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Conditions for Model Matching of Switched Asynchronous Sequential Machines with Output Feedback

Jung–Min Yang

Abstract—Solvability of the model matching problem for input/output switched asynchronous sequential machines is discussed in this paper. The control objective is to determine the existence condition and design algorithm for a corrective controller that can match the stable-state behavior of the closed-loop system to that of a reference model. Switching operations and correction procedures are incorporated using output feedback so that the controlled switched machine can show the desired input/output behavior. A matrix expression is presented to address reachability of switched asynchronous sequential machines with output equivalence with respect to a model. The presented reachability condition for the controller design is validated in a simple example.

Keywords—Asynchronous sequential machines, corrective control, model matching, input/output control.

I. Introduction

ASYNCHRONOUS sequential machines, or clockless logic circuits, as they are often called, are hardware/software systems that operate sequentially with no global synchronizing clock [1]. Since first invented in mid 1950's [2], asynchronous sequential machines have been used in various areas as an important building block of the system, e.g., digital systems [3], communication networks [4], parallel computation [5] etc. Corrective control is a novel automatic control theory developed exclusively for asynchronous machines. It utilizes the unique feature of asynchronous machines that the speed of their transient transitions is very fast (in zero time, ideally). As long as the stable reachability is guaranteed from a given state to a desired state, a corrective controller can generate a control input sequence that drives the considered machine towards the desired state. With the aforementioned capability, corrective control has been successfully applied to compensating the stable-state behavior of a given asynchronous machine with various faulty behavior [6]–[9].

In this paper, we address the model matching problem of switched asynchronous sequential machines. The switched systems are a kind of hybrid systems that consist of several submachines and a rule that coordinates switching operations between them. Due to their importance in both theoretical and practical applicability, the study of switched systems has drawn a great attention, especially in the field of linear systems [10]. In event-driven dynamics, however, few studies on switched systems have been reported so far. Notable among them are switched Boolean networks for gene regulatory networks [11] and control of switched asynchronous sequential machines by [12].

In [12], the problem of model matching for switched asynchronous sequential machines is investigated in the framework of corrective control wherein submachines have the characteristics of input/state machines, namely the present state is given as the output. In this study, we focus our concern on the case that submachines have the form of input/output machines whose output value is different from the present state. In contrast to the case of switched machines with input/state submachines, the closed-loop system does not have to match the behavior of the model in terms of the input/state specification. Instead, model matching is regarded as complete if the controlled machine provides the same output as that of the reference model in response to a given external input. The necessary and sufficient reachability condition for the existence of an appropriate corrective controller will be addressed based on corrective control theory and asynchronous mechanism. Note that a discussion on detailed procedures of controller construction is omitted in this paper and left as a further research (see [7] for prior works on input/output corrective control of single asynchronous machines).

The rest of this work is structured as follows. Section II provides a modeling formalism of input/output switched asynchronous machines and the problem statement for model matching. In Section III, stable reachability of input/output switched asynchronous machines is described in terms of Boolean matrices and the condition for corrective controllability is addressed. A simple example is provided in Section IV to validated the presented scheme and theorem. Finally, Section V concludes the paper.

II. Preliminaries

A. Switched Asynchronous Sequential Machines

Let us consider a switched asynchronous sequential machine Σ with m submachines. Assume that each submachine is a single input/output asynchronous sequential machine, namely the present output of the machine is different from the state value. Σ is represented as

$$\Sigma = \{\Sigma_i | i \in M\}$$
$$\Sigma_i = (A, Y, X, x^0, f_i, h_i)$$

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Jung–Min Yang is with the is with the School of Electronics Engineering, Kyungpook National University, Daegu 41566, Republic of Korea (e-mail: jmyang@knu.ac.kr).

where $M := \{1, \ldots, m\}$, Σ_i is the ith submachine, A is the input set, Y is the output set, X is the state set with n states, $x^0 \in X$ is the initial state, $f_i : X \times A \to X$ is the state transition function partially defined on $X \times A$, and $h_i : X \to Y$ is the output function. Since every submachine is assumed to have an equal operational domain, the input, state, and output sets of Σ_i are the same for every $i \in M$; only f_i and h_i differ from one another.

Each submachine Σ_i is operated according to the characteristics of a single asynchronous sequential machine, that is, it is not governed by any synchronizing clock and the state transition is executed only in response to changes of external inputs. A state–input pair $(x, v') \in X \times A$ is a stable pair of Σ_i if $f_i(x, v') = x$ and x is a stable state. If $f_i(x, v') \neq x$, on the other hand, x is a transient state and (x, v') is a transient pair. Note that x may be stable or transient depending on the value of the present input. Denote by

$$U_i(x) := \{v \in A | f_i(x, v) = x\}$$

the set of external inputs that make a stable pair with x in Σ_i. Owing to the absence of a synchronizing clock, Σ_i stays at a stable pair (x, v') indefinitely. If the input v' changes to another value v for which (x, v) is a transient pair, Σ_i engages in a series of transient transitions

$$\begin{aligned} f_i(x, v) &= x_1 \\ f_i(x_1, v) &= x_2 \\ &\vdots \end{aligned}$$

where v remains fixed. Assuming no infinite cycles, Σ_i reaches the *next stable state* x_k such that $x_k = f_i(x_k, v)$ at the end of the chain with k transient transitions. Since the transition speed of asynchronous sequential machines is very fast, the meaningful behavior of asynchronous sequential machines may be described merely in terms of stable states. To this end, we introduce the stable recursion function s as [6]:

$$\begin{aligned} s_i : X \times A &\to X \\ s_i(x, v) &= x' \end{aligned}$$

where x' is the next stable state of a valid state–input pair (x, v). A chain of transient transitions from a stable state to its next stable state, as represented by s_i, is termed a stable transition. The domain of s_i can be expanded to $X \times A^+$ in a natural way as follows, where A^+ is the set of all nonempty strings of characters in A.

$$\begin{aligned} s_i(x, v_1 v_2 \cdots v_k) = s_i(s_i(x, v_1), v_2 \cdots v_k), \\ v_1 v_2 \cdots v_k \in A^+. \end{aligned}$$

B. Problem Statement

A control configuration for the switched asynchronous sequential machine Σ is shown in Fig. 1 where C is the corrective controller, D is the demultiplexer, P is the multiplexer, $v \in A$ is the external input, $u \in A$ is the control signal, $\sigma \in M$ is the switching signal, and $y_i \in Y$, $i \in M$, is the output of Σ_i. Let Σ_c denote the closed-loop system consisting of C, D, P, and Σ.

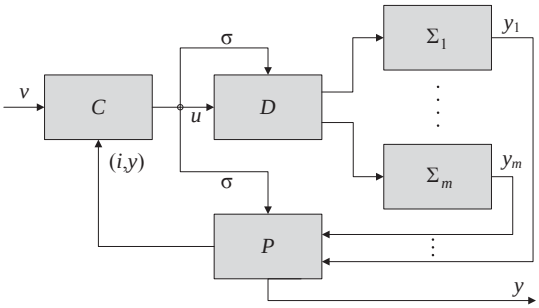

Fig. 1 Control configuration for the switched asynchronous sequential machine Σ

Since C is also designed in the form of an asynchronous sequential machine, Σ_c has the asynchronous mechanism. C provides Σ with u or σ, either of which is generated at a time, but not simultaneously. D plays the role of determining the *active submachine* whose dynamics is manifested by Σ. Upon receiving σ, D selects Σ_σ as the active submachine and delivers the present control signal u to Σ_σ. Hence changing σ equals the activation of switching operation. P receives m output feedback values from all submachines $\Sigma_1, \ldots, \Sigma_m$ and selects y, the feedback value generated by Σ_σ. P forwards y and $i \in M$, the index of the active submachine, to C.

The control objective is to present the existence condition and design algorithm for a corrective controller C that matches the stable-state behavior of Σ_c to that of a reference model

$$\Sigma' = (A, Y, Z, z^0, f', h').$$

Note that Σ' has the same input and output set as those of Σ whereas its state set

$$Z = \{z_1, \ldots, z_r\}$$

differs from X. Accordingly, f' and h' have mapping relations $f' : Z \times A \to Z$ and $h' : Z \to Y$. We also assume that h' is an injective function, namely each $z_p \in Z$ corresponds to a distinctive output value.

We regard that model matching between Σ_c and Σ' is accomplished if two machines show the same input/output behavior, i.e., for an external input, they provide an identical output value. Moreover, each submachine Σ_i can serve as structural redundancy of Σ in model matching control. In the case of input/output control for single asynchronous machines [7], model matching is infeasible if the considered machine does not have enough reachability to realize the matched behavior with Σ'. However, the switched machine Σ can change its mode to another submachine whenever the active submachine does not have the required reachability, and can active the correction procedure to take the new active submachine toward the desired state.

To avoid unpredictable behaviors caused by the absence of a synchronizing clock, we assume that Σ_c always preserves the principle of fundamental mode operations [13] whereby a variable must change its value when both C and Σ are in stable states, and no two or more variables can be altered simultaneously.

III. Main Result

A. Skeleton Matrices

We first introduce several skeleton matrices that are needed to describe stable reachability of switched machines for model matching control.

Definition 1: Given $\Sigma' = (A, Y, Z, z^0, f', h')$ with the stable recursion function s', the *one-step skeleton matrix* $S^1(\Sigma')$ is an $r \times r$ matrix whose (p, q) entry is defined as

$$S^1_{p,q}(\Sigma') = \begin{cases} 1 & \exists v \in A \text{ such that } s'(x_p, v) = x_q \\ 0 & \text{otherwise} \end{cases}$$

$S^1(\Sigma')$ epitomizes stable reachability of the model Σ' via unit input characters. Note that we have to consider only one-step stable reachability of Σ' since model matching control will be activated upon the transmission of an input character to Σ and Σ'.

Definition 2: Given $\Sigma_i = (A, Y, X, x^0, f_i, h_i)$ and $\Sigma' = (A, Y, Z, z^0, f', h')$, the *output equivalence list* of Σ_i with respect to Σ' is $E(\Sigma_i, \Sigma') := \{E^i_1, \ldots, E^i_r\}$ where

$$E^i_p := \{x \in X | h_i(x) = h'(z_p)\}, \ p = 1, \ldots, r.$$

$E^i_p \in X$ represents the subset of X every element of which has the same output as $z_p \in Z$. If Σ_i and Σ' stay at $x \in E^i_p$ and z_i, respectively, they are said to have output equivalence.

Definition 3: Given the output equivalence list $E(\Sigma_i, \Sigma') = \{E^i_1, \ldots, E^i_r\}$, the *fused skeleton matrix* $\Delta(i)$ is an $r \times r$ matrix whose (p, q) entry is defined as

$$\Delta_{p,q}(i) = \begin{cases} 1 & \forall x \in E^i_p, \exists x' \in E^i_q \text{ and } t \in A^+ \\ & \text{such that } s_i(x_p, t) = x_q \\ 0 & \text{otherwise} \end{cases}$$

$\Delta(i)$ depicts stable reachability of Σ_i in terms of elements of $E(\Sigma_i, \Sigma')$. $\Delta_{p,q}(i) = 1$ if every state of E^i_p can reach at least a state of E^i_q via a chain of stable transitions ($s_i(x_p, t) = x_q$). Note that the final state of E^i_q is unspecified since stable reachability between elements of $E(\Sigma_i, \Sigma')$ is measured with respect to outputs, that is, any state of E^i_q shows identical output characteristic.

Switching capability of Σ implies the ability of Σ to change its mode from a submachine to another submachine at a specific stable state. In [12], a constraint is imposed on the switching operation that as the result of switching, the active submachine always takes the same state possessed by the previous submachine. In this study, we generalize the switching operation by relaxing the foregoing constraint. In other words, the new active submachine does not necessarily transfer to the same state at which the old one has stayed before switching. To address the switching relation between two submachines, we define the following matrix.

Definition 4: $W(i, j)$, the *switching incidence matrix of two submachines* Σ_i *and* Σ_j, is an $n \times n$ matrix whose (p, q) entry is

$$W_{p,q}(i, j) = \begin{cases} 1 & \Sigma \text{ switches the mode from } \Sigma_i \text{ at } x_p \\ & \text{to } \Sigma_j \text{ at } x_q \\ 0 & \text{otherwise} \end{cases}$$

$W(i, j)$ represents switching capability of Σ in the most general way, that is, the state of the present submachine may differ from the previous one after switching. The motivation for introducing $W(i, j)$ stems from the fact that some switched machines have multiple submachines that share the same system module to compose the state space. As the switching operation depends on this implementation restraint, the next state may be different from the previous one.

Note that for switching from Σ_i at x_p to Σ_j at x_q, there must exist an external input $a \in A$ that makes a stable pair with both x_p of Σ_i and x_q of Σ_j, i.e.,

$$W_{p,q}(i, j) = 1 \Rightarrow U_i(x_p) \cap U_j(x_q) \neq \varnothing. \tag{1}$$

Under the principle of fundamental mode operations, Σ_i should stay at the stable state x_p at the moment that the switching signal σ changes. Hence the present control signal is $u \in U_i(x_p)$. Moreover, u must also make a stable pair with x_q in Σ_j, namely $u \in U_j(x_q)$; otherwise Σ_j could not maintain x_q upon completion of the switching operation. However, the condition $u \in U_j(x_q)$ may not be always valid since u is determined only by the past state trajectory of Σ_i. Still, as long as $U_i(x_p) \cap U_j(x_q) \neq \varnothing$ is held true, C can achieve the switching operation by changing the control signal to $u' \in U_i(x_p) \cap U_j(x_q)$ right before transmitting the switching signal $\sigma = j$. In this sense, (1) is a requisite for guaranteeing consistent switching.

While the switching incidence matrix $W(i, j)$ is defined based on states, it can be transformed into a matrix based on the entries of the output equivalence list as:

Definition 5: Let $E(\Sigma_i, \Sigma')$ and $E(\Sigma_j, \Sigma')$ be the output equivalence ist of Σ_i and Σ_j with respect to Σ', and let $W(i, j)$ be as defined in Definition 4. $G(i, j)$, the *switching incidence matrix of* Σ_i *and* Σ_j *with output equivalence with respect to* Σ', is an $r \times r$ matrix whose (p, q) entry is

$$G_{p,q}(i, j) = \begin{cases} 1 & \forall x_{p'} \in E^i_p, \exists x_{q'} \in E^i_q \text{ s.t. } W_{p',q'}(i, j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

In short, $\Delta(i)$ shows stable reachability of single submachines and $G(i, j)$ provides switching capability of Σ between different submachines, both represented in terms of output equivalence with respect to Σ'. The following definition combines stable reachability and switching capability of Σ in one matrix expression.

Definition 6: $\mathscr{C}^k(\Sigma)$, the *one-step switching skeleton matrix of* Σ *with output equivalence with* Σ' ($k = 1, 2, \ldots$), is an $rm \times rm$ matrix recursively defined as

$$\mathscr{C}^1(\Sigma) := \begin{pmatrix} \Delta(1) & G(1,2) & \cdots & G(1,m) \\ G(2,1) & \Delta(2) & \cdots & G(2,m) \\ \vdots & \vdots & \vdots & \vdots \\ G(m,1) & \cdots & \cdots & \Delta(m) \end{pmatrix}$$

$$\mathscr{C}^k(\Sigma) := \mathscr{C}^{k-1}(\Sigma) \times_{\mathscr{B}} \mathscr{C}^1(\Sigma)$$

where '$\times_{\mathscr{B}}$' denotes the Boolean product of two Boolean matrices where logic AND and OR are used instead of multiplication and plus operations in the matrix product.

Definition 7: $\Pi(\Sigma)$, the *switching skeleton matrix of* Σ *with output equivalence with* Σ', is an $rm \times rm$ matrix defined as

$$\Pi(\Sigma) := \sum_{k=1}^{nm-1} {}_{+_{\mathscr{B}}} \mathscr{C}^k(\Sigma)$$

where '$+_{\mathscr{B}}$' denotes the Boolean addition of two matrices.

$\mathscr{C}^k(\Sigma)$ shows whether an element of the output equivalence list of a submachine can be reachable from another element of the output equivalence list of another submachine in exactly k steps. Here, "one-step" implies that Σ takes either one switching operation or a chain of stable transitions. $\Pi(\Sigma)$ is a generalized description of stable reachability for the switched asynchronous sequential machine Σ interpreted in term of output equivalence with respect to the given model Σ'. Not only does $\Pi(\Sigma)$ represent stable reachability within each submachine, it also elucidates whether a state of a submachine can be reached from another state of a different submachine by a combination of stable transitions and switching operations wherein both start and final state are represented as entries of the corresponding output equivalence list. Since Σ has nm states in total, any state in Σ can be reached within $nm-1$ steps of switching and correction procedures. Hence $\mathscr{C}^1(\Sigma), \ldots, \mathscr{C}^{nm-1}(\Sigma)$ are sufficient to express the entire reachability of Σ.

Note that $E^i_j \in E(\Sigma_i, \Sigma')$, or the jth entry of the output equivalence list of Σ_i, has the jth position of the ith block of $\mathscr{C}^k(\Sigma)$ and $\Pi(\Sigma)$. Denoting by $\theta^i_j \in \{1, \ldots, rm\}$ the index of E^i_j, we have

$$\theta^i_j = (i-1)r + j. \tag{2}$$

B. Model Matching

Remind that for Σ' with its state set $Z = \{z_r, \ldots, z_r\}$, the output equivalence list of submachine Σ_i is denoted by $E(\Sigma_i, \Sigma') = \{E^i_1, \ldots, E^i_r\}$, $i = 1, \ldots, m$. Hence for each $z_j \in Z$, we have m equivalent subsets of states, namely $E^1_j, \ldots, E^m_j$.

Definition 8: Given Σ and Σ', Φ, a *subordinate state list with output equivalence with respect to* Σ', is

$$\Phi := \{x_j \in E^{i_j}_j | i_j \in M, j = 1, \ldots, r\}.$$

$K(\Phi)$, the skeleton matrix of Φ, is an $r \times r$ matrix whose (p,q) entry is defined as ($p, q \in \{1, \ldots, r\}$)

$$K_{p,q}(\Phi) := \Pi_{p',q'}(\Sigma)$$

where $\Pi(\Sigma)$ is the switching skeleton matrix of Σ with output equivalence with Σ' introduced in Definition 7, and $p' = \theta^{i_p}_p$ and $q' = \theta^{i_q}_q$ (see (2)).

Φ consists of r states, each state x_j taken from an entry $E^{i_j}_j$ of the output equivalence list of Σ_{i_j}. i_j implies that submachine Σ_{i_j} having the entry $E^{i_j}_j$ may differ from each z_j. In other words, Φ represents a collection of r states that are output equivalent with Z, while elements of Φ may belong to different submachines.

Using Φ and the skeleton matrices defined in the previous discussion, we now present the existence condition for a corrective controller that solves the model matching problem between Σ_c and Σ'.

Theorem 1: Given the switched asynchronous sequential machine Σ and the model Σ', a corrective controller C in Fig. 1 exists that matches the stable-state behavior of Σ_c to that of Σ' if and only if a subordinate state list $\Phi = \{x_j \in E^{i_j}_j | i_j \in M, j = 1, \ldots, r\}$ exists for which

$$K(\Sigma') \leqslant K(\Phi). \tag{3}$$

Condition (3) means that Σ possesses a subordinate state list $\Phi = \{x_1, \ldots, x_r\}$ whose stable reachability is greater than the model Σ'. Provided that (3) is valid, assume $K_{p,q}(\Sigma') = 1$ for some $p, q \in \{1, \ldots, r\}$, which implies that Σ' has a stable transition from z_p to z_q. But since $K(\Sigma') \leqslant K(\Phi)$, $K_{p,q}(\Phi) = 1$. According to Definition 8, the latter leads to $\Pi_{p',q'}(\Sigma)$ with $p' = (i_p - 1)r + p$ and $q' = (i_q - 1)r + q$, which means that $x_p \in \Phi$ of Σ_{i_p} can be reached to $x_q \in \Phi$ of Σ_{i_q} through a sequence of switching operations and correction procedures. Hence we can design a controller module that realizes the corresponding feedback path. After constructing each controller module for all stable transitions of Σ', the overall controller C is completed by assembling each controller module using *join* operation [8] that combines two corrective controller modules. A detailed algorithm for constructing C is omitted in this study.

IV. Example

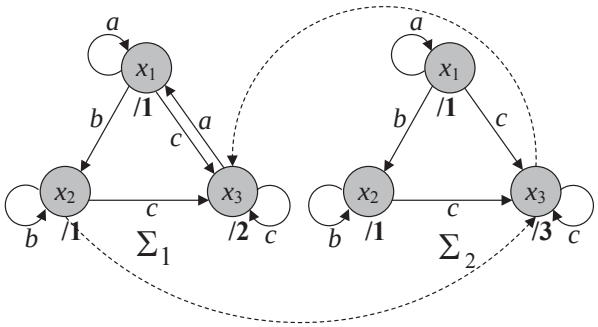

Fig. 2 State flow diagram of $\Sigma = \{\Sigma_1, \Sigma_2\}$

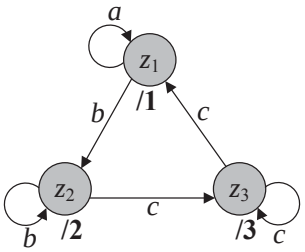

Fig. 3 State flow diagram of Σ'

Consider a simple switched asynchronous sequential machine $\Sigma = \{\Sigma_1, \Sigma_2\}$ ($M = \{1, 2\}$) shown in Fig. 2, where $X = \{x_1, x_2, x_3\}$ with $x^0 := x_1$, $A = \{a, b, c\}$, and $Y = \{1, 2, 3\}$. The output of each state is marked after dash '/' in the figure. For simplicity, we set $f_i(x, v) = s_i(x, v)$ for all $i = 1, 2$ and $(x, v) \in X \times A$. Solid arrows represent state transitions in submachines and dashed arrows switching

capability between Σ_1 and Σ_2. Fig. 3 shows the state flow diagram of the model Σ, where the state set is $Z = \{z_1, z_2, z_3\}$.

First, we derive the output equivalence list of Σ_1 and Σ_2 with respect to Σ'.

$$E(\Sigma_1, \Sigma') = (E_1^1, E_2^1, E_3^1) = (\{x_1, x_2\}, \{x_3\}, \varnothing)$$
$$E(\Sigma_2, \Sigma') = (E_1^2, E_2^2, E_3^2) = (\{x_1, x_2\}, \varnothing, \{x_3\}).$$

Next, referring to Figs. 2 and 3, we compute the fused skeleton matrix and switching incidence matrix as follows.

$$\Delta(1) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\Delta(2) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$
$$G(1,2) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$G(2,1) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

The one-step switching skeleton matrix $\mathscr{C}^1(\Sigma)$ is derived as

$$\mathscr{C}^1(\Sigma) = \begin{pmatrix} \Delta(1) & G(1,2) \\ G(2,1) & \Delta(2) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The one-step skeleton matrix $S^1(\Sigma')$ of the model Σ' is computed as

$$S^1(\Sigma') = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

To determine the existence of a corrective controller for model matching between Σ and Σ', we compute $\mathscr{C}^k(\Sigma)$, $k = 2, \ldots, 5$, so as to derive the switching skeleton matrix $\Pi(\Sigma)$ according to Definitions 6 and 7.

$$\Pi(\Sigma) = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Referring to the above result, take a subordinate state list Φ as follows.

$$\Phi = \{x_1 \in E_1^1, x_3 \in E_2^1, x_3 \in E_3^2\}.$$

The corresponding index set of Φ is $\{1, 2, 6\}$. Finally, by Definition 8, we derive the skeleton matrix $K(\Phi)$ of Φ as

$$K(\Phi) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

For instance, $K_{1,3}(\Phi) = \Pi_{1,6}(\Sigma) = 1$ and $K_{3,2}(\Phi) = \Pi_{6,2}(\Sigma) = 1$. Clearly, since Φ satisfies condition (3), by Theorem 1 a corrective controller C can be designed that solves the model matching problem between Σ and Σ'.

V. Summary

We have investigated controllability of a class of switched asynchronous sequential machines. When switched asynchronous sequential machines are endowed with submachines having the form of input/output machines, their reachability are described in terms of output equivalence with respect to a given model. We have presented that stable reachability and switching capability of switched asynchronous sequential machines can be represented by matrix expressions, which leads to the existence condition for a corrective controller solving the model matching problem. The examination of the controller existence has been practised in the case study.

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