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Extracting the Coupled Dynamics in Thin-Walled Beams from Numerical Data Bases

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***Abstract*—**In this work we use the Discrete Proper Orthogonal Decomposition transform to characterize the properties of coupled dynamics in thin-walled beams by exploiting numerical simulations obtained from finite element simulations. The outcomes of the will improve our understanding of the linear and nonlinear coupled behavior of thin-walled beams structures. Thin-walled beams have widespread usage in modern engineering application in both large scale structures (aeronautical structures), as well as in nano-structures (nano-tubes). Therefore, detailed knowledge in regard to the properties of coupled vibrations and buckling in these structures are of great interest in the research community. Due to the geometric complexity in the overall structure and in particular in the cross-sections it is necessary to involve computational mechanics to numerically simulate the dynamics. In using numerical computational techniques, it is not necessary to over simplify a model in order to solve the equations of motions. Computational dynamics methods produce databases of controlled resolution in time and space. These numerical databases contain information on the properties of the coupled dynamics. In order to extract the system dynamic properties and strength of coupling among the various fields of the motion, processing techniques are required. Time- Proper Orthogonal Decomposition transform is a powerful tool for processing databases for the dynamics. It will be used to study the coupled dynamics of thin-walled basic structures. These structures are ideal to form a basis for a systematic study of coupled dynamics in structures of complex geometry.

***Keywords*—**Coupled dynamics, geometric complexity, Proper Orthogonal Decomposition (POD), thin walled beams.

I. Introduction

BEAMS, especially thin-walled ones, form the basic components in many types of complex structures. They are widespread in almost all engineering fields, used as a basic structural element in civil engineering, as rotating blades in turbines and helicopter, as stiffeners in aircraft fuselage and aeronautical structures, and as basic members in robots, various machinery, space and marine multi-body mechanisms, and recently in nanostructures. As a result for their wide usage, cross sections of various geometries have been invented to meet various design requirements. Thin-walled beams with complex cross sections forms a class of technologically important structures encountered in many sectors of mechanical sciences and biological systems.

Basic issues dealing with nonlinear coupled dynamics and normal modes of vibrations in linear and nonlinear thin-walled beams are topics of increasing interest in modern engineering sciences. The main factors that affect the coupled dynamics in a thin-walled beam are the following: (1) the complexity of the geometry of the cross section: Perpendicular force acting on the beam will produce flexural motion coupled to torsion motion. (2) large displacements and rotations and (3) The complexity of loading conditions, for example, if the aerodynamic loads acting on a wing do not coincide with the elastic axis, motions with combined bending and twisting will be induced.

The coupled dynamics of a thin-walled beam can be very complicated if all the sources inducing coupling (cross-section geometry, loading, and geometric nonlinearity) act simultaneously. Clearly high fidelity modeling and reliable computational models are needed to predict the dynamics of a thin-walled beam structure. Classical methods involving analytic computations are valid only for very small motions for structures of very simple geometries. The majority of the available analytical solutions deal with coupled vibrations in beams by considering the linear inertial coupling that comes out of beams cross section geometry. At the same time, they ignore the geometric nonlinear coupling due to the effect of large displacements and/or rotations. The most obvious drawback of the classical linear methods is their weakness to provide an effective way to compute the dynamics of complicated structural systems composed of multiple beams, or combination of beams and other structural members.

The limitations of the analytical computations methods (linear and perturbations) formed an impetus to develop powerful numerical computational methods where the modern electronic digital computer plays a crucial role in regard to high speed and low cost of computations. For example, the Finite Element method can be used to derive discrete computational models to simulate with controlled accuracy and resolution in time and space the dynamics of complicated structures.

When simulating the dynamics of a structural system using a numerical computational method (FE), numerical databases are produced. In this numerical databases accurate information on the system dynamic properties, such as the Normal Modes of vibration and coupled dynamics are available but hidden in the numerical database. Without suitable processing for the numerical databases, it is very difficult to extract the hidden information. This leads to a search for methods to process numerical solutions databases and try to extract important properties. Here we use the Proper Orthogonal Decomposition transform to process these databases. The POD is a powerful tool for obtaining optimum spatial and temporal information, and providing bases for model reduction of nonlinear structural systems [6]-[9].

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II. Literature Review

Bishop and Johnson [3] classified the thin-walled beams structures based on their cross-section geometry and the location of the shear center into the following categories:

(1) Fig. 2 (a) depicts a cross-section symmetric about two perpendicular directions; the shear center s and the centroid c are coincident. Examples are rectangular and circular cross sections beams. In such case the bending and the torsional vibrations are nearly independent (decoupled) for a uniform beam moving at sufficiently small strain energy level. The linear vibration properties for the axial, transverse and torsion can be obtained from the classical beam theories (Bernoulli-Euler and Timoshenko). However, geometric nonlinearity is going to couple all elementary motions if the displacements and rotations of the cross-section are large.

(2) Fig. 2 (b) depicts a cross-section symmetric about one direction; the shear center s and centroid c lie on the same horizontal coordinate axis but separated by a non-zero distance. A channel cross section is such an example.

(3) Fig. 2 (c) depicts a cross-section without any symmetry; it is the general case in which the shear center s and the centroid c are separated by distances e_1 and e_2. In this case, the bending and torsional vibrations interact (coupled) due to the no coincidence of the shear and centroid centers.

Several analytic models have been derived to describe aspects of the coupled dynamics in beams. Timoshenko derived the governing differential equations. Dokumaci [4] obtained an exact solution for the coupled bending and torsional for the Timoshenko beam neglecting the shear deflection, the rotary inertia and the warping effect. The solution was obtained by assuming harmonic motion of radian frequency ω, to reduce the partial differential equation into a coupled ordinary differential equation. Jun Li [10], using d'Alembert principle, derived the bending-torsion coupled governing equation of motion for an axially loaded Timoshenko beam including the shear deformation, the rotary inertia and the warping stiffness. The differential equation for coupled vibrations can be written in the form:

$$\begin{aligned} & m\frac{\partial^2 \upsilon}{\partial t^2} - me_1\frac{\partial^2 \phi}{\partial t^2} - kGA\left(\frac{\partial^2 \upsilon}{\partial x^2} - \frac{\partial \theta}{\partial x}\right) + P\left(\frac{\partial^2 \upsilon}{\partial x^2} - e_1\frac{\partial^2 \phi}{\partial x^2}\right) = 0 \\ & \rho I\frac{\partial^2 \theta}{\partial t^2} - EI_2\frac{\partial^2 \theta}{\partial x^2} - kGA\left(\frac{\partial \upsilon}{\partial x} - \theta\right) = 0 \\ & GJ\frac{\partial^2 \phi}{\partial x^2} - P\left(\frac{I_s}{m}\frac{\partial^2 \phi}{\partial x^2} - e_1\frac{\partial^2 \upsilon}{\partial x^2}\right) + me_1\frac{\partial^2 \upsilon}{\partial t^2} - I_s\frac{\partial^2 \phi}{\partial t^2} - E\Gamma\frac{\partial^4 \phi}{\partial x^4} = 0 \end{aligned} \quad (1)$$

where $\upsilon(x,t)$ is the transverse displacement in the y-direction, $\theta(x,t)$ is the rotation about the z-axis (shearing), and $\phi(x,t)$ is the torsional rotation about the x-axis. Clearly this model describes the coupling due to the bending motion of the beam in the y direction, which will excite the torsion ϕ as well as the shearing θ. EI_2 is the flexural rigidity in the y direction, m is the mass per unit length, GJ is the torsional rigidity, $E\Gamma$ is the warping rigidity, r is the radius of gyration of the beam cross section and P is the static axial loading. The dynamic transfer matrix method was used in order to solve this coupled system to determine the coupled natural frequency and mode shapes.

For the third case where no cross-section symmetry exists, the shear center s and the centroid c are separated by distances e_1 and e_2 in the both directions. Vlasov [12] developed a torsion theory in which restrained warping is included. This theory is also called "warping torsion" or "non-uniform torsion". In Vlasov's theory the torsion is not constant along the x-axis. Based on Vlasov model for the coupled flexural torsional vibrations including the rotary inertia, shear deformation and warping stiffness are in the below differential equation for coupled vibrations:

$$\begin{aligned} & EI_2\frac{\partial^4 U_2}{\partial x^4} + m\frac{\partial^2 U_2}{\partial t^2} - me_1\frac{\partial^2 R_1}{\partial t^2} - \rho I_2\frac{\partial^4 U_2}{\partial x^2 \partial t^2} = 0, \\ & EI_3\frac{\partial^4 U_3}{\partial x^4} + m\frac{\partial^2 U_3}{\partial t^2} + me_2\frac{\partial^2 R_1}{\partial t^2} - \rho I_3\frac{\partial^4 U_3}{\partial x^2 \partial t^2} = 0, \\ & E\Gamma_1\frac{\partial^4 R_1}{\partial x^4} - GJ\frac{\partial^2 R_1}{\partial x^2} + m\left(e_2\frac{\partial^2 U_3}{\partial t^2} - e_1\frac{\partial^2 U_2}{\partial t^2}\right) + \rho\left(I_s\frac{\partial^2 R_1}{\partial t^2} - \Gamma_1\frac{\partial^4 R_1}{\partial x^2 \partial t^2}\right) = 0. \end{aligned} \quad (2)$$

where $U_2(x,t)$ and $U_3(x,t)$ denote the flexural motion of the beam in the y- and z-directions, and $R_1(x,t)$ denote the torsional rotation about the x-axis. The terms EI_2, EI_3 represent respectively the flexural rigidities about the y and z directions. Term I_s represents the polar moment of inertia of the cross-section about the shear center, J is the torsional constant, Γ_1 is the warping constant, A is the cross sectional area, and ρ is the mass density.

In this model, strong interaction between the two bending motions and the torsion will exist. The coupling exists due to the location of the shear center in distance from the location of the center of mass. The complexity of the geometry is lumped into two parameters.

Many approaches have been developed to solve this coupled equation. Friberg [5] derived the dynamic stiffness matrix, and Tanaka and Bercin [11] used symbolic algebra techniques to solve it and to obtain the coupled frequencies and mode shapes for different boundary conditions. The Classic Modal Analysis approach was implemented by assuming a known spatial behavior of the beams for the two lateral displacements and the torsional rotation as:

$$U_2(x,t) = u_2(x)\sin(\omega t),$$
$$U_3(x,t) = u_3(x)\sin(\omega t), \quad (3)$$
$$R_1(x,t) = r_1(x)\sin(\omega t)$$

where ω is the circular frequency. The spatially distributed amplitudes and the frequency are the new unknowns [11].

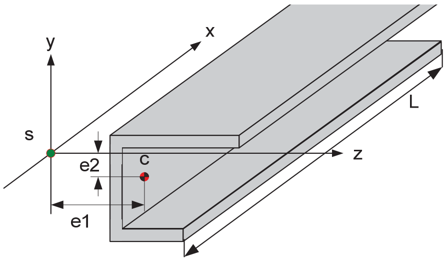

Fig. 1 Thin-walled bean with arbitrary cross-section, the shear center and the centroid are not coincident

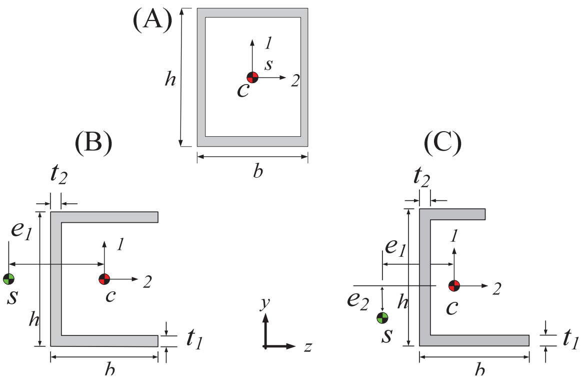

Fig. 2 Symmetry properties of cross-sections: (a) double symmetry, (b) single symmetry, (c) no symmetry

It is well known that accurate solutions for the thin walled beam dynamics can be obtained based on the computational methods represented by the Finite element methods (FEA) [2]. The numerically obtained solution for the free vibration of the beam contains information on the modal properties that is hidden in the numerical database. Proper operation (post-processing) is necessary in order to extract the dynamics properties. We claim that such a proper operation is the Time-POD transform.

III. POD Analysis of Rods

The method of Proper Orthogonal Decompositions (POD) for coupled fields is a powerful tool since it can process the database of information (numerical solution) and extract a small number of optimum spatio-temporal patterns or modes. Each pattern is the product of a function of time and a function of space. Here will apply the POD method to analyze the free motions of thin walled cantilever beam to extract the coupled dynamics from the numerical solution obtained from FEA.

We briefly present the POD method for coupled dynamic in thin walled beam. The POD version presented here is based on the Karunen-Loeve expansion developed to process stochastic processes in statistics and fluid mechanics. Let the augmented vector:

$$\vec{V}(s,t) = \begin{bmatrix} \vec{U}(s,t) \\ \vec{R}(s,t) \end{bmatrix} = \begin{bmatrix} U_2(s,t) \\ U_3(s,t) \\ R_1(s,t) \end{bmatrix} \quad (4)$$

represents the spatiotemporal evolution of a motion over the time interval $t \in [T_1, T_2]$. The motion can be expressed as follows:

$$\vec{V}(s,t) = <\vec{V}(s,t)>(s) + \vec{v}(s,t) \quad (5)$$

$$<\vec{V}(s,t)>(s) \equiv \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} V(s,t)dt \quad (6)$$

where the first term in (5) is the average of the motion and the second term is its fluctuation. We assume that the fluctuation of the motion can be expanded in a series of modes, that is:

$$\vec{v}(s,t) = \sum_{m=1}^{M} A_m(t)\sqrt{\lambda_m}\vec{\Phi}_m(s) \quad (7)$$

A generic mode is characterized by amplitude $A_m(t)$, a shape or spatial pattern $\sqrt{\lambda_m}\vec{\Phi}(s)$, and energy content λ_m. This expansion shall be called Proper Orthogonal Decomposition if the amplitudes and energies are the eigen-functions and eigen-vectors to the following eigen-problem:

$$\int_{T_1}^{T_2} C(t,\tau)A_m(\tau)d\tau = \lambda_m A_m(t) \quad (8)$$

The term $C(t,\tau)$, called the time autocorrelation operator, is defined by the autocorrelation in time operation:

$$C(t,\tau) \equiv \int_0^L \bar{v}(s,t)\bar{v}(s,\tau)ds = \int_0^L \sum_{m=1}^{3} \{V_m(s,t) - <V_m>(s)\}\{V_m(s,\tau) - <V_m>(s)\}ds \quad (9)$$

The shapes are obtained via the projection operation:

$$\vec{\Psi}_m(s) \equiv \sqrt{\lambda_m}\vec{\Phi}_m(s) = \int_{T_1}^{T_2} \vec{v}(s,t)A_m(t)dt \quad (10)$$

The amplitudes as well as the shapes form orthonormal sets of functions. For this reason the expansion is called Proper Orthogonal (PO). Moreover, the trace of the autocorrelation operator is energy like quantity of the motion since it is a quadratic form of the displacements and rotations. A motion is characterized uniquely by the POD energy spectrum, which is the set:

$$\Lambda \equiv \{\lambda_m\}_{m=1}^{\infty} \quad (11)$$

The constants λ_m are called energies since their sum adds up to be the trace of the autocorrelation operator. That a motion involves in general coupling between translational displacement and rotational displacement is reflected by the fact that the shape of a POD mode ($\overline{\Psi}_m(s)$) is a vector composed of the three elements, that is:

$$\overline{\Psi}_m(s) \equiv \begin{bmatrix} \vec{\chi}_m(s) \\ \overline{\Psi}_m(s) \end{bmatrix} \equiv \begin{bmatrix} \chi_{2m}(s) \\ \chi_{3m}(s) \\ \Psi_{1m}(s) \end{bmatrix} \tag{12}$$

where, $\chi_{2m}(s)$, $\chi_{3m}(s)$ are the bending POD mode shape in y and z directions respectively and $\Psi_{1m}(s)$ is the POD mode shape for the torsion about x axis.

Clearly the shape of a POD mode reveals how the displacement filed is coupled with the rotation field.

By computing the POD modes for a motion, we can determine how the motion couples in different patterns. The POD approach can further form the bases to identify the dominant degrees-of-freedom by calculating the norm (participation) of each mode component. The norms of the components of a POD mode:

$$\begin{aligned} C_{2m} &= \chi_{2m}(s) = \sqrt{\frac{2}{L}\int_0^L (\chi_{2m}(s))^2 ds} \\ C_{3m} &= \chi_{3m}(s) = \sqrt{\frac{2}{L}\int_0^L (\chi_{3m}(s))^2 ds} \\ C_{4m} &= \Psi_{1m}(s) = \sqrt{\frac{2}{L}\int_0^L (\Psi_{1m}(s))^2 ds} \end{aligned} \tag{13}$$

Using the above approach to process the finite element solution after rearranged it into a database of snapshots shall lead to property identification of the coupled beam. More information can be found in [6]-[9].

TABLE I
PROPERTY DETAILS FOR THE BEAM STUDIED IN THE EXAMPLE

Symbol	Meaning	Value
EI_1	Bending rigidity	$6.75023\text{x}10^6\ Nm^2$
EI_2	Bending rigidity	$2.89392\text{x}10^7\ Nm^2$
kGA	Torsional rigidity	$8.83721\text{x}10^8\ N$
$E\Gamma$	Warping rigidity	$45471.7\ Nm^4$
I_s	Polar moment of inertia	$32.41\ Kg.m$
L	Beam length	$2.0\ m$
e_1	Distance between shear center s and the centroid	$0.06350784\ m$
e_2	Distance between shear center s and the centroid	$-0.08040439\ m$
b	Beam width	$0.3m$
h	Beam height	$0.2m$

IV. Applying the POD to Extract the Coupled Vibrations in Linear Thin Walled Beams

The introduced POD-based methodology will be used to process the numerical solutions generated by the FE computational model. We consider the free vibration of the arbitrary cross-section thin-walled beam depicted in Fig. 1. The beam geometric and material properties are presented in Table I. In order to follow a systematic way in the analysis, we go through the following steps:

1. The natural frequencies and mode shapes for the clamped-free beam boundary condition will be obtained by solving the coupled (2).
2. Then, the free vibration for the same case will be obtained by numerically solving the FE model for the same beam. The FE model is composed of 40 two-node finite elements. The free motion is excited by applying an impulsive force at the free end of the beam pointing in the negative y direction. The simulation for the beam dynamics is produced by numerical time integration of the FE model. The solution is recorded every $dt = 0.0001$ second for a total time of 0.5 seconds.
3. Finally, the numerical solution (database) is arranged in snapshots matrix and transformed into Proper Orthogonal modes by using the Time POD transform. Comparison between the analytical solution and the POD results will be discussed.

A. POD Transform of the FE Numerical Database

In the discussion about the POD for coupled dynamics we have seen that the POD method characterizes in principle the dynamics of an infinite-degrees-of-freedom system in terms of a finite number of proper orthogonal modes. Each POD mode is characterized by the fraction of the energy of the motion it carries, a function of time $A_m(t)$ characterizing the mode distribution over the time domain, and a vector function of space $\mathbf{\Phi}_m(s)$ characterizing the spatial distribution of the coupled fields.

In the dynamic analysis of the cantilever thin-walled beam, a concentrated impact force was applied at its tip in order to excite the dynamics. The impact force excites many modes of vibration that will interact synchronously: some must be slow-related to the low order modes of vibration, and some quite fast-related to the higher order modes. Also, due to the geometric properties of the beam, the following couplings are expected to take place: (1) coupling between bending and torsional motions, (2) coupling between the two bending motions, and (3) coupling between the high and low order modes of vibration.

The POD transform for the discrete dynamics reveals that the energy is distributed over a finite number of POD modes. Fig. 3 reveals that the first four modes contain about 99.99% of the total “energy”. The first POD mode dominates with energy content at 98.7 %. This is the main property of the POD technique: namely its ability to compute from data the mathematical structure of the finite number of orthogonal modes that compose a motion. The PO modes describe

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optimally the degrees-of-freedom that a motion activates. We mention that a continuous flexible structure can be resolved into an infinite number of degrees-of-freedom.

The fact that the beam motion is dominant by one POD mode is important for many reasons. It is useful because it will help focus on the dominant dynamics, and it provides weighting factors that can be used to obtain more practical reduced models for the beam. As example for this beam, the reduced model can reach a high accuracy by involving the first POD mode because the latter contains 98.7 % of the energy of the motion.

The plot for the amplitude of the first POD reveals that the temporal behavior of the first POD mode in an oscillation dominated by one harmonic. The FFT transform, Fig. 4, determines a large amplitude harmonic with frequency at 37.1 Hz, plus another relatively higher frequency equal to 94.37 Hz but with small amplitude. Comparing the obtained frequency by the one from the analytic solution, it turns out that the dominant frequency is very close to the first torsional natural frequency of the beam (Table II). This result is quite important as far as the physical meaning of the POD modes is concerned.

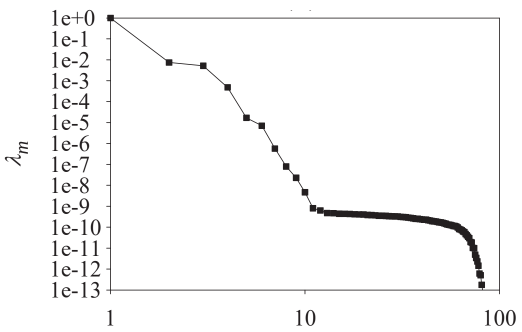

Fig. 3 The energy distribution for the POD modes. A finite number of modes contain more than 99.9 % of the energy

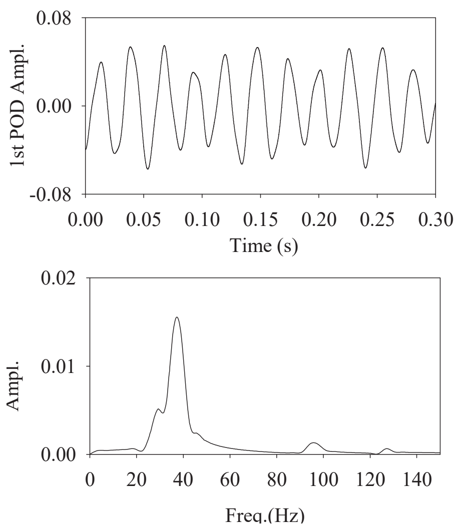

Fig. 4 The first POD amplitude: amplitude (upper), the FFT analysis (lower)

TABLE II
COMPARISON BETWEEN THE ANALYTICAL SOLUTION FREQUENCIES AND THE FIRST POD MODE FREQUENCIES

No	Natural frequencies from analytic solution	Frequencies of first POD amplitude
1	37.49 Hz	37.1 Hz
2	69.09 Hz	95.69 Hz

It is important to know the kind of motion that generated the frequencies of the first POD mode, without depending on the analytical solution. In order to do so, we examine the shapes of the first POD mode. Plots for the first POD mode shape components are shown in Fig. 5. The magnitudes of the mode components (norms) indicate the level of participation of each component in the total mode dynamics. It is obvious that the torsional component of the mode dominates over the two flexural components Fig. 6 (b).

The norms of the components of the POD will provide a direct measure about the participation of each component in the mode. The norms and the normalized POD mode are shown in Fig. 6. The norm of the torsional component (C41) is 0.982287 whereas the norms of the bending comments are respectively 0.294505 and 0.284605. This means the dominant frequency of 37.10 Hz is generated essentially from the torsional vibration. But because of the asymmetry in the cross-section geometry, the two transverse motions are directly coupled to the torsional motion. And this goes with the fact that in the asymmetric beam cross-section, any force applied to the beam will excite torsional motion. In fact the applied force can be presented as pure bending force acting on the shear center s and a moment about the x axis applied at the centroid. Clearly this will lead to strong coupling between the torsion and the bending modes.

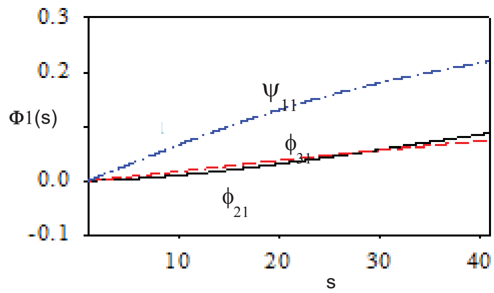

Fig. 5 The components of the shape for the first POD mode, the torsional component is dominant

We must also not forget that for this case the total torsional strains are the sum of those due to pure torsion and those due to wrapping. And as it is well known for the thin-walled beam, warping has strong effect on the coupled natural frequencies [1].

The second POD mode with energy 0.007417 has the amplitude shown in Fig. 7. The dominant frequency in this mode is at 27.34 Hz, and the other two are at 37.10 Hz and 95.69 Hz. The dominant frequency is identical to the first coupled bending vibration mode of the beam. Again we concluded that by the help of the results obtained from the analytical solution. The first normal mode of vibration has frequency at 27.34 Hz. In this mode we can see that the amplitude has more than one frequency, which means that this motion is coupled. The interesting in this result is that the POD represents the coupling between the slow and fast dynamics.

Again if we compare the results we again from the POD analysis and the result from solving the linear coupled problem we can see that the error is acceptable, Table III. This enhances the conclusion we made before that the proposed method can extract the natural frequencies from processing the free dynamics.

The normalized mode shapes and their norms (Fig. 8) reveal that the dominant motion is the bending vibration of the beam in the y direction Also it is revealed that this motion is coupled to torsional motion about the x-axis and with smaller amount to bending vibration in the z direction.

Up to this point by analyzing the first two POD modes, we managed to gain a full understanding for the dominant dynamics, the strength of the coupling and their effect on the beam behavior.

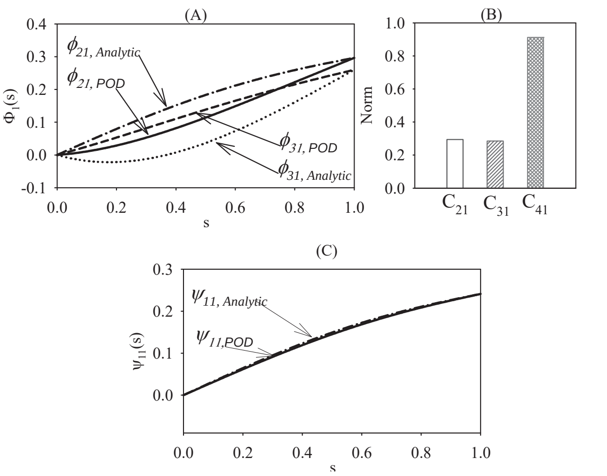

Fig. 6 (a) The shape of the first POD mode: normalized components of POD mode vs. components of the first linear mode, (b) The norms of the components of the POD mode, and (c) The dominant component and the identical linear component

The third POD mode contains a smaller amount of energy, which is 0.005173. The third POD amplitude shown in Fig. 9, the dominant frequencies in this mode are the 126.9108 Hz and the 97.62367 Hz. By involving the norms of the POD mode shape components, Fig. 11, we realize that the 97.62367 Hz is the frequency of the second bending mode of the beam in the z direction, while the 126.9108 Hz is the second normal mode of the beam in torsion. The conclusion is that the third POD mode represents a coupling between the torsion and the bending at higher modes.

The fourth POD mode has a very small amount of energy, 0.0004789. The dominant component is the bending vibration in the z direction.

The results from the beam analysis give evidence on the capabilities of the POD method to analyze deeply a complicated dynamic behavior. The interesting result is the fact that the PO modes are somehow related to the linear normal modes of the coupled system. This relation is not very clear at this point.

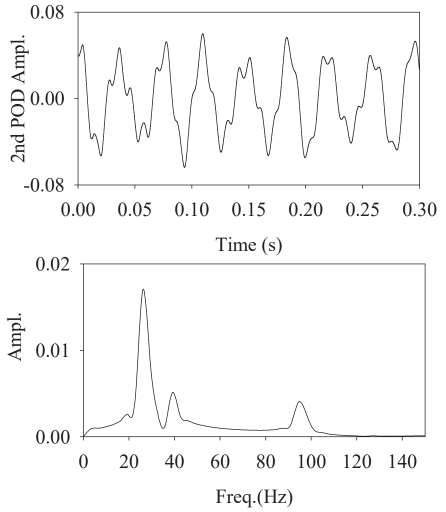

Fig. 7 The second POD mode: amplitude (upper), the FFT analysis (lower)

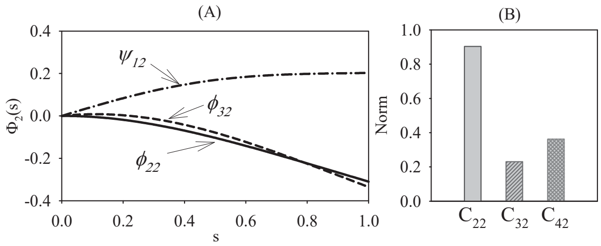

Fig. 8 (a) The normalized components of the second POD mode and (b) their norms

TABLE III
COMPARISON BETWEEN THE ANALYTICAL SOLUTION FREQUENCIES AND THE SECOND POD MODE FREQUENCIES

No	Freq. Analytic Solution	Freq. First POD amplitude
1	27.19 Hz	27.34 Hz
2	37.49 Hz	37.1 Hz
3	96.09 Hz	95.69z

B. Examine the Effect of Warping on the Beam Dynamics

We mentioned that in thin-walled beam with axis symmetry, if a pure torsion will act over the beam section, and the cross section of the beam is free to warp, then the warping will take place without causing any axial or shearing strains. While in the case where the beam cross section has no axis of symmetry, the warping will vary along the beam due to the torsion and hence there will be axial and/or bending strains. The spatial rate of change of the angle of twist will not be constant, but will vary along the axis of the beam, as result, the stresses, and thus the strains, are considered to depend exclusively on beam internal forces caused by the applied loading. The membrane force and bending moments are related to the centroid axis while the torsional moment and shear forces are related to the shear center axis. One of the most important computational difficulties is the accurate calculation under non uniform torsional loading of the beam response when it exhibits a significant cross sectional warping. Vlasov torsion theory, in which warping is included. The torsion is not constant along the x-axis. The rotations of the beam cross-section follow the following differential equation:

$$E\Gamma \frac{\partial^4 \phi}{\partial x^4} - GJ \frac{\partial^2 \phi}{\partial x^2} = M_x \qquad (14)$$

where GJ is the torsional stiffness, $E\Gamma$ is the warping stiffness and M_x is the distributed torsion moment along the beam and Γ is the warping constant.

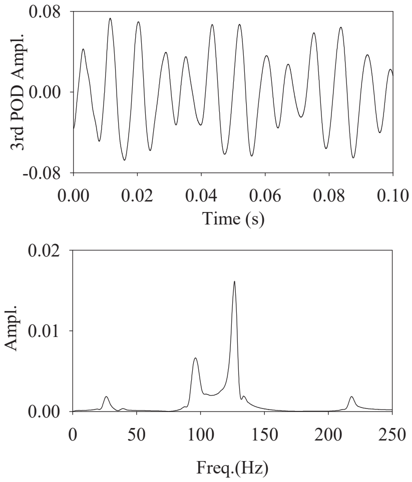

Fig. 9 (a) The third POD mode: amplitude, and (b) FFT analysis

TABLE IV
COMPARISON BETWEEN THE ANALYTICAL SOLUTION FREQUENCIES AND THE THIRD POD MODE FREQUENCIES

No	Freq. Analytic Solution	Freq. First POD amplitude
1	96.09 Hz	95.69 Hz
2	126.14 Hz	126.91 Hz

In this part we will use the POD method to investigate the warping effect on the beam dynamics. This will be done by processing the dynamics including the warping effect using the POD and compare to the beam dynamic without warping.

Fig. 12 compares the POD spectra for a case without warping and the same case including the warping effect. We notice that warping does not affect significantly the energy fraction of the dominant mode; but it does affect significantly the energy distribution over higher modes. Now, the effect of warping will be investigated by monitoring the norms of the components of the POD modes. In the dominant mode components, Fig. 13 the warping effect reduced the magnitudes of norms of the two bending and torsion components. The warping appears to have significant value of $C_{w1} = 0.58$. The importance of this result is that we could determine quantity and quality of the warping effect on the total beam dynamics.

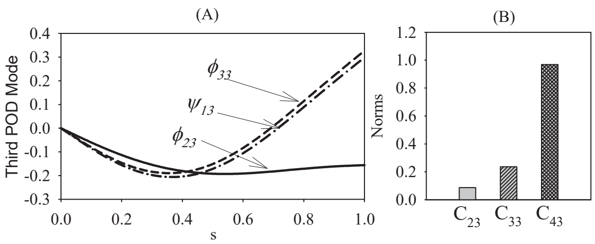

Fig. 10 The third POD mode: the normalized components (A), and their norms (B).

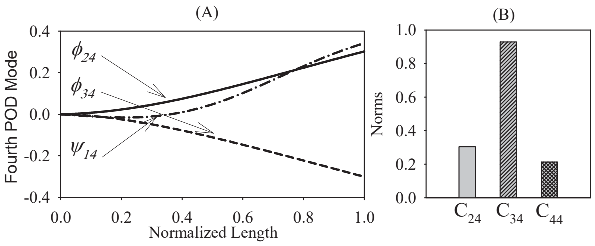

Fig. 11 The fourth POD mode: (a) the normalized components, and (b) the norms

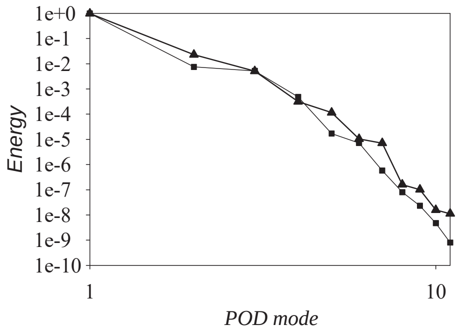

Fig. 12 Normalized energy distribution in the POD modes, for the case without warping (rectangles), and the case with warping (triangles)

In the second mode, Fig. 14, the warping has no significant effect on the dominant component (bending in y direction); whereas small effect appears on the torsional component.

In the third mode, Fig. 15, we can see that the warping component dominates. In particular, warping lowered considerably the torsional component and less the other components. In the fourth mode, Fig. 16, the warping has affects slightly the components of the mode. The general conclusion is that the effect of the warping is more prominent in the torsion component of the POD modes.

V. CONCLUSION

Using the Discrete Time-POD transform we processed finite element numerical simulations of free motions of thin-walled beams to find the interesting result that the motions are characterized by a small number of POD modes. For each POD mode, interesting information on the coupling between displacement and rotation fields is extracted. Because the norms of the components of shapes of the POD modes quantify the level of coupling between the fields, we used them to examine the effect of warping on the coupled dynamics. We found that warping has significant effect on the POD modes in which the dominant motion is the torsion, while is has less effect on the POD modes where the motion is dominated by bending modes.

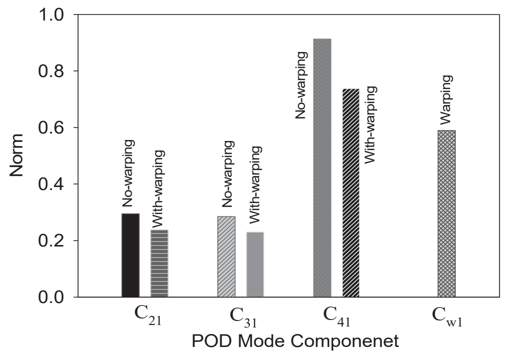

Fig. 13 Effect of warping on the norms of the first POD mode shape, the warping reduces the norms of the bending and the torsion

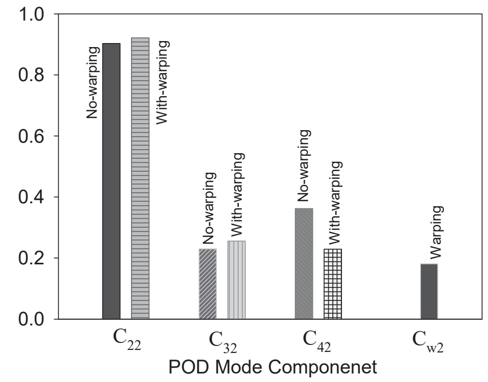

Fig. 14 The effect of warping on the norms of the second POD mode shape

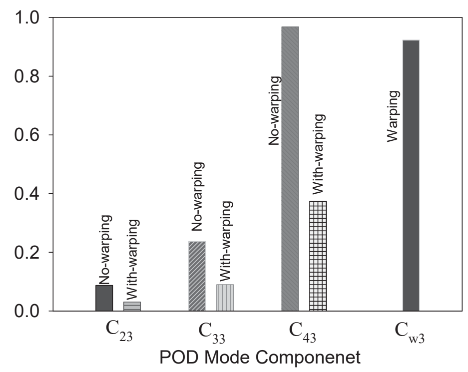

Fig. 15 The effect of warping on the norms of the third POD mode shape, the warping reduced the torsional motion

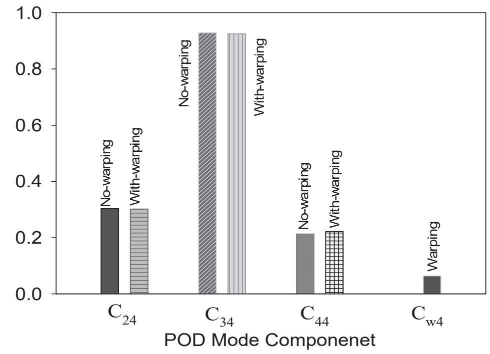

Fig. 16 The effect of warping on the norms of the fourth POD mode shape

Time proper orthogonal decomposition transform can be used to process finite element simulations of high fidelity models to characterize coupling in generic motions of thin-walled beams of arbitrary cross-section.

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