the number of cars in recent years, parking problem is becoming more and more serious. The main reason for this problem is that the traditional parking lot does not meet the great demand due to some disadvantages such as a large amount of land occupation, difficulty of finding parking space, low efficiency of parking and so on. To solve this problem, we should build a more efficient parking system and we find that the automated storage/retrieval (AS/R) system can provide an efficient support. And many scholars have done much research work about this topic these years [1]-[5].

In our system, the parking area is cylindrical and the tower crane is located at the middle of the bottom of the parking area. Three kinds of movements can be done simultaneously to the S/R machine for the storage or retrieval request. In the vertical direction, the S/R machine can move up and down and in the horizontal direction, the S/R machine makes a rotation movement and a radial movement simultaneously. Depending on these characteristics, the car has access to visit any parking space in the system.

In Section II, some problem descriptions are made including assumptions and notations. Some mathematical travel time models of the S/R machine for the single command under the random storage assignment rule is derived in Section III. And in order to verify the rationality of the model, we compare the model with discrete case in Section IV. Finally, we make some conclusions and suggestions for future works in Section V.

II. Assumptions and Notions

The parking system has a tower type S/R machine which is located at the middle of the bottom can do three kinds of movements simultaneously, and in general, we assume I/O point is located at the lowest point of the parking area. Fig. 1 shows the general shape of the parking system.

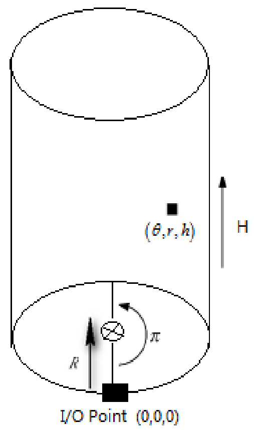

Fig. 1 The general shape of the parking system

Some assumptions are similar in literature [1]-[5]. We just emphasize the following points:

1. The tower crane type S/R machine can rotate clockwise and counterclockwise and three kinds of movements can be done. In the vertical direction, the S/R machine can move up and down and the S/R machine make a rotation movement and a radial movement simultaneously in the horizontal direction. The speed in each direction is a known constant.
2. All parking spaces are the same size and each different car is regarded as a unit load. Therefore all parking spaces are candidates for storing any car.

The following notions are used in this paper.

H : The height of the system.

R : The radius of the system.

S_v : Vertical speed of the S/R machine.

S_c : Rotation speed of the S/R machine.

S_r : Radial speed of the S/R machine.

t_v : The time from I/O point to the top of the system in vertical direction, $t_v = H/S_v$.

t_c : The time to rotate π rad from I/O point, $t_c = \pi/S_c$.

t_r : The time from I/O point to the center of system in horizontal direction, $t_r = R/S_r$.

T_h : $Max\{t_c, t_r\}$.

T : $Max\{T_h, t_v\}$.

b : $\min\{t_c/T_h, t_r/T_h\} \quad 0 \le b \le 1$.

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c : $\min\{T_h/T, t_v/T\}$ $\quad 0 \le c \le 1$.

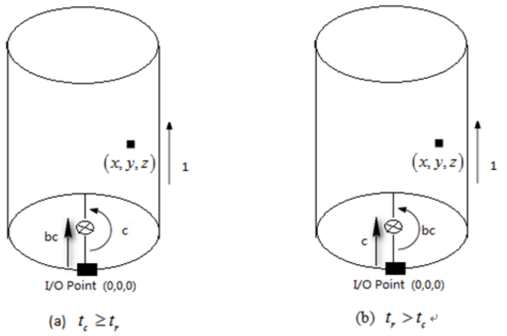


Fig. 2 The continuous representation of the parking area in case 1

III. Travel Time Model for Single Command Cycle

For the single command cycle, the expected travel time equals to the two times of the travel time between I/O point and the random parking space according to the literature [4]. In many papers about AS/R systems, the authors often regard their system as a continuous pace in time and based on it, developing the mathematical travel time model. For example, in the literature [1]-[3], the authors assumed the rack to be square in time and got the normalized system. Similarly, we assume the parking area to be cylinder in time. Because of the S/R machine rotating in both clockwise and counterclockwise, without loss of generality, we just consider the right-hand side half parking area only in the following paper. And we use *(x, y, z)* to represent a parking space in our system to be storage or retrieval, where x is the rotation movement in normalized time, y is the radial movement in normalized time, z is vertical movement in normalized time. Now, let $t_{x,y,z}$ denote the travel time from I/O point to the random parking space. According to the characteristics of the S/R machine, we can get $t_{x,y,z}$ is as:

$$t_{x,y,z} = Max\{x, y, z\} \tag{1}$$

For the random storage policy, we can easily assume the coordinate locations to be uniformly distributed. And we can see that there are some differences travel models due to the inequality of t_v, t_c and t_r. According to our analysis, it can be classified into two cases. The two cases are:

- Case 1: $t_v > T_h$.
- Case 2: $t_v \le T_h$.

And these cases can be split some sub-case again. More details, we will discuss in the following paper.

In case 1, according to the inequality of t_c and t_r , two sub-cases we can get:

- Case 1 - 1 : $t_v > t_c > t_r$.
- Case 1 - 2 : $t_v > t_r > t_c$.

Fig. 2 shows the continuous representation of the parking area in case 1.

Let $G(t)$ denote the probability that the travel time to the location (x, y, z) is no more than or equal to t. Thus,

$$G(t) = \Pr(t_{x,y,z} \le t) = F_x(t)F_y(t)F_z(t) \tag{2}$$

where F_x, F_y and F_z are the distribution function of x, y, and z, respectively. And in case 1-1, x, y, and z must satisfy these conditions:

$$\begin{cases} 0 \le x \le c \\ 0 \le y \le bc \\ 0 \le z \le 1 \end{cases} \tag{3}$$

The appendix part provides the procedures of getting the conditions (3). We can easily get the distribution function of x, y, z respectively as:

$$\begin{cases} F_x(t) = \dfrac{t}{c} \qquad 0 \le t \le c \\ F_y(t) = \dfrac{area \;\; for \;\; y \le t}{area \; of \; whole \;\; parking \; space} \\ = \dfrac{(bc)^2 - (bc-t)^2}{(bc)^2} = \dfrac{2bct - t^2}{(bc)^2} \qquad 0 \le t \le bc \\ F_z(t) = t \qquad 0 \le t \le 1 \end{cases} \tag{4}$$

According to (2), (4) we get $G(t)$ as:

$$G(t) = \begin{cases} \dfrac{2bct^3 - t^4}{b^2c^3} \quad 0 \le t \le bc \\ \dfrac{t^2}{c} \qquad bc \le t \le c \\ t \qquad c \le t \le 1 \end{cases} \tag{5}$$

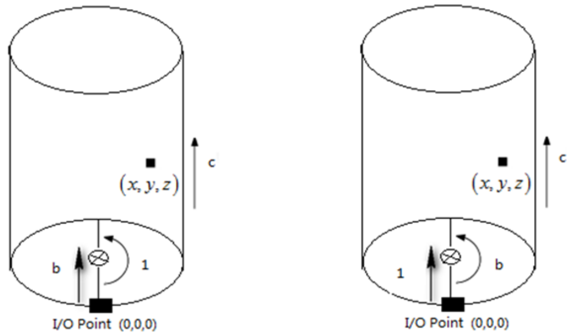
Therefore, the expected travel time $E(SC)$ in this case is as:

$$E(SC) = 2\int_0^1 tG'(t)dt = 1 + \frac{c^2}{3} + \frac{1}{15}b^3c^2 \tag{6}$$
$$0 \le b \le 1, 0 \le c \le 1$$

In case 1-2, x, y, and z must satisfy the conditions:

$$\begin{cases} 0 \le X \le bc \\ 0 \le Y \le c \\ 0 \le Z \le 1 \end{cases} \tag{7}$$

As the conditions changed, the distribution function of x and y are changed as:



(a) $t_c > t_r > t_v$
(b) $t_c > t_h > t_v$
(c) $t_r > t_c > t_v$
(d) $t_r > t_v > t_c$

Fig. 3 The continuous representation of the parking area in case 2

$$\begin{cases} F_x(t) = \dfrac{t}{bc} & 0 \le t \le bc \\ F_y(t) = P_r(Y \le t) = \dfrac{c^2 - (c-t)^2}{c^2} & \\ = \dfrac{2ct - t^2}{c^2} & 0 \le t \le c \end{cases} \tag{8}$$

Therefore, we can get $G(t)$ as:

$$G(z) = \begin{cases} \dfrac{2ct^3 - t^4}{bc^3} & 0 \le t \le bc \\ \dfrac{2ct^2 - t^3}{c^2} & bc < t \le c \\ t & c < t \le 1 \end{cases} \tag{9}$$

Thus, the expected travel time in this case is as:

$$E(SC) = 2\int_0^1 tG'(t)dt = 1 + \frac{c^2}{6} + \frac{b^3c^2}{3} - \frac{b^4c^2}{10} \tag{10}$$
$$0 \le b \le 1, 0 \le c \le 1$$

In case 2, according to the inequality of t_v, t_c and t_r, four sub-cases we can get:

- Case 2 - 1 : $t_c > t_r > t_v$.
- Case 2 - 2 : $t_r > t_c > t_v$.
- Case 2 - 3 : $t_c > t_v > t_r$.
- Case 2 - 4 : $t_r > t_v > t_c$.

Fig. 3 shows the continuous representation of the parking area in case 2. In this case, the procedures of getting the expected travel time is similar to the earlier one, so we do not give details of the procedures in this paper and we display the result of each case in Table II in the appendix part.

IV. Comparisons with Discrete Case

We have derived some mathematical travel time models for the single command cycle under the random storage policy in Section III. And we compare the results of these continuous models with the ones from a corresponding discrete parking space to emphasize the reality of the above results. In the discrete system we assume that the parking space is same size and the shape of it is square where the length of an edge is 3 m. And $S_v = 20m/\min$, $S_r = 35m/\min$, $S_c = \pi\, rad/\min$. Some results are presented in Table I. In Table I, we assumed $H = 15m$. We can get the expected travel time in the continuous model and the discrete case respectively for the different radius. For example, if R is 30 m, the expected travel time in continuous model equals to 1.2434 min and in discrete case equals to 1.2900 min. From Table I, we can observe that the continuous model shows a good satisfactory performance with the largest percentage deviation being 11.3242% and the closer b and c are, the smaller the deviation will be.

V. Conclusions

In this paper, we mainly analyze an automated parking system where the storage and retrieval requests are performed by a tower crane. And some travel time models in the continuous system are derived for the single command under the randomized storage assignment rule. Finally, we compare these models with discrete case to emphasize the reality of these models and confirm that the continuous models display a satisfactory performance.

In the future, we can extend the paper to the following case. Firstly, using different storage assignment rule, for example, class based. Secondly a dwell point strategy can be studied.

TABLE I
Results of the Fixed H

R	No. of parking spaces	b	c	Discrete case	Continuous model	% Deviation
35	6288	1.0000	0.7500	1.4198	1.2757	11.3000
30	4560	0.8571	0.7500	1.2900	1.2434	3.7485
25	3072	0.7143	0.7500	1.1765	1.2199	3.5598
20	1904	0.5714	0.7500	1.0677	1.2041	11.3242

APPENDIX

According to $t_v \ge t_c \ge t_r$, we can get the value of the T_h, T, b and c respectively as follows:

$$T_h = Max\{t_c, t_r\} = t_c$$

$$T = Max\{t_h, T_h\} = t_h$$

$$b = \min\left\{\frac{t_c}{T_h}, \frac{t_r}{T_h}\right\} = \frac{t_r}{T_h}$$

$$c = \min\left\{\frac{T_h}{T}, \frac{t_h}{T}\right\} = \frac{T_h}{T}$$

Let (x', y', z') denote a storage or retrieval point in the un-normalized dimension. According to the assumption and the previous analysis, we get x', y' and z' must satisfy the conditions:

$$\begin{cases} 0 \le x' \le t_c \\ 0 \le y' \le t_r \\ 0 \le z' \le t_h \end{cases} \tag{11}$$

Now we make a process that normalize the (x', y', z') to the (x, y, z), and after the normalization we get the relationship between (x', y', z') and (x, y, z) as:

$$\begin{cases} x = \dfrac{x'}{T} \\ y = \dfrac{y'}{T} \\ z = \dfrac{z'}{T} \end{cases} \tag{12}$$

According to (11), (12) we know that x, y, and z must satisfy the conditions:

$$\begin{cases} 0 \le x = \dfrac{x'}{T} \le \dfrac{t_c}{T} = \dfrac{T_h}{T} = c \\ 0 \le y = \dfrac{y'}{T} \le \dfrac{t_r}{T} = \dfrac{ct_r}{T_h} = bc \\ 0 \le z = \dfrac{z'}{T} \le \dfrac{t_h}{T} = 1 \end{cases} \Rightarrow \begin{cases} 0 \le x \le c \\ 0 \le y \le bc \\ 0 \le z \le 1 \end{cases}$$

In other cases, the procedure of deriving the conditions of the x, y and z is similar to this.

TABLE II
RESULTS OF DIFFERENT CASES

Case	Conditions	Distribution function	$G(t)$	$E(SC)$
$t_c > t_r > t_h$	$\begin{cases} 0 \le x \le 1 \\ 0 \le y \le b \\ 0 \le z \le c \\ b > c \end{cases}$	$\begin{cases} F_x(t) = t \quad 0 \le t \le 1 \\ F_y(t) = \dfrac{2bt - t^2}{b^2} \quad 0 \le t \le b \\ F_z(t) = \dfrac{t}{c} \quad 0 \le t \le c \end{cases}$	$\begin{cases} \dfrac{2bt^3 - t^4}{b^2 c} \quad 0 \le t \le c \\ \dfrac{2bt^2 - t^3}{b^2} \quad c \le t \le b \\ t \quad b \le t \le 1 \end{cases}$	$1 + \dfrac{b^2}{6} + \dfrac{c^3}{3b} - \dfrac{c^4}{10b^2}$
$t_r > t_c > t_h$	$\begin{cases} 0 \le x \le b \\ 0 \le y \le 1 \\ 0 \le z \le c \\ b > c \end{cases}$	$\begin{cases} F_x(t) = \dfrac{t}{b} \quad 0 \le t \le b \\ F_y(t) = 2t - t^2 \quad 0 \le t \le 1 \\ F_z(t) = \dfrac{t}{c} \quad 0 \le t \le c \end{cases}$	$\begin{cases} \dfrac{2t^3 - t^4}{bc} \quad 0 \le t \le c \\ \dfrac{2t^2 - t^3}{b} \quad c \le t \le b \\ 2t - t^2 \quad b \le t \le 1 \end{cases}$	$\dfrac{2}{3} + \dfrac{2b^2}{3} - \dfrac{b^3}{6} + \dfrac{c^3}{3b} - \dfrac{c^4}{10b}$
$t_c > t_v > t_r$	$\begin{cases} 0 \le x \le 1 \\ 0 \le y \le b \\ 0 \le z \le c \\ c > b \end{cases}$	$\begin{cases} F_x(t) = t \quad 0 \le t \le 1 \\ F_y(t) = \dfrac{2bt - t^3}{b^2} \quad 0 \le t \le b \\ F_z(t) = \dfrac{t}{c} \quad 0 \le t \le c \end{cases}$	$\begin{cases} \dfrac{2bt^3 - t^4}{b^2 c} \quad 0 \le z \le b \\ \dfrac{t^2}{c} \quad b \le t \le c \\ t \quad c \le t \le 1 \end{cases}$	$1 + \dfrac{b^3}{15c} + \dfrac{c^2}{3}$

Case	Conditions	Distribution function	$G(t)$	$E(SC)$
$t_r > t_v > t_c$	$\begin{cases} 0 \le x \le b \\ 0 \le y \le 1 \\ 0 \le z \le c \\ c > b \end{cases}$	$\begin{cases} F_x(t) = \frac{t}{b} & 0 \le t \le b \\ F_y(t) = 2t - t^2 & 0 \le t \le 1 \\ F_z(t) = \frac{t}{c} & 0 \le t \le c \end{cases}$	$\begin{cases} \frac{2t^3 - t^4}{bc} & 0 \le t \le b \\ \frac{2t^2 - t^3}{c} & b \le t \le c \\ 2t - t^2 & c \le t \le 1 \end{cases}$	$\frac{2}{3} - \frac{b^4}{10c} + \frac{b^3}{3c} - \frac{c^3}{6} + \frac{2c^2}{3}$

REFERENCES

[1] W. H. Hausman, L. B., Schwarz, and S. C. Graves, “Optimal Storage Assignment in Automatic Warehousing Systems”, Management Science, vol. 22, no. 2, pp. 629-638, 1976.
[2] L. B. Schwarz, S. C. Graves, and, S.C. and W. H. Hausman, “Scheduling Policies for Automatic Warehousing Systems: Simulation Results”, AIIE Transtration, vol. 10, no. 3, pp. 260-270, 1978.
[3] Y. A. Bozer, and J. A. White, “Travel-time Models for Automated Storage/Retrieval Systems”, IIE Transactions, vol. 16, no. 4, pp. 329-338, 1984.
[4] H. Hwang, and S. B. Lee, “Travel-time models considering the operating characteristics of the storage and retrieval machine”, International Journal of Production Research, vol. 28, no. 10, pp. 1779-1789, 1990.
[5] S. G. Koh, B. S. Kim, and B. N. Kim, "Travel time model for the warehousing system with a tower crane S/R machine", Computers & Industrial Engineering, vol. 43, no. 3, pp. 495-507, 2002.