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Contaminant Transport Modeling Due to Thermal Diffusion Effects with the Effect of Biodegradation

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Abstract—The heat and mass transfer characteristics of contaminants in groundwater subjected to a biodegradation reaction is analyzed by taking into account the thermal diffusion (Soret) effects. This phenomenon is modulated mathematically by a system of partial differential equations which govern the motion of fluid (groundwater) and solid (contaminants) particles. The numerical results are presented graphically for different values of the parameters entering into the problem on the velocity profiles of fluid, contaminants, temperature and concentration profile.

Keywords—Heat and mass transfer, Soret number, porous media.

I. Introduction

CONTAMINATION of groundwater by chemical solvents, microorganisms and petroleum products coming from leaking pipelines and tanks, hazardous spills and surface waste deposits is a problem of increasing public concern. The contaminants migrate through the subsurface, and eventually reach the water table.

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Convective heat transfer in porous media has been a subject of great interest for the last several decades. The research activities in this field has been evaluated because of a broad range of applications such as pollutant dispersion in groundwater, geophysical, thermal and insulating engineering, petroleum reservoirs, ground hydrology and solar power collector [1]-[5].

Recently, studies on heat and mass transfer problems on free convection flow of a fluid saturated Darcy porous medium have received considerable attention because of numerous applications, such as migration of moisture through air contained in fibrous insulations, dispersion of chemical contaminants through water saturated soil, spreading of pollutants, water movements in reservoirs, building science and convection in the earth's crust.

The Coupling between transport of heat and mass takes place due to influence of density variations with temperature and concentration. However, there are circumstances when direct coupling between temperature and concentration is possible when the cross diffusion (Soret effect) is not negligibly small. The Soret effect refers to the mass flux produced by temperature gradients. The Soret effect plays a significant role in the problems concerning in contaminant transport in groundwater, exploitation of geothermal reservoirs, etc.

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Flows driven by buoyancy effects of thermal diffusion is encountered mostly in movement of contaminants in groundwater. Convective flows with temperature and concentration differences where the transfer of heat and mass takes place have been studied extensively. The influence of contaminant particles on the flow of a viscous fluid has several important applications. Saffman [6] initiated and investigated the study of dusty fluids and discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Migration of moisture air contained in fibrous insulations and grain storage installations and the dispersion of chemical contaminants through water-saturated soil, super coveting geothermics are investigated by Dunn and Hardee [7].

The laminar flow which arises in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species was investigated by Gebhart and Pera [8]. A fundamental study of the phenomenon of natural convection heat and mass transfer near a vertical surface embedded in a fluid saturated porous medium was analyzed by Bejan and Khair [9]. Beg and Makinde [10] extended their study with species diffusion in a Darcian porous medium channel using numerical solution.

The present investigation is to study systematically and numerically the laminar flow of an incompressible fluid (groundwater) due to thermal and mass diffusion and biodegradation reaction. The highly coupled non-linear partial differential equations governing the physical system are first reduced using perturbation technique to the ordinary differential equations and is converted into system of six simultaneous equations. These equations are then solved numerically by stream function method to obtain velocity, temperature and concentration profiles for various physical parameters.

II. Mathematical Formulation

In Cartesian co-ordinate system, we consider unsteady laminar flow of a viscous, incompressible fluid (groundwater) of uniform cross section h, lying below the porous layer and above the impermeable layer. Initially at $t \leq 0$, the region is assumed to be at the same temperature T_0 and concentration C_0. When $t > 0$, the temperature of the region is instantaneously raised to T_w and concentration raised to C_w which oscillate with time and is therefore maintained constant. Let x axis be along the fluid flow and y axis perpendicular to it. The schematic diagram of the problem is shown in Fig. 1.

A. Assumptions

The governing equations are written based on the following assumptions:
(i) The contaminant particles are solid, spherical, non-conducting, and equal in size and uniformly distributed in the flow region.
(ii) The density of contaminants is constant and the temperature between the particles is uniform throughout the motion.
(iii) The interaction between the particles, chemical reaction between the particles and fluid has been considered.
(iv) The volume occupied by the particles per unit volume of the mixture, (i.e., volume fraction of contaminants) and mass concentration have been taken into consideration.

B. Governing Equations of the Flow

The governing equations of continuity, momentum, energy and species are:
For fluid particle:

$$\frac{\partial u_f}{\partial x}+\frac{\partial v_f}{\partial y}=0 \tag{1}$$

$$\begin{aligned}&\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial x}+v_f\frac{\partial u_f}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u_f}{\partial x^2}+\frac{\partial^2 u_f}{\partial y^2}\right)\\&+\frac{KN}{p}(u_s-u_f)+g\beta_T(T_f-T_0)+g\beta_T(C_f-C_0)-\frac{\nu}{k_1}u_f\end{aligned} \tag{2}$$

$$\begin{aligned}&\frac{\partial v_f}{\partial t}+u_f\frac{\partial v_f}{\partial x}+v_f\frac{\partial v_f}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v_f}{\partial x^2}+\frac{\partial^2 v_f}{\partial y^2}\right)\\&+\frac{KN}{p}(v_s-v_f)-\frac{\nu}{k_1}v_f\end{aligned} \tag{3}$$

$$\begin{aligned}&\rho_{C_p}\left(\frac{\partial T_f}{\partial t}+uf\frac{\partial T_f}{\partial x}+vf\frac{\partial T_f}{\partial y}\right)=K_T\left(\frac{\partial^2 T_f}{\partial x^2}+\frac{\partial^2 T_f}{\partial y^2}\right)\\&+\frac{N_mC_p}{\tau_t}(T_s-T_f)\end{aligned} \tag{4}$$

$$\begin{aligned}&\frac{\partial C_f}{\partial t}+u_f\frac{\partial C_f}{\partial x}+v_f\frac{\partial C_f}{\partial y}=D_1\left(\frac{\partial^2 C_f}{\partial x^2}+\frac{\partial^2 C_f}{\partial y^2}\right)\\&+\frac{D_1k_T}{T_m}\left(\frac{\partial^2 T_f}{\partial x^2}+\frac{\partial^2 T_f}{\partial y^2}\right)\end{aligned} \tag{5}$$

For solid particles:

$$\frac{\partial u_s}{\partial x}+\frac{\partial v_s}{\partial y}=0 \tag{6}$$

$$\frac{\partial u_s}{\partial t}+us\frac{\partial u_s}{\partial x}+vs\frac{\partial u_s}{\partial y}=\frac{k}{m}(u_f-u_s)-\frac{\nu}{k_1}u_s \tag{7}$$

$$\frac{\partial v_s}{\partial t}+vs\frac{\partial v_s}{\partial x}+vs\frac{\partial v_s}{\partial y}=\frac{k}{m}(v_f-v_s)-\frac{\nu}{k_1}v_s \tag{8}$$

$$NC_m\left(\frac{\partial T_s}{\partial t}+u_s\frac{\partial T_s}{\partial x}+v_s\frac{\partial T_s}{\partial y}\right)=-\frac{NC_p}{\tau_T}(T_s-T_f) \tag{9}$$

$$\begin{aligned}&\frac{\partial C_s}{\partial t}+u_s\frac{\partial C_s}{\partial x}+v_s\frac{\partial C_s}{\partial y}=D_2\left(\frac{\partial^2 C_s}{\partial x^2}+\frac{\partial^2 C_s}{\partial y^2}\right)\\&+\frac{D_2K_T}{T_m}\left(\frac{\partial^2 T_s}{\partial x^2}+\frac{\partial^2 T_s}{\partial y^2}\right)-\gamma(C_s-C_0)\end{aligned} \tag{10}$$

where u_f,v_f is the velocity of the fluid particle, u_s,v_s is the velocity of solid particles, ρ is the density of the fluid, ρ is the pressure, K is the Stokes resistance coefficient, N is the number density of the solid particles, g is the gravitational acceleration, β_T is the thermal expansion coefficient, β_c is coefficient of expansion with concentration, ν is the kinematic viscosity, k_1 is the permeability of the porous medium, T_f is the temperature of the fluid, T_s is the temperature of solid particles, T_0 is the initial temperature, C_p is the specific heat of the fluid, C_m is the specific heat of solid particles, K_T is the thermal conductivity of the fluid, τ_T is the thermal relaxation time of the solid particles, m is the mass of the solid particles, C_f is the concentration of the fluid, C_s is the concentration of the solid particles, D_1 is the coefficient of mass diffusivity of the fluid, D_2 is the coefficient of mass diffusivity of solid particles, γ is the biodegradation reaction constant.

The initial and boundary conditions to the problem are:

When t=0, u_f=0, v_f=0, u_s=0, v_s=0, $T_f=T_0$, $T_s=T_0$, $C_f=C_0$, $C_s=C_0$

$$\left.\begin{aligned}u_f=0,\ u_s=0,\ v_f=0,\ v_s=0,\ T_f=T_a,\\ T_s=T_b,\ \frac{\partial C_f}{\partial y}=0,\ \frac{\partial C_s}{\partial y}=0\end{aligned}\right\}\quad y=0 \tag{11}$$

$$\left.\begin{aligned}&\frac{\partial u_f}{\partial y}=\frac{\alpha}{\sqrt{k_1}}u_f,\ \frac{\partial u_s}{\partial y}=\frac{\alpha}{\sqrt{k_1}}u_s,\ v_f=v_a,\ v_s=v_b,\\&\frac{\partial T_f}{\partial y}=\frac{\alpha}{\sqrt{k_1}}T_f,\ \frac{\partial T_s}{\partial y}=\frac{\alpha}{\sqrt{k_1}}T_s,\ \frac{\partial C_s}{\partial y}=0,\\&C_f=c_a,\ C_s=c_b\end{aligned}\right\}\quad y=h \tag{12}$$

The problem is non-dimensionalized by substituting the following non-dimensional quantities,

$$\begin{aligned}&u_f^*=\frac{u_f}{v_0},\ u_s^*=\frac{u_s}{v_0},\ t^*=\frac{tv_0^2}{\nu},\ v_f^*=\frac{v_f}{v_0},\ v_s^*=\frac{v_s}{v_0},\\&x^*=\frac{v_0x}{\nu},\ y^*=\frac{v_0y}{\nu},\theta_f^*=\frac{T_f-T_0}{T_w-T_0},\ \theta_s^*=\frac{T_s-T_0}{T_w-T_0},\\&\phi_f^*=\frac{C_f-C_0}{C_w-C_0},\ \phi_s^*=\frac{C_s-C_0}{C_w-C_0},\ p^*=\frac{p}{\rho v_0^2}\end{aligned} \tag{13}$$

Substituting the above non-dimensional quantities (13) in the governing equations (1)-(12), (after removing asterisks), we get

$$\begin{aligned}&\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial x}+v_f\frac{\partial u_f}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u_f}{\partial x^2}+\frac{\partial^2 u_f}{\partial y^2}\\&+G_f(u_s-u_f)+Gr\theta_f+Gm\phi_f-\sigma^2u_f\end{aligned} \tag{14}$$

$$\begin{aligned}&\frac{\partial v_f}{\partial t}+u_f\frac{\partial v_f}{\partial x}+v_f\frac{\partial v_f}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial^2 v_f}{\partial x^2}+\frac{\partial^2 v_f}{\partial y^2}\\&+G_f(v_s-v_f)-\sigma^2v_f\end{aligned} \tag{15}$$

$$\begin{aligned}&\frac{\partial \theta_f}{\partial t}+u_f\frac{\partial \theta_f}{\partial x}+v_f\frac{\partial \theta_f}{\partial y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta_f}{\partial x^2}+\frac{\partial^2 \theta_f}{\partial y^2}\right)\\&+\frac{f}{\Lambda}(\theta_s-\theta_f)\end{aligned} \tag{16}$$

$$\frac{\partial \phi_f}{\partial t}+u_f\frac{\partial \phi_f}{\partial x}=\frac{1}{Sc_1}\left(\frac{\partial^2 \phi_f}{\partial x^2}+\frac{\partial^2 \phi_f}{\partial y^2}\right)+Sc_1\frac{\partial^2 \phi_f}{\partial y^2} \quad (17)$$

$$\frac{\partial u_s}{\partial t}+u_s\frac{\partial u_s}{\partial x}+v_s\frac{\partial u_s}{\partial y}=\frac{1}{G_p}(u_f-u_s)-\sigma^2 u_s \quad (18)$$

$$\frac{\partial v_s}{\partial t}+u_s\frac{\partial v_s}{\partial x}+v_s\frac{\partial v_s}{\partial y}=\frac{1}{G_p}(v_f-v_s)-\sigma^2 v_s \quad (19)$$

$$\frac{\partial \theta_s}{\partial t}+u_s\frac{\partial \theta_s}{\partial x}+v_s\frac{\partial \theta_s}{\partial y}=-RL_o(\theta_s-\theta_f) \quad (20)$$

$$\frac{\partial \phi_s}{\partial t}+u_s\frac{\partial \phi_s}{\partial x}=\frac{1}{Sc_2}(\frac{\partial^2 \phi_s}{\partial x^2}+\frac{\partial^2 \phi_s}{\partial y^2})+Sr_2\frac{\partial^2 \phi_s}{\partial y^2}-D_1\phi_s \quad (21)$$

The initial and boundary conditions reduce to

$$u_f=0,\ u_s=0,\ v_f=0,\ v_s=0,\ \theta_f=0,\ \theta_s=0,\ \phi_f=0,\ \phi_s=0\ at\ t=0$$

$$u_f=0,\ u_s=0,\ v_f=0,\ v_s=0,\ \theta_f=T_{a_f},\ \theta_s=T_{b_s},\ \frac{\partial \phi_f}{\partial y}=0,\ \frac{\partial \phi_s}{\partial y}=0,\ at\ y=0 \quad (22)$$

$$\left.\begin{aligned}&\frac{\partial u_f}{\partial y}=\alpha\sigma u_f,\ \frac{\partial u_s}{\partial y}=\alpha\sigma u_s,\ v_f=v_{b_f},\ v_s=v_{c_s},\\ &\frac{\partial \theta_f}{\partial y}=\alpha\sigma(\theta_f+T_{c_f}),\ \ \frac{\partial \theta_s}{\partial y}=\alpha\sigma(\theta_s+T_{c_s}),\\ &\phi_f=c_{a_f},\ \phi_s=c_{b_s}\end{aligned}\right\}\ \text{y=1} \quad (23)$$

where, Grashof number: $Gr=\frac{g\beta_t\nu(T_w-T_0)}{v_o^3}$ Modified Grashof number: $Gm=\frac{g\beta_c\nu(C_w-C_0)}{v_0^3}$ Porosity Parameter: $\sigma=\frac{\nu}{\sqrt{k_1}v_0}$ Fluid particle parameter: $G_f=\frac{KN\nu}{\rho v_0^2}$ Particle mass parameter: $G_p=\frac{mv_0^2}{\nu k}$ Prandtl number: $Pr=\frac{\mu C_p}{k_T}$ Time relaxation parameter: $\Lambda=\frac{\tau_t v_0^2}{\nu}$ Mass concentration of solid particle: $f=\frac{Nm}{\rho}$ Schmidt number of the fluid: $Sc_1=\frac{\nu}{D_1}$ Schhmidt number of solid particle: $Sc_2=\frac{\nu}{D_2}$ Soret number of the fluid: $Sr_1=\frac{D_1 k_T(T_w-T_0)}{T_m\nu(C_w-C_0)}$ Soret number of solid particle: $Sr_2=\frac{D_2 k_T(T_w-T_0)}{T_m\nu(C_w-C_0)}$ Temperature relaxation time parameter: $L_0=\frac{\rho h^2}{\mu\tau_t}$ Dimensionless biodegradation reaction parameter:

$$D_1=\frac{\gamma_\nu}{v_0^2}$$
$$R=\frac{C_p}{C_m}$$

III. Method of Solution

To get the solution of the system of equations (14)-(21) subject to the boundary conditions (22) and (23), we assume the following perturbation method for small geometric parameter (i.e. $\epsilon << 1$) as:

$$u_i(x,y,t)=u_{0_i}(y)+\epsilon e^{\mathbf{i}(\lambda x+\omega t)}u_{1_i}(y)+o(\epsilon^2) \quad (24)$$

$$v_i(x,y,t)=\epsilon e^{\mathbf{i}(\lambda x+\omega t)}u_{1_i}(y)+o(\epsilon^2) \quad (25)$$

$$p(x,y,t)=p_0(x)+\epsilon e^{\mathbf{i}(\lambda x+\omega t)}p_1(y)+o(\epsilon^2) \quad (26)$$

$$\theta_i(x,y,t)=\theta_{0_i}(y)+\epsilon e^{\mathbf{i}(\lambda x+\omega t)}\theta_{1_i}(y)+o(\epsilon^2) \quad (27)$$

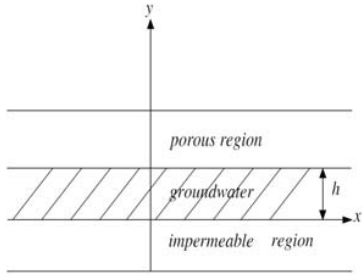

Fig. 1 Physical Configuration

$$\phi_i(x,y,t)=u_{0_i}(y)+\epsilon e^{\mathbf{i}(\lambda x+\omega t)}\phi_{1_i}(y)+o(\epsilon^2) \quad (28)$$

where, $i = f,s$ represents the fluid and solid particle respectively, **i** represents the imaginary part.

Considering the real part and neglecting the higher order terms (order of ϵ^2) we proceed our problem. Substituting (24)-(28) in (14)-(21) and collecting the coefficient of various power of ϵ on both sides, we obtain the following set of equations:

A. Base State Equations

$$\frac{\partial^2 u_{0_f}}{\partial y^2}-g_3u_{0_f}+Gr\theta_{0_f}+Gm\phi_{0_f}+g_1=0 \quad (29)$$

$$\frac{1}{Pr}\frac{\partial^2\theta_{0_f}}{\partial y^2}+\frac{f}{\Lambda}\left(\theta_{0_s}-\theta_{0_f}\right)=0 \quad (30)$$

$$\frac{1}{Sc_1}\frac{\partial^2\phi_{0_f}}{\partial y^2}-Sr_1\frac{\partial\theta_{0_f}}{\partial y}=0 \quad (31)$$

$$\frac{1}{G_p}u_{0_f}-\left(\frac{1}{G_p}+\sigma^2\right)u_{0_s}=0 \quad (32)$$

$$RL_0\left(\theta_{0_s}-\theta_{0_f}\right)=0 \quad (33)$$

$$\frac{1}{Sc_2}\frac{\partial^2\phi_{0_s}}{\partial y^2}+Sr_2\frac{\partial^2\theta_{0_s}}{\partial y^2}+D_1\phi_{0_s}=0 \quad (34)$$

The corresponding boundary condition becomes

$$u_{0_f}=0, u_{0_s}=0,\ \theta_{0_{f=T_{b_f}}},\ \frac{\partial\phi_{0_f}}{\partial y}=0, \frac{\partial\phi_{0_s}}{\partial y}=0\ at\ y=0 \quad (35)$$

$$\frac{\partial u_{0_f}}{\partial y}=\alpha\sigma u_{0_f}, \frac{\partial u_{0_s}}{\partial y}=\alpha\sigma u_{0_s},\ \frac{\partial\theta_{0_f}}{\partial y}=\alpha\sigma(\theta_{0_f}+T_{c_f}),\ \phi_{0_f}=c_{a_f},\ \phi_{0_s}=c_{b_s}\ at\ y=1 \quad (36)$$

The solutions for zeroth-order velocity, temperature and concentration of both fluid particle and solid particles are given by

$$u_{0_f}=Ae^{\sqrt{g_3}y}+Be^{-\sqrt{g_3}y}+\frac{1}{g_3}\left(g_4\frac{y^2}{2}+Grc_1y+Grc_2+Gmc_4+g_1+g_4\right) \quad (37)$$

$$u_{0_s}=\frac{u_{0_f}}{1+G_p\sigma^2} \quad (38)$$

$$\theta_{0_f} = c_1 y + c_2 \quad (39)$$

$$\theta_{0_s} = \theta_{0_f} \quad (40)$$

$$\phi_{0_f} = Sr_1 Sc_1 c_1 \frac{y^2}{2} + c_4 \quad (41)$$

$$\phi_{0_s} = \frac{c_{a_s}}{c_{0_s}\sqrt{D_1 Sc_2}} cos[\sqrt{D_1 Sc_2}]y \quad (42)$$

where,

$$g_1 = -\frac{\partial p}{\partial x}; \quad g_2 = -\frac{\partial p}{\partial y};$$

$$g_3 = \sigma^2 + G_f + \frac{G_f}{1+G_p\sigma^2}; \quad g_4 = -GmSr_1Sc_1;\ c_1 = \frac{\alpha}{1-\sigma\alpha};$$

$$c_2 = T_{b_f};\ c_{a_f} = Sr_1Sc_1c_1;\ T_{b_f} = \frac{T_a - T_0}{T_w - T_0};\ T_{c_f} = \frac{T_0\nu}{\nu_0(T_w - T_0)};$$

$$A = -B - \frac{1}{g_3}\left(Grc_2 + Gmc_4 + g_1 + g_4\right);$$

$$B = \frac{-1}{(\sqrt{g_3} + \alpha\sigma)\cosh(\sqrt{g_3})} \left(\left(\alpha\sigma - \frac{1}{g_3}(Grc_2 + Gmc_4 + g_1 + g_4)e^{\sqrt{g_3}}\right)\right) + \frac{1}{g_3}\left(\frac{3}{2}g_4 + Grc_1 + Grc_2 + Gmc_4 + g_1\right) + \frac{1}{\sqrt{g_3}}e^{\sqrt{g_3}}(Grc_2 + Grc_4 + g_1 + g_4) - \frac{1}{g_3}(g_4 + Grc_1);$$

B. First Order Equations

$$\frac{\partial^2 u_{1_f}}{\partial y^2} + \left(\omega\tan(\lambda x + \omega t) - G_f + u_{0_f}\lambda\tan(\lambda x + \omega t) - \lambda^2 - \sigma^2\right) u_{1_f} + \lambda p_1 tan(\lambda x + \omega t) + G_f u_{1_s} - Gr\theta_{1_f} - G_m\phi_{1_f} = 0 \quad (43)$$

$$\frac{\partial^2 v_{1_f}}{\partial y^2} + \left(\omega tan(\lambda x + \omega t) + u_{0_f} \lambda\tan(\lambda x + \omega t) - \lambda^2 tan(\lambda x + \omega t) - \sigma^2\right) v_{1_f} - g_2 + G_f(v_{1_s} - v_{1_f}) = 0 \quad (44)$$

$$\frac{1}{Pr}\frac{\partial^2\theta_{1_f}}{\partial y^2} + \left(\omega tan(\lambda x + wt) - \frac{1}{Pr}\lambda^2 - \frac{f}{\lambda}\right)\theta_{1_f} + \lambda tan(\lambda x + \omega t)\theta_{0_f}u_{1_f} - v_{1_f}\frac{\partial\theta_{0_f}}{\partial y} + \frac{f}{\lambda}\theta_{1_s} = 0 \quad (45)$$

$$\frac{1}{Sc_1}\frac{\partial^2\phi_{1_f}}{\partial y^2} + Sr_1\frac{\partial\theta_{1_f}}{\partial y} + \left(\omega tan(\lambda x + \omega t) + u_{0_f}\lambda tan(\lambda x + \omega t) + \frac{1}{Sc_1}\lambda^2\right)\phi_{1_f} = 0 \quad (46)$$

$$\omega tan(\lambda x + \omega t) + \left(\lambda tan(\lambda x + \omega t)u_{0_s} - \frac{1}{G_p} - \sigma^2\right) u_{1_s} - v_{1_s}\frac{\partial u_{0_s}}{\partial y} + \frac{1}{G_f}u_{1_f} = 0 \quad (47)$$

$$\left(\omega tan(\lambda x + \omega t) + \lambda tan(\lambda x + \omega t)u_{0_s} - \sigma^2 - 1\right) v_{1_s} - v_{1_f} = 0 \quad (48)$$

$$\left(\omega tan(\lambda x + \omega t) + \lambda tan(\lambda x + \omega t)u_{0_s} + RL_0\right)\theta_{1_s} - v_{1_s}\frac{\partial\theta_{0_s}}{\partial y} - RL_0\theta_{1_f} = 0 \quad (49)$$

$$\left(\omega tan(\lambda x + \omega t) - \lambda tan(\lambda x + \omega t)u_{0_s} - \frac{1}{Sc_2}\lambda^2 + D_2\right) \phi_{1_s} + \frac{1}{Sc_2}\frac{\partial^2\phi_{1_s}}{\partial y^2} + Sr_2\theta_{1_s} = 0 \quad (50)$$

The solution for the above equations of velocity, temperature and concentration, after introducing the streamfunction ψ in the following form,

$$u_{1_f} = -\frac{\partial\psi_{1_f}}{\partial y}, u_{1_s} = -\frac{\partial\psi_{1_s}}{\partial y}, v_{1_f} = \frac{\partial\psi_{1_f}}{\partial x}, v_{1_s} = \frac{\partial\psi_{1_s}}{\partial x}$$

and eliminating the pressure perturbations we get the equation

$$\psi_{1_f}^{''''} + \left(\omega tan(\lambda x + \omega t) + u_{0_f}\lambda tan(\lambda x + \omega t) - \lambda^2 - B_f - \sigma^2 + \lambda^2 tan^2(\lambda x + \omega t)\right)\psi_{1_f}^{''} - \left(u_{0_f}^{''}\lambda tan(\lambda x + \omega t) + \omega\lambda^2 tan^3(\lambda x + \omega t) + u_{0_f}\lambda^3 tan^3(\lambda x + \omega t) - \lambda^4 tan^3 (\lambda x + \omega t) - \sigma^2\lambda^2 tan^2(\lambda x + \omega t) + B_f\lambda^2 tan^2(\lambda x + \omega t)\right)\psi_{1_f} + B_f\lambda^2 tan^2(\lambda x + \omega t)\Psi_{1_s} + B_f\psi_{1_s}^{''} - Gr\theta_{1_f}^{'} - Gm\phi_{1_f}^{'} = 0 \quad (51)$$

$$\frac{1}{Pr}\frac{\partial^2\theta_{1_f}}{\partial y^2} + \left(\omega tan(\lambda x + \omega t) - \frac{1}{Pr}\lambda^2 - \frac{f}{\Lambda}\right)\theta_{1_f} - \lambda tan (\lambda x + \omega t)\theta_{0_f} + \psi_{1_f}^{'} + \lambda tan(\lambda x + \omega t)\psi_{1_f}\theta_{0_f}^{'} + \frac{f}{\Lambda}\theta_{1_s} = 0 \quad (52)$$

$$\frac{1}{Sc_1}\phi_{1_f}^{''} + S_{r_1}\theta_{1_f}^{'} + \left(\omega tan(\lambda x + \omega t) + u_{0_f}\lambda tan(\lambda x + \omega t) + \frac{1}{Sc_1}\lambda^2\right)\phi_{1_f} = 0 \quad (53)$$

$$\left(\lambda tan(\lambda x + \omega t)u_{0_s} + \frac{1}{B_p} - \sigma^2\right)\psi_{1_s}^{'} - \left(\omega tan(\lambda x + \omega t) + \lambda tan(\lambda x + \omega t) + u_{0_s} - 1 - \sigma^2 + \lambda tan(\lambda x + \omega t)u_{0_s}^{'}\right) \lambda tan(\lambda x + \omega t)\psi_{1_s} + \frac{+}{1}B_p\psi_{1_f}^{'} - \lambda tan(\lambda x + \omega t)\psi_{1_f} = 0 \quad (54)$$

$$\left(\omega tan(\lambda x + \omega t) + \lambda tan(\lambda x + \omega t)u_{0_s} + RL_0\right)\theta_{1_s} + \lambda\tan (\lambda x + \omega t)\psi_{1_s}\theta_{0_s}^{'} - RL_0\theta_{1_f} = 0 \quad (55)$$

$$\frac{1}{Sc_2}\phi_{1_s}^{''} + \left(\omega tan(\lambda x + \omega t) - \lambda tan(\lambda x + \omega t)u_{0_s} - \frac{1}{Sc_2}\lambda^2 + D_1\right)\phi_{1_s} + Sr_2\theta_{1_s} = 0 \quad (56)$$

where the prime ($'$) denotes differentiation with respect to y.

Equations (51)-(56) are solved numerically with the corresponding boundary conditions

$$\psi_{1_f} = 0,\ -\psi_{1_f}^{'} = 0,\ -\psi_{1_s}^{'} = 0,\ \theta_{1_f} = T_{b_f},\ \phi_{1_f}^{'} = 0,\ \phi_{1_s}^{'} = 0\ at\ y = 0$$

$$\psi^{''}_{1_f} = \alpha\sigma\psi^{'}_{1_f}, \quad \psi_{1_f} = v_c, \ \theta^{'}_{1_f} = \alpha\sigma\theta_{1_f},$$
$$\phi_{1_f} = c_{a_f}, \phi_{1_s} = c_{a_s} \quad at \quad y = 1 \quad (57)$$

Solving (51)-(56) we get ψ_{1_f}, ψ_{1_s}, θ_{1_f}, θ_{1_s}, ϕ_{1_f} and ϕ_{1_s}. Differentiating ψ_{1_f} and ψ_{1_s} with respect to y, we get u_{1_f} and u_{1_s}. The sum of the base part and perturbed part gives the required velocity of fluid particle u_f and velocity of solid particle u_s respectively as:

$$u_f = u_{0_f} + \epsilon e^{\cos{(\lambda x + \omega t)}} u_{1_f}$$
$$u_s = u_{0_s} + \epsilon e^{\cos{(\lambda x + \omega t)}} u_{1_s}$$

Similarly the sum of the base part and perturbed part of temperature (θ_f , θ_s) and concentration (ϕ_f , ϕ_s) of fluid and solid particles are respectively given below:

$$\theta_f = \theta_{0_f} + \epsilon e^{\cos{(\lambda x + \omega t)}} \theta_{1_f}$$
$$\theta_s = \theta_{0_s} + \epsilon e^{\cos{(\lambda x + \omega t)}} \theta_{1_s}$$
$$\phi_f = \phi_{0_f} + \epsilon e^{\cos{(\lambda x + \omega t)}} \phi_{1_f}$$
$$\phi_s = \phi_{0_s} + \epsilon e^{\cos{(\lambda x + \omega t)}} \phi_{1_s}$$

IV. Results and Discussions

In order to study the behavior of fluid velocity (u_f), solid velocity (u_s), temperature (θ_f) and concentration ((ϕ_f),(ϕ_s)) fields, a comprehensive numerical computation is carried out for various values of the parameter that describe the flow characteristics, and the results are presented in terms of graphs. Figs. 2-7 present the effect of thermal Grashof number, fluid parameter and mass Grashof number on the velocity distributions of fluid particle and solid particle, respectively. For various values of Grashof number (Gr) the velocity profiles for fluid and solid particle are plotted in Figs. 2 and 5. The Grashof number signifies the relative effect of the thermal buoyancy force to the viscous force. It is observed that there is a decrease in velocity field with increase in Grashof number. This is due to the fact that the pressure of contaminant particles causes a resistive force which slows down the movement of the fluid and in turn decreases the fluid and solid particle velocity.

From Figs. 3 and 6 the following observations are made: The velocity profiles of both the fluid and solid particles decrease with increase in fluid particle parameter (G_f) for the case of $G_f < 1$. Also for G_f=1 and $G_f > 1$, the velocity profiles of the fluid and contaminant particle are negative. The differences of the velocity are greater in the case of contaminants than fluid phase, when the mass concentration of the solid particle changes. This is due to the presence of contaminants. It is clear that the contaminant particles have an influence in changing the velocity of the fluid when compared with clean fluid profiles.

It is clear from Figs. 4 and 7, that an increase in mass Grashof number leads to enhance the velocity profiles because it reduces the drag force. This is due to the inclusion of the buoyancy effects. The presence of the buoyancy effects, complicate the problem, by coupling of the flow problem with thermal and mass problem.

The effect of the Prandtl number Pr on the temperature profile is shown in Fig. 8. The central reason behind this effect is that the heat absorption causes a decrease in the thermal energy of the fluid and solid particle. This is consistence with the well known fact that the rate of cooling is faster with increasing Prandtl number for both fluid and contaminant particle.

The concentration profiles for different values of the Schmidt number for the fluid and solid particles are displayed through Figs. 9 and 11, respectively. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. It is seen that the concentration profile decreases with increase of Schmidt number, in both the fluid and contaminant particle. This causes the concentration buoyancy effects to decrease a yielding eradication in the fluid and contaminants. Hence, the concentration boundary layer becomes thinner, velocity and at the same time the concentration diffusion species depletes. However, the reduction in mass diffusion causes the decrease in the concentration.

The Soret number, Sr, defines the effect of the temperature gradients including significant mass diffusion effects. Figs. 10 and 12 reveal that the concentration profile slackens with the reckoning of Sr. From this illustration it is noted that the heat is generated due to buoyancy force, which induces the flow rate to the booming in the concentration profile for both the fluid and contaminant phase. Also it is noted that an rising in the Soret number due to the contribution of temperature gradients to species diffusion aggregates the concentration, and consequently enhances the same (concentration profile).

Fig. 13 shows the effect of biodegradation reaction parameter D_1 on the concentration for contaminant particle. It is observed that the concentration decreases with increase in biodegradation reaction parameter. The reason behind the effect is due to hydrodynamic dispersion occurence, which results in the faster travelling molecules of biodegradation which results in a decrease in the concentration profile. Further the bacterial growth and acclimation increase the rate of degradation, which results in a decrease in the concentration.

V. Conclusion

This study focused on the behavior of a source contaminant and toxic intermediates under physical and biological conditions including groundwater flow, biodegradation and Soret effect. In contaminated sites, the physical, chemical, biological and geological conditions determining groundwater contamination is more complicated. Micro organism populations can vary with time and location, and these variation may change overall biodegradation of organic contaminants. Thus these site-specific factors should be integrated in characterizing the contaminant migration and in evaluating the natural attenuation processes, its potential health hazards and also the evaluation of potential plume development at contaminated sites.

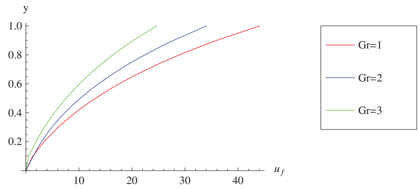

Fig. 2 Velocity of fluid particle for different Grashof number

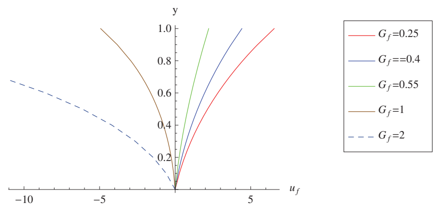

Fig. 3 Velocity of fluid for different fluid particle parameter

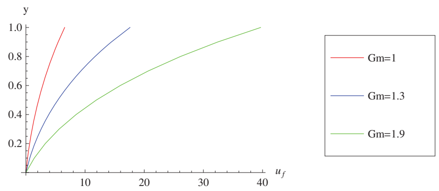

Fig. 4 Velocity of fluid particle for different mass Grashof number

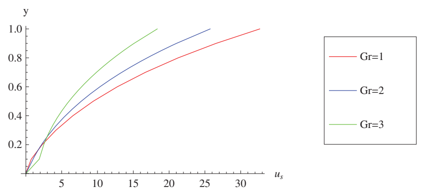

Fig. 5 Velocity of solid particle for different Grashof number

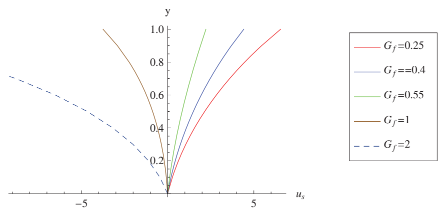

Fig. 6 Velocity of solid particle for different fluid particle parameter

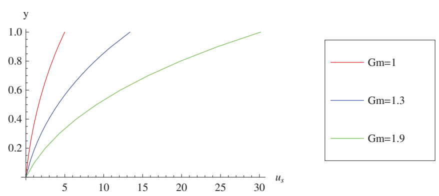

Fig. 7 Velocity of solid particle for different mass Grashof number

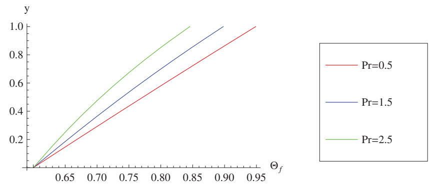

Fig. 8 Temperature of fluid particle for different Prandtl number

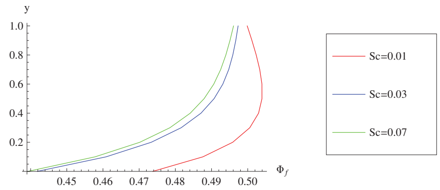

Fig. 9 Concentration of fluid particle for different Schmidt number

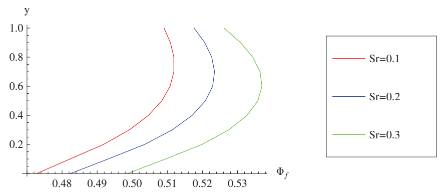

Fig. 10 Concentration of fluid particle for different Soret number

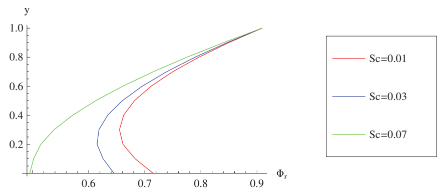

Fig. 11 Concentration of solid particle for different Schmidt number

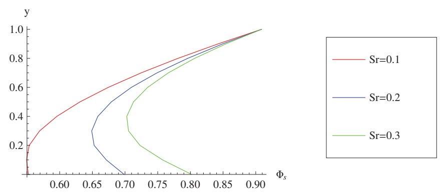

Fig. 12 Concentration of solid particle for different Soret number

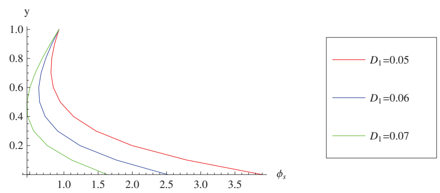

Fig. 13 Concentration of solid particle for different biodegradation parameter

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