

# Control-Oriented Enhanced Zero-Dimensional Two-Zone Combustion Modelling of Internal Combustion Engines

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**Abstract**—This paper investigates an efficient combustion modeling for cycle simulation of internal combustion engine (ICE) studies. The term “efficient model” means that the models must generate desired simulation results while having fast simulation time. In other words, the efficient model is defined based on the application of the model. The objective of this study is to develop math-based models for control applications or shortly control-oriented models. This study compares different modeling approaches used to model the ICEs such as mean-value models, zero dimensional, quasi-dimensional, and multi-dimensional models for control applications. Mean-value models have been widely used for model-based control applications, but recently by developing advanced simulation tools (e.g. Maple/MapleSim) the higher order models (more complex) could be considered as control-oriented models. This paper presents the enhanced zero-dimensional cycle-by-cycle modeling and simulation of a spark ignition engine with a two-zone combustion model. The simulation results are cross-validated against the simulation results from GT-Power package and show a good agreement in terms of trends and values.

**Keywords**—Two-zone combustion, control-oriented model, wiebe function, internal combustion engine.

## I. INTRODUCTION

IN the recent decades due to the stringent emission limits and demand for improving fuel consumption, the automotive researchers have been trying to develop new methodologies to control powertrain characteristics. An ICE plays the most important role in the powertrain, which deals with emission gases and fuel consumption issues. Therefore, an efficient control-oriented ICE model significantly assists automotive industries to improve their products in terms of emission, fuel consumption, and drivability. The term “efficient model” implies that the models must provide desired simulation results together with fast simulation time. The fast simulation time feature is a key factor to categorize the model as a control-oriented model. However, the concept of control-oriented or the dynamic model with fast simulation time has been defined by considering computers/software progress. The advanced simulation tools (e.g. Maple/MapleSim [1]) are able to solve complex multi-domain dynamic systems more efficiently and faster than old programming tools. Fig. 1 shows different approaches of ICEs. As indicated in Fig. 1, the ICE model includes more and more physical parameters (or more details)

by approaching from the left to the right of the diagram. The first approach, the empirical method, uses experimental data to simulate the ICE model. The empirical modeling approach is usually represented by a group of look-up tables. Look-up tables are very fast, but are not physics-based, and cannot be adapted for different models with different parameters. The second approach, the power balancing, is a combination of empirical data and physics-based equations. Mean-value engine modeling is an example of using both physics-based equations (e.g. mass and energy conservation equations) and look-up tables (e.g. combustion model). In the other words, mean-value engine models [2] are the intermediate level ICE models which include more physics-based details than simplistic look-up tables, but are significantly simpler models than the large cycle-by-cycle engine models. The combinations of the physics-based components and look-up tables, with low computational effort, make the mean-value engine model suitable for control applications. The third approach includes simulations of the combustion model as well as thermodynamic equations. For instance, a two-zone combustion model with flame propagation through the combustion chamber can be categorized in this group. The quasi-dimensional or enhanced zero-dimensional modeling approach is also efficient control-oriented model, which includes more physics-based equations and can predict simulations at each crank angle with fast simulation time [3], [4]. The multi-dimensional models, which are placed in the last and most complex group in Fig. 1, are able to spatially capture the variation of in-cylinder variables (e.g. pressure and temperature) at each crank angle. The models in this group are very detailed, but computationally inefficient. Therefore, multi-dimensional models (e.g. computational-fluid-dynamics (CFD)) are not suitable plant models for control applications.

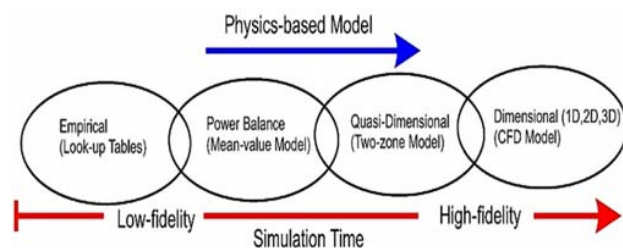


Fig. 1 Different approaches of ICE models

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The focus of this paper is on the third ICE modeling approach, quasi-dimensional, enhanced zero-dimensional ICE modeling approach. The four-stroke operation, including two-zone combustion model, in this group is fully physics-based model with one empirical correlation equation which is called Wiebe function. The Wiebe function [5] is used to empirically calculate the burned volume fraction. The proposed two-zone combustion model is not fully quasi-dimensional model due to the empirical Wiebe function in the two-zone combustion model. Fig. 2 shows the place of the proposed model in between zero-dimensional and quasi-dimensional modelling approach. Non-predictive and semi-predictive models are significantly faster than fully predictive models [6]. However, the semi-predictive (or enhanced zero-dimensional) models represent more physics-based equations than the non-predictive models. The proposed two-zone combustion with turbulent flame propagation is categorized in the semi-predictive group, because the mass-burned model is enhanced by replacing it with the flame propagation model. The burned volume fraction is explicitly represented as an empirical Wiebe function, but the burned mass fraction is calculated from the physics-based equations.

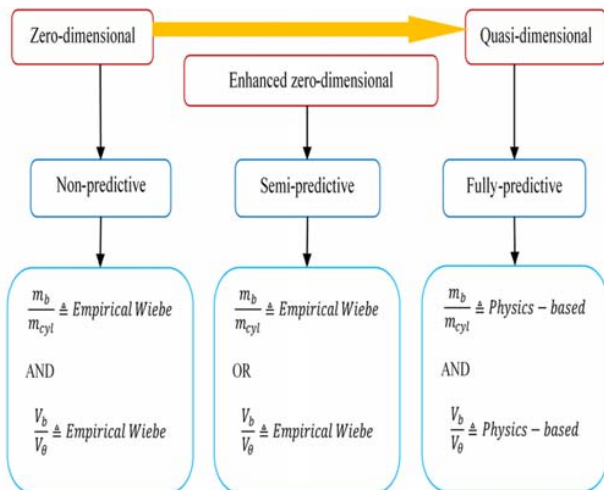


Fig. 2 From zero-dimensional to quasi-dimensional two-zone combustion modelling approach

## II. MODEL DEVELOPMENT

The objective of this paper is to develop efficient spark ignition engine model to generate cycle-by-cycle simulation results based on four-stroke operation, while having fast simulation time (in terms of CPU time). Fig. 3 depicts the cycle simulation procedure, which is derived from the thermodynamic Otto cycle [7]. The most complex aspect in simulating four-stroke operation is modeling combustion process. The combustion model is developed based on the two-zone theory with turbulent flame propagation, which is named enhanced zero-dimensional combustion model in this paper. The full description of all thermodynamic four-stroke operation is provided in the PhD dissertation by Adibi-Asl [8].

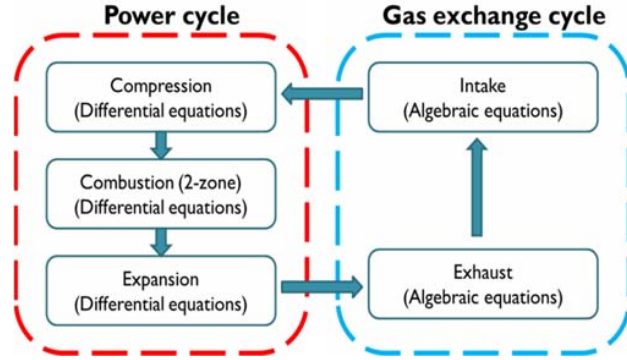


Fig. 3 Four-stroke cycle simulation procedure

In a two-zone combustion modeling approach, the combustion chamber is split into burned and unburned zones [9]. The flame propagation (entrained mass) is moving from burned toward unburned zone during combustion as shown in Fig. 4. The flame propagation model in two-zone combustion has been mathematically presented in the different literature [10], [11]. However, the Blizzard-Keck model [10] is employed in this study. The flame's shape is assumed spherical, the spark plug is placed at the center, and the entrained zone area is assumed small in comparison with the burned and unburned zones.

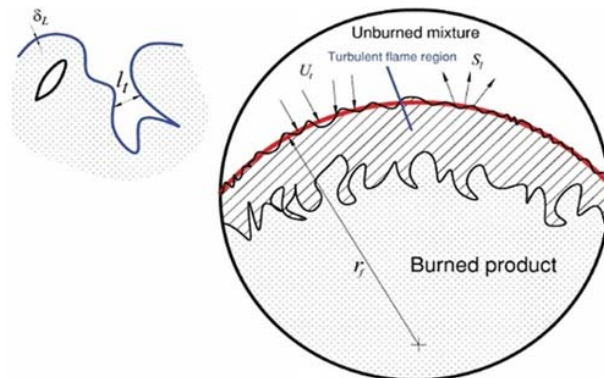


Fig. 4 Schematic of two-zone combustion chamber

The two-zone combustion equations consist of five differential-algebraic-equations (DAEs), (1)-(5), which are solved at each time step (or fraction of a crank angle). The entrained mass variation ( $\dot{m}_e$ ) is proportional to the flame front speed, laminar speed ( $S_L$ ), and characteristic speed of eddies ( $U_t$ ). The eddies' length which wrinkles around the flame front is shown in Fig. 4. The time duration to burn the eddies are represented by  $l_t$  and  $\tau_b$ . The unburned density ( $\rho_u$ ) is defined as the ratio between unburned mass and unburned volume ( $\rho_u = \frac{m_u}{V_u}$ ). The flame front area ( $A_f$ ) is calculated from flame volume based on the spherical flame propagation assumption. Geometrically, the sum of the burned volume ( $V_b$ ) and unburned volume ( $V_u$ ) at any crank angle must be equal to the total instantaneous volume of the cylinder ( $V_\theta$ ). Similarly, the sum of burned mass ( $m_b$ )

and unburned mass ( $m_u$ ) is equal to the total in-cylinder mass ( $m_{cyl}$ ) aspirated during the intake stroke, from intake valve open (IVO) to intake valve close (IVC). The four DAEs (1)-(4), along with the empirical Wiebe function (5), a characteristic S-shape function representing the time history of mass/volume fraction burned, are used to calculate values of the burned mass ( $m_b$ ), unburned mass ( $m_u$ ), entrained mass ( $m_e$ ), burned volume ( $V_b$ ), and unburned volume ( $V_u$ ) at each time step. In the Wiebe function, the parameter  $\theta_{sc}$  is the crank angle at the start of combustion, and  $a, m$ , and  $BD$  are parameters to be identified from a source of experimental data.

$$dm_e/dt = \rho_u A_f (U_t + S_t) \quad (1)$$

$$dm_b/dt = \rho_u A_f S_t + (m_e - m_b)/\tau_b \quad (2)$$

$$V_b + V_u = V_\theta \quad (3)$$

$$m_b + m_u = m_{cyl} \quad (4)$$

$$V_b/V_\theta = 1 - \exp\left(-a\left(\theta - \theta_{sc}/BD\right)^{m+1}\right) \quad (5)$$

### III. SIMULATION RESULTS AND DISCUSSIONS

The mathematical formulation (five DAEs) of the two-zone combustion engine model is integrated with four-stroke thermodynamic equations which consist of differential equations during power cycle and algebraic equations during gas exchange process, and emission prediction model [8]. The whole system is simulated in MapleSim [1] software. GT-Power package [6] is employed for cross-validation purposes, since the GT-Power models are calibrated and validated with experimental results. The single cylinder parameters and values are provided in Table I.

The mass fraction is the key variable which significantly affects the in-cylinder thermodynamic properties such as in-

cylinder pressure, burned and unburned temperature. Fig. 5 shows the burned mass fraction of the model in MapleSim and GT-Power model (reference model) during the combustion process. The S-shapes of both curves along with the start and the points are closely matched. The empirical parameters in the Wiebe function must be tuned at each operating point. The homotopy optimization technique is used to identify the Wiebe function parameters [12]. The apparent heat release, the amount of chemical energy released during the combustion process, is shown in Fig. 6.

TABLE I  
SINGLE CYLINDER SPARK IGNITION ENGINE PARAMETERS AND VALUES

Parameter	Value
Engine Speed	3000 [rpm]
Stroke length	0.1 [m]
Bore diameter	0.085 [m]
Connecting rod length	0.17 [m]
Compression ratio	10:1
In-cylinder trapped mass during power cycle	0.0005 [kg]
Cylinder wall temperature	470 [K]
Ambient temperature	290 [K]
Average pressure	1 [bar]
Average Combustion burned duration	80 [degrees]
Start of Combustion (spark advance angle)	30 BTDC [degrees]
Empirical Wiebe parameters (a,m)	(2.1, 1.2)

TABLE II  
SIMULATION TIME

Model	Fixed time step (Implicit Euler Solver) (STEP SIZE = 0.0001)	Variable time step (Rosenbrock Stiff Solver) (Absolute tolerance = 0.0001)
Single-cylinder thermodynamic and combustion model	0.032 [s]	0.031 [s]
Single-cylinder thermodynamic, combustion, and emission model	0.093 [s]	0.035 [s]

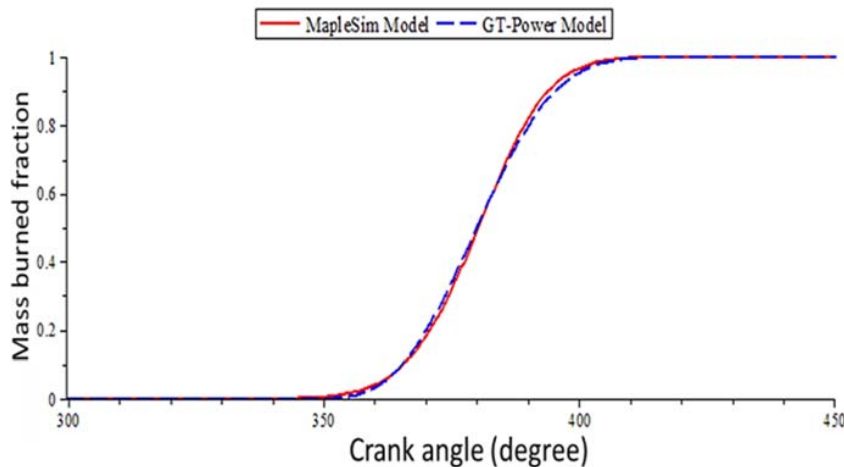


Fig. 5 Mass burned fraction during combustion

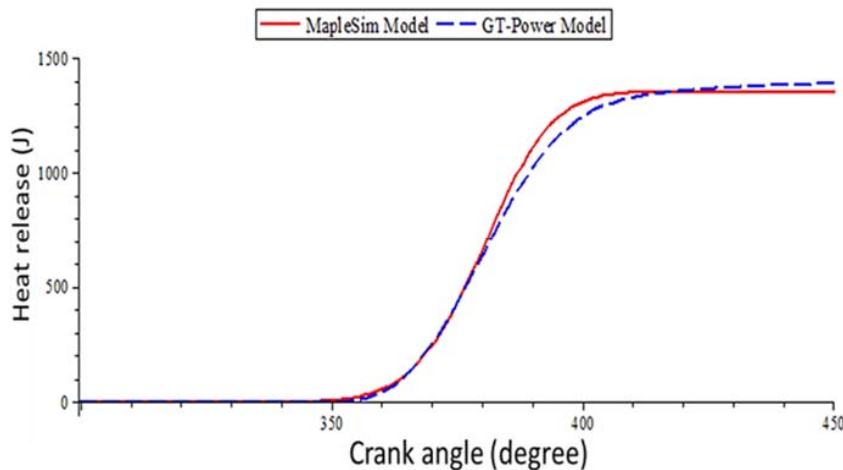


Fig. 6 Heat release during combustion

The simulations are executed on a 64-bit Windows 7 computer with Intel (R) Core (TM) Duo 3.33 GHz CPU. The simulation time for four-stroke operation, or two fully crank rotations, is defined as (6), where “ $n$ ” and “ $\Delta t$ ” represent engine speed in rpm and time duration in seconds. For instance, the simulation time for one cycle (or 720 degrees of crank rotation) at 3000 rpm is about 0.04 seconds.

$$\Delta t = 120/n \quad (6)$$

Table II represents the CPU time that takes to simulate one cycle at 3000 rpm or 0.04 seconds of single cylinder spark ignition engine model with different solvers’ set-up. The simulations results are generated in MapleSim 6 for both fixed and variable time step. The CPU time is less than one cycle time (0.04 seconds) at 3000 rpm. In the other words, the model could be used for many iterations application such as feedback controller design, optimizations, and hardware-in-the-loop (HIL) simulations.

The variable time step is usually faster than fixed time step, since the Rosenbrock stiff solver in MapleSim manages the sizing of time step. For instance, the solver loosens the time step when it is needed and consequently the simulation is faster. Moreover, adding more look-up tables and nonlinear algebraic equations to the model (e.g. emission sub-model [8]) dramatically slows down the simulation time with fixed time step.

#### IV. CONCLUSIONS

This paper categorized different modeling approaches to simulate ICEs. The stringent emission limits from governments along with the tight competition among automotive manufacturers to improve fuel consumption, have forced the automotive industry to develop new control strategies. The efficient control-oriented ICE model development with an acceptable level of complexity and fast simulation time can significantly help the automotive industry to come up with more reliable model-based control strategies. The enhanced zero-

dimensional (also called semi-predictive) two-zone combustion model is developed in MapleSim environment. The cycle-by-cycle spark ignition engine model in this paper is able to generate simulation results at each crank angle. The simulation time for both thermodynamic equations (including two-zone combustion model) and emission model, which is a set of complex differential-algebraic-equations (DAEs), shows that the CPU time (integration time) is faster than cycle time.

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