International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:9, No:11, 2015

Reliability Analysis of Computer Centre at Yobe State University Using LRU Algorithm

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***Abstract*—**In this paper, we focus on the reliability and performance analysis of Computer Centre (CC) at Yobe State University, Damaturu, Nigeria. The CC consists of three servers: one database mail server, one redundant and one for sharing with the client computers in the CC (called as a local server). Observing the different possibilities of the functioning of the CC, the analysis has been done to evaluate the various popular measures of reliability such as availability, reliability, mean time to failure (MTTF), profit analysis due to the operation of the system. The system can ultimately fail due to the failure of router, redundant server before repairing the mail server and switch failure. The system can also partially fail when a local server fails. The failed devices have restored according to Least Recently Used (LRU) techniques. The system can also fail entirely due to a cooling failure of the server, electricity failure or some natural calamity like earthquake, fire tsunami, etc. All the failure rates are assumed to be constant and follow exponential time distribution, while the repair follows two types of distributions: i.e. general and Gumbel-Hougaard family copula distribution.

***Keywords*—**Reliability, availability Gumbel-Hougaard family copula, MTTF, internet data center.

I. Introduction

RELIABILITY of a system plays a significant role in operations of industry and organization. It involves effective methods to improve the reliability and availability of complex systems under different failure and repair policies. The system reliability has been extensively studied and used by various authors like Govil [2], Gupta and Sharma [9] Cui and Lirong [6] and many others. They have discussed the reliability characteristics of complex systems by taking several failures and one repair policy. Examine the present scenario with the complexity of advanced technology and modern demands of the networking system; it is necessary to study the computer center that has become an essential requirement of usual life. Singh et al. [11] have investigated the reliability characteristic for Internet data center with a redundant server including a main mail server. In continuation to the

study of Internet data center, Rawal et al. [5] have discussed the reliability of Internet Data Centre having one mail server and one redundant server especially for the use of Internet. In this paper, the authors have put their attention toward many other factors which were not taken into account in the earlier study, but still, they have left many necessary parameters. For example, the authors in this paper do not consider the connectivity and sharing (of files and data) with clients' computers.

In this article, we study the functioning of Computer Centre (CC) at Yobe State University, Damaturu, Nigeria, under various repair and maintenance policies using the Least Recently Used (LRU) algorithm. The function of CC is to provide the Internet to whole the University and provide labs (for all the computing related activities) to all the students of the University. The CC consists of three servers: one database (mail server), one redundant server and one for sharing (files and data) with the client computers in the CC (called as a local server). The CC is having 100 computers, two routers, and five switches. All systems are interconnected by the local server. The CC has two types of failure: partial failure and complete failure. Whenever local server fails, all 100 computers become disconnected from the Internet but working for other kinds of use. However, all other systems which are directly connected to either mail server or redundant server are unaffected by this. Whenever the mail server fails, the redundant server comes into function automatically by a switchover device. The switch-over device is instantaneous and automatic. The system can fail due to the following:

I. Failure of redundant server before repair of the central server
II. Failure of local server
III. Failure of switch
IV. Failure of router
V. Failure of cooling system
VI. Failure due to natural calamity like earthquake or fire etc.

The system will be in complete failure mode if a redundant server fails before repairing of the main mail server. The system will be in degraded mode: (i) when the central mail server fails completely, and the redundant server is in the partial failure mode, (ii) local server fails. The failed systems are repaired according to Least Recent Used (LRU) algorithm. The idea behind the use of this algorithm is that if the server which has been ideal for a long time needs not to be repaired first after it fails. Since there may be the possibility that it may not be used for a long time in future too.

The authors in [1], [3], [4], [9] have studied the reliability measures of a system, with different types of failures and one

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type of repair. However, there are many situations in real life systems where more than one repair is possible between two adjacent transition states. When this possibility exists, the reliability of the system can analyze with the help of copula [10]. The authors [7], [8], [12]-[14], [16]-[18] have studied the reliability models with different types of failure, and different types of repair are employing Copula distribution.

They have concluded that reliability of system improves by using Copula. M. Ram et al. [7] have discussed the reliability of a system with different failure rates and common cause failure under the preemptive resume policy with the concept of Gumbel-Hougaard family copula distribution. References [12], [14] discussed the reliability analysis of a system which has two subsystems under k-out-of-n: G; policy using Copula distribution. Waiting repair policy also play a significant role in reliability theory, whenever the repair is employed to a failed unit and mean time the other operating unit of the system fails, then recently failed unit has to wait for getting repair until it is not a priority unit. Authors [8], [15] studied the reliability characteristics of complex repairable systems using Copula distribution.

Therefore, about the earlier models discussed, here we have considered Computer Centre in which we highlighted improvement of reliability due to two different repair facilities available between adjacent states, i.e. the initial state and complete failed states. All failure rates are assumed to be constant and follow an exponential distribution. The repair follows general and Gumbel-Hougaard family copula distributions.

This paper is organized as follows: Section II describes the notation and assumptions used in the article. Section III illustrates the state transition diagram of the model. Section IV explains the mathematical formulation and solution of the model. Finally, we conclude in Section V.

II. Notations and Assumptions

A. Notations

t	Time variable on time scale.
s	Laplace transform variable.
$\lambda_1 / \lambda_2 / \lambda_C$	Failure rates for Main mail server/ Redundant server/ local server.
$\lambda_S / \lambda_R / \lambda_{CL}$	Failure rates for switch/ Router/ Natural calamity like earthquake Tsunami are suddenly getting fire etc.
$\phi_1(x)/ \phi_2(x)/\phi_c(x)/ \mu_0(x)$	General repair rates for Main mail server/ Redundant server/ local server/ repair rate for complete failed states.
$P_i(t)$	The notation, $P_i(t)$ represents the probability the probability of the system to be in state S_i at instant's' for i =0 to 9.
$\overline{P}(s)$	Laplace transformation of P (t).
$P_j(x, t)$	The state transition probability that the system is in state S_o for j=1 to 8; the system is under repair and elapsed repair time lies in interval x, x+Δx,
$E_p(t)$	Notation for expected profit during the interval [0, t).
K_1, K_2	Revenue and service cost per unit time respectively.
$\mu_0(x)=C_\theta(u_1(x),u_2(x))$	The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula is given as $C_\theta(u_1(x),u_2(x)) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where θ is a parameter.

B. Assumptions

The following assumptions are taken throughout the study of the mathematical model.

i. Initially, the system is in S_0 state where all servers are in good condition.
ii. When the main mail server fails, the redundant server takes over the load, and repair is assigned to the failed main mail server.
iii. When the local server fails, the client computers in CC are disconnected from local server and waits for repair. However, there is no effect on other systems which are connected directly from the main or redundant server. This needs fast repairing, i.e. Copula distribution is employed to repair.
iv. The system waits for repair if repair facility is not available; as soon as the repair service is available, the repairing is employed to the failed unit.
v. During repair, the preference to the server is given in the order of Least Recently Used (LRU), because the server which has been ideal for a long time needs not to be repaired first after it fails. Since there may be the possibility that it may not be used for a long time in future too.
vi. All failure rates are assumed to be constant.
vii. A switch failure, router failure, cooling failure, and failure due to natural calamity needs fast repairing and Gumbel- Hougaard copula distribution is employed for repairing complete failed states.
viii. Repaired system works like a new, and the repair does not damage anything.

III. Formulation and Solution of Mathematical Model

A. Mathematical Formulation of Model

By the probability of considerations and continuity of arguments, the following set of difference-differential equations governing the present mathematical model can be obtained as:

$$\left[\frac{\partial}{\partial t}+\lambda_1+\lambda_S+\lambda_R+\lambda_2+\lambda_C\right]P_0(t)=\int_0^\infty \varphi_1(x)P_1(x,t)dx +\int_0^\infty \varphi_2(x)P_2(x,t)dx+\int_0^\infty \varphi_C(x)P_C(x,t)dx+ \int_0^\infty \mu_0(x)P_7(x,t)dx+\int_0^\infty \mu_0(x)P_4(x,t)dx+ \int_0^\infty \mu_0(x)P_8(x,t)dx+\int_0^\infty \mu_0(x)P_5(x,t)dx+ \int_0^\infty \mu_0(x)P_6(x,t)dx \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R+\phi_1(x)\right]P_1(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_R+\lambda_S+\lambda_{CL}+\lambda_1+\phi_2(x)\right]P_2(x,t)=0 \tag{3}$$

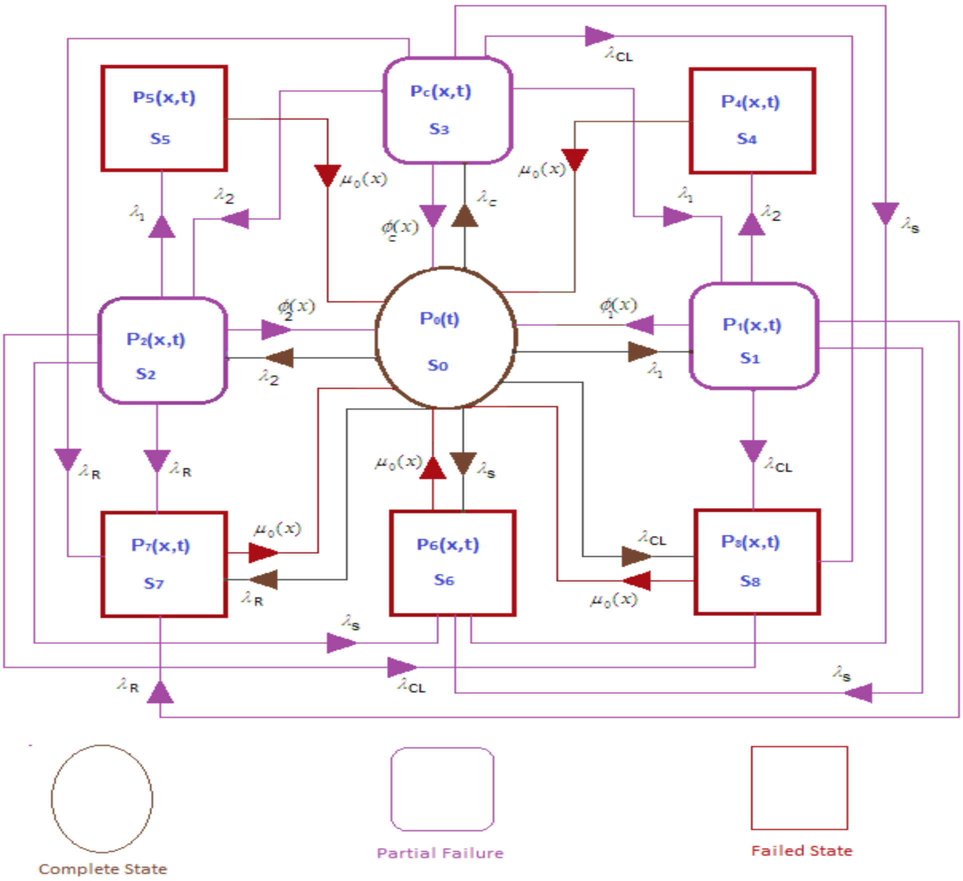

Fig. 1 State Transition Diagram of Model

IV. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_S+\lambda_{CL}+\lambda_R+\phi_C(x)\right]P_C(x,t)=0 \quad (4)$$

V. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_4(x,t)=0 \quad (5)$$

VI. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_5(x,t)=0 \quad (6)$$

VII. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_6(x,t)=0 \quad (7)$$

VIII. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_7(x,t)=0 \quad (8)$$

IX. $$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_8(x,t)=0 \quad (9)$$

Boundary conditions:

$$P_1(0,t)=\lambda_1(P_0(t)+P_C(0,t)) \quad (10)$$
$$P_2(0,t)=\lambda_2(P_0(t)+P_C(0,t)) \quad (11)$$
$$P_C(0,t)=\lambda_C P_0(t) \quad (12)$$
$$P_4(0,t)=\lambda_2 P_1(0,t) \quad (13)$$
$$P_5(0,t)=\lambda_1 P_2(0,t) \quad (14)$$
$$P_6(0,t)=\lambda_S(P_0(t)+P_1(0,t)+P_C(0,t)+P_2(0,t)) \quad (15)$$
$$P_7(0,t)=\lambda_R(P_0(t)+P_1(0,t)+P_C(0,t)+P_2(0,t)) \quad (16)$$
$$P_8(0,t)=\lambda_{CL}(P_0(t)+P_1(0,t)+P_C(0,t)+P_2(0,t)) \quad (17)$$

B. Solution of the Model

Taking Laplace transformation of (1)-(17) and using these with help of initial condition, $P_0(t)=1$ and other state probabilities are zero at t:

$$\begin{aligned}&[s+\lambda_1+\lambda_S+\lambda_R+\lambda_2+\lambda_C+\lambda_{CL}]\bar{P}_0(s)=1+\int_0^\infty\varphi_1(x)\bar{P}_1(x,s)dx+\\&\int_0^\infty\varphi_2(x)\bar{P}_2(x,s)dx+\int_0^\infty\varphi_C(x)\bar{P}_C(x,s)dx\\&+\int_0^\infty\mu_0(x)\bar{P}_6(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_7(x,s)dx+\\&\int_0^\infty\mu_0(x)\bar{P}_5(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_4(x,s)dx+\\&\int_0^\infty\mu_0(x)\bar{P}_8(x,s)dx\end{aligned} \quad (18)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R+\phi_1(x)\right]\bar{P}_1(x,s)=0 \quad (19)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_R+\lambda_S+\lambda_{CL}+\lambda_1+\phi_2(x)\right]\bar{P}_2(x,s)=0 \quad (20)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_S+\lambda_{CL}+\lambda_R+\phi_C(x)\right]\overline{P}_C(x,s)=0 \quad (21)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_4(x,s)=0 \quad (22)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_5(x,s)=0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_6(x,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_7(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_8(x,s)=0 \quad (26)$$

Laplace transform of boundary conditions:

$$\overline{P}_1(0,s)=\lambda_1(\overline{P}_0(s)+\overline{P}_C(0,s)) \quad (27)$$

$$\overline{P}_2(0,s)=\lambda_2(\overline{P}_0(s)+\overline{P}_C(0,s)) \quad (28)$$

$$\overline{P}_C(0,s)=\lambda_C\overline{P}_0(s) \quad (29)$$

$$\overline{P}_4(0,s)=\lambda_2\overline{P}_1(0,s) \quad (30)$$

$$\overline{P}_5(0,s)=\lambda_1\overline{P}_2(0,s) \quad (31)$$

$$\overline{P}_6(0,s)=\lambda_S(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)+\overline{P}_C(0,s)) \quad (32)$$

$$\overline{P}_7(0,s)=\lambda_R(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)+\overline{P}_C(0,s)) \quad (33)$$

$$\overline{P}_8(0,s)=\lambda_{CL}(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)+\overline{P}_C(0,s)) \quad (34)$$

Solving the (19)-(26) with help of (27)- (34) and then using in (18), one may have:

$$\overline{P}_0(s)=\frac{1}{D(s)} \quad (35)$$

$$\overline{P}_1(s)=\frac{\lambda_1(1+\lambda_C)}{D(s)}\frac{(1-S_{\phi_1}(s+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R))}{(s+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R)} \quad (36)$$

$$\overline{P}_2(s)=\frac{\lambda_2(1+\lambda_C)}{D(s)}\frac{(1-S_{\phi_2}(s+\lambda_1+\lambda_{CL}+\lambda_S+\lambda_R))}{(s+\lambda_1+\lambda_{CL}+\lambda_S+\lambda_R)} \quad (37)$$

$$\overline{P}_C(s)=\frac{\lambda_C}{D(s)}\frac{(1-S_{\phi_C}(s+\lambda_1+\lambda_2+\lambda_S+\lambda_{CL}+\lambda_R))}{(s+\lambda_1+\lambda_2+\lambda_S+\lambda_{CL}+\lambda_R)} \quad (38)$$

$$\overline{P}_4(s)=\frac{\lambda_1\lambda_2(1+\lambda_C)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (39)$$

$$\overline{P}_5(s)=\frac{\lambda_1\lambda_2(1+\lambda_C)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (40)$$

$$\overline{P}_6(s)=\frac{\lambda_S(1+(\lambda_1+\lambda_2)(1+\lambda_C)+\lambda_C)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (41)$$

$$\overline{P}_7(s)=\frac{\lambda_R(1+(\lambda_1+\lambda_2)(1+\lambda_C)+\lambda_C)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (42)$$

$$\overline{P}_8(s)=\frac{\lambda_{CL}(1+(\lambda_1+\lambda_2)(1+\lambda_C)+\lambda_C)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (43)$$

$$\begin{aligned}D(s)&=s+\lambda_1+\lambda_2+\lambda_S+\lambda_R+\lambda_C+\lambda_{CL}-\\&[\lambda_1(1+\lambda_C)S_{\varphi_1}(s+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R)+\\&2\lambda_1\lambda_2(1+\lambda_C)S_{\mu_0}(s)+\\&\lambda_2(1+\lambda_C)S_{\varphi_2}(s+\lambda_1+\lambda_{CL}+\lambda_S+\lambda_R)+\\&\lambda_C S_{\varphi_C}(s+\lambda_1+\lambda_2+\lambda_{CL}+\lambda_S+\lambda_R)+\\&2\lambda_1\lambda_2(1+\lambda_C)S_{\mu_0}(s+\lambda_S(1+(\lambda_1+\lambda_2)(1+\lambda_C)\\&+\lambda_C)S_{\mu_0}(s)+\lambda_{CL}(1+(\lambda_1+\lambda_2)(1+\lambda_C)+\lambda_C)S_{\mu_0}(s)\\&+\lambda_R(1+(\lambda_1+\lambda_2)(1+\lambda_C)+\lambda_C)S_{\mu_0}(s)]\end{aligned}$$

The sum of Laplace transformations of the state probabilities when the system is in up state and in failed state at any time is as follows:

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_C(s) \quad (44)$$

$$\overline{P}_{down}(s)=1-\overline{P}_{up}(s) \quad (45)$$

IV. Particular Cases

A. Availability Analysis

For particular cases the study of availability is focus on following cases: when repair follows exponential distribution setting:

$$S_{\mu_0}(s)=\frac{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}},\quad \overline{S}_{\phi_i}(s)=\frac{\phi_i}{s+\phi_i}$$

Taking the values of different parameters as, λ_1=0.05, λ_2 = 0.03, λ_C = 0.04, λ_S = 0.045, λ_R = 0.12, λ_{CL} = 0.01, Ø, θ = 1, x = 1, in (44), then taking inverse Laplace transform, one can obtain:

Availability= - 0.031027e (-1.47000 t) - 0.00461e (-1.06200t)
+ 0.00580 e (-2.7339 t) +0.03705 e (-1.172639t)
+ 0.001129 e (-1.10096 t)-0.010098 e (-1.074933 t)
+1.00400 e (-0.08812t) (46a)

Taking λ_C = 0, i.e. local server is not in existence and for same values of failure rates of parametric values in (44), and taking inverse Laplace transform the expression for availability is given as in (46 b).

Availability= 0 .00.3318 e (-1.0700t) +0.006305 e (-2.73530 t)
+0.012187 e (-1.16938 t) - 0.020156 e (1.090609t)
+ 0.99835 e (-0 . 043207 t) (46b)

For, the time t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, …; units of time, one may get different values of Availability with the help of (45A), (45b) and as shown in Fig. 2.

B. Reliability Analysis of the System

Assuming all repairs in (44) equal to zero and then taking inverse Laplace transform, one may get an expression for the reliability of system and for given values of failure rates λ_1=0.05, λ_2=0.03, λ_C = 0.04, λ_S = 0.045, λ_R = 0.012 and λ_{CL} = 0.010 in (44), we get (46 a) and (46b):

$$\text{Reliability}=0.57778e^{(-0.09700\,t)}+0.44572e^{(-0.11700\,t)}+e^{(-0.14700t)}-1.0235e^{(-0.18700t)} \tag{46c}$$

TABLE I
AVAILABILITY VARIATION WITH RESPECT TO TIME T

Time (t)	Availability	
	A1	**A2, ($\lambda_C = 0$)**
0	1.000	1.000
1	0.919	0.955
2	0.842	0.915
3	0.771	0.877
4	0.706	0.840
5	0.646	0.804
6	0.592	0.770
7	0.542	0.738
8	0.496	0.707
9	0.454	0.677
10	0.416	0.639

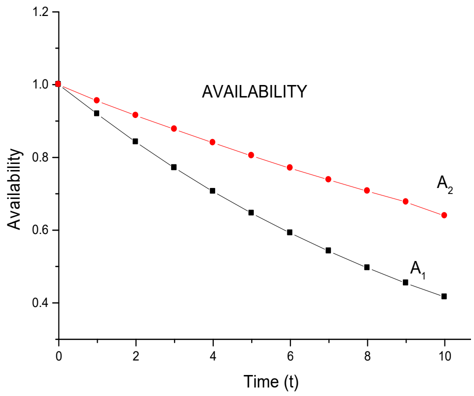

Fig. 2 Availability as a function of time t

TABLE II
VARIATION OF RELIABILITY WITH RESPECT TO TIME T

Time(t)	Reliability
0	1.000
1	0.935
2	0.870
3	0.805
4	0.742
5	0.682
6	0.625
7	0.570
8	0.520
9	0.473
10	0.430

C. Mean Time to Failure (MTTF) Analysis

Taking all repairs zero and then taking limit, $s \to 0$, one can obtain the expression for mean time to failure(MTTF) as:

$$MTTF=\frac{1}{(\lambda_1+\lambda_S+\lambda_R+\lambda_2+\lambda_C+\lambda_{CL})}\left(1+\frac{\lambda_1(1+\lambda_C)}{(\lambda_S+\lambda_R+\lambda_2+\lambda_{CL})}+\frac{\lambda_2(1+\lambda_C)}{(\lambda_1+\lambda_S+\lambda_R+\lambda_{CL})}+\frac{\lambda_C}{(\lambda_1+\lambda_S+\lambda_R+\lambda_2+\lambda_{CL})}\right) \tag{47}$$

Setting λ_1=0.05, λ_2=0.03, λ_C=0.015, λ_S=0.045, λ_R=0.012, λ_{CL}=0.010 and varying λ_1, λ_2, λ_C, λ_S, λ_R, λ_{CL} one by one respectively as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 in (47), one can obtain the variation of (MTTF) with respect to failure rates as shown in Fig. 4.

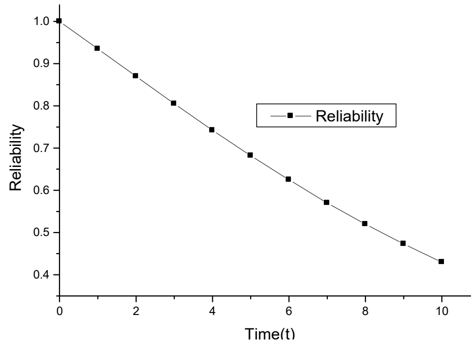

Fig. 3 Reliability as function of time t

TABLE III
VARIATION OF MTTF WITH RESPECT TO FAILURE RATES

Failure Rate	MTTF λ_1	MTTF λ_2	MTTF λ_C	MTTF λ_S	MTTF λ_R	MTTF λ_{CL}
0.1	6.17	5.39	6.19	2.98	2.195	2.157
0.2	7.11	5.88	6.59	1.32	2.195	1.065
0.3	7.79	6.36	6.81	0.75	1.077	0.634
0.4	8.26	6.72	6.95	0.48	0.640	0.421
0.5	8.61	7.0	7.05	0.34	0.424	0.300
0.6	8.87	7.22	7.12	0.25	0.301	0.224
0.7	9.08	7.40	7.17	0.19	0.225	0.174
0.8	9.24	7.54	7.21	0.15	0.175	0.139
0.9	9.38	7.66	7.25	0.12	0.140	0.114
1.0	9.49	7.76	7.28	0.10	0.112	0.094

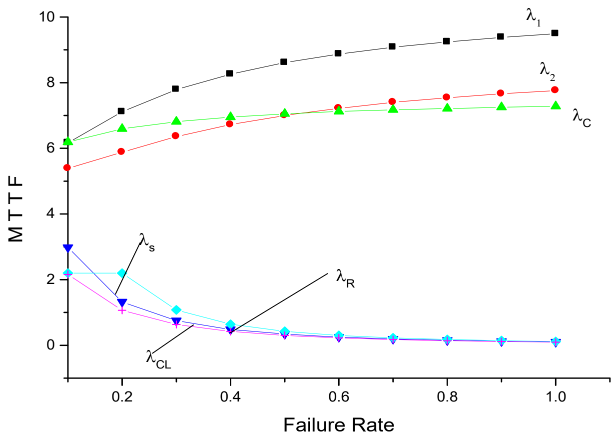

Fig. 4 MTTF as a function of failure rates

D. Cost Analysis

Assuming that the service facility be always available, then expected profit during the interval [0, t) is;

$$E_p(t) = K_1\int_0^t P_{up}(t)dt - K_2 t \tag{48}$$

For the same set of the parameter as in (47), one can obtain (49 a) and (49 b) respectively. Therefore, expected profit in interval [0, 1) can be obtained by the expression;

$$E_P(t) = K_1\,(0.02111\, e^{(-1.4700\, t)} + 0.004337\, e^{(-1.0620\, t)} - 0.002122\, e^{(-2.73392\, t)} - 0.031597\, e^{(-1.172639t)} + 0.001025\, e^{(-1.100961\, t)} + 0.0093942 e^{(-1.07493\, t)} - 11.393674\, e^{(-.0.0881199\, t)}\, 11.392\;\; K_2 t \qquad (49a)$$

Setting $K_1 =$ 1and $K_2 =$ 0.50, 0.40, 0.30, 0.20 and 0.01 respectively and varying t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 units of time, the results for expected profit in [0, t) can be obtain as shown in Fig. 5.

TABLE IV
EXPECTED PROFIT WITH RESPECT OF TIME T

Time(t)	Expected profit for K_1 =1				
	K_2=0.5	K_2=0.4	K_2=0.3	K_2=0.2	K_2=0.1
0	0.0	0.0	0.0	0.0	0.0
1	0.459	0.559	0.659	0.759	0.859
2	0.839	1.039	1.239	1.439	1.639
3	1.145	1.445	1.745	2.045	2.345
4	1.383	1.783	2.183	2.583	2.983
5	1.558	2.058	2.558	3.058	3.558
6	1.677	2.277	2.877	3.477	4.077
7	1.743	2.443	3.143	3.843	4.543
8	1.762	2.562	3.362	4.162	4.962
9	1.737	2.637	3.537	4.437	5.337
10	1.672	2.672	3.672	4.672	5.672

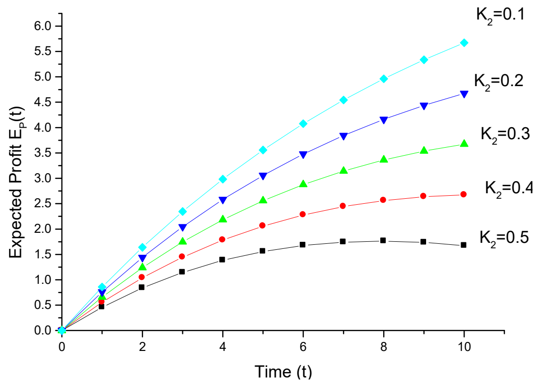

Fig. 5 Expected profit for various values of time t

TABLE V
EXPECTED PROFIT WHEN LOCAL SERVER FAILURE IS IGNORED

Time(t)	Expected profit for K_1 =1, (λ_C =0)				
	K_2=0.5	K_2=0.4	K_2=0.3	K_2=0.2	K_2=0.1
0	0.0	0.0	0.0	0.0	0.0
1	0.493	0.593	0.693	0.793	0.893
2	0.976	1.176	1.376	1.576	1.776
3	1.450	1.750	2.050	2.350	2.650
4	1.913	2.313	2.713	3.113	3.513
5	2.366	2.866	3.367	3.866	4.366
6	2.810	3.410	4.010	4.610	5.210
7	3.243	3.943	4.643	5.343	6.043
8	3.666	4.466	5.266	6.066	6.866
9	4.080	4.980	5.880	6.780	7.680
10	4.483	5.483	6.483	7.483	8.483

Expression for expected profit corresponding to non-existence of local server:

$$E_P(t) = K_1(0.0024495 e^{(-1.0620t)} - 0.0022444 e^{(-2.7351t)} - 0.01043095 e^{(-1.167348t)} + 0.0149474 e^{(-1.09035t)} - 93.3081 e^{(-0.01072t)} + 93.3033) - K_2 t \qquad (49b)$$

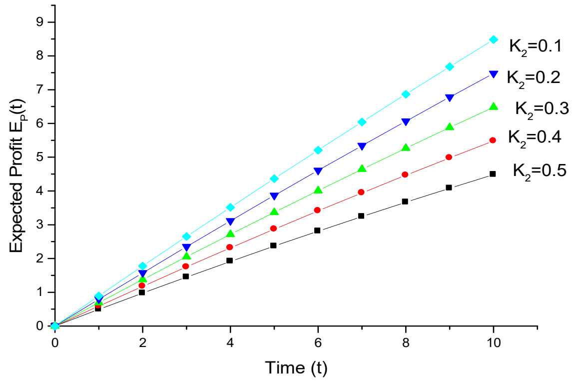

Fig. 6 Expected profit for *(λ_C =0)*

V. RESULT DISCUSSION AND CONCLUSIONS

Fig. 2 provides information how the availability of the complex repairable system changes on the time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_1 = 0.05$, $\lambda_2 = 0.03$, $\lambda_c = 0.045$, $\lambda_s = 0.040$, $\lambda_R = 0.012$, $\lambda_{CL} = 0.01$ availability of the system decreases and ultimately becomes steady to the value zero after a sufficient long interval of time. Consequently, one can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model. Availability of system increases as the parameter $\lambda c = 0$ failure of a local server is ignored. In Fig. 3 provides the variation in reliability of the non-repairable system. Fig. 4, yields the mean-time-to-failure (M.T.T.F.) of the system on variation in λ_1, λ_2, λ_C, λ_S, and λ_R and λ_{CL} respectively when the other parameters have fixed as constant. The variation in MTTF corresponding to failure rates λ_1, λ_2, λ_C, is increasing but corresponding to failure rates λ_S, λ_R, λ_{CL} it is decreasing, which gives the information regarding responsible factor for the proper functioning of the system. When revenue cost per unit time K_1 is fixed at 1, service costs K_2 = 0.5, 0.4, 0.30, 0.20, 0.10; profit has been calculated, and results are demonstrated by graphs in Figs 3-5. A critical examination from Figs. 5 and 6 reveals that expected profit increases at the time when the service cost K_2 fixed at a minimum value. Expected profit increases when a failure in local server is ignored. Finally, one can observe that as service cost increase, profit decrease. In general, for low service cost, expected profit is high in comparison to high service cost.

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