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Modelling Medieval Vaults: Digital Simulation of the North Transept Vault of St Mary, Nantwich, England

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***Abstract*—**Digital and virtual heritage is often associated with the recreation of lost artefacts and architecture; however, we can also investigate works that were not completed, using digital tools and techniques. Here we explore physical evidence of a fourteenth-century Gothic vault located in the north transept of St Mary's church in Nantwich, Cheshire, using existing springer stones that are built into the walls as a starting point. Digital surveying tools are used to document the architecture, followed by an analysis process to hypothesise and simulate possible design solutions, had the vault been completed. A number of options, both two-dimensionally and three-dimensionally, are discussed based on comparison with examples of other contemporary vaults, thus adding another specimen to the corpus of vault designs. Dissemination methods such as digital models and 3D prints are also explored as possible resources for demonstrating what the finished vault might have looked like for heritage interpretation and other purposes.

***Keywords*—**Digital simulation, heritage interpretation, medieval vaults, virtual heritage, 3D scanning.

I. Introduction

ST MARY'S church is a fourteenth-century chapel of ease in Nantwich, Cheshire, a town in England made wealthy from the salt trade in the Middle Ages. Its chancel is covered with an ambitious lierne vault, completed in the late fourteenth-century [1]. Additionally, there is evidence of an original intention to vault at least the outer bay of the north transept (Fig. 1), which can be seen in the springer stones that were built into the walls but not developed into a full rib vault (Fig. 2). A similarly uncompleted vault over the crossing was finished with timber lierne ribs by George Gilbert Scott between 1854 and 1861 [1]. The aim of this paper is to show the possibilities offered by digital technologies to document the surviving remains of the north transept vault and postulate simulations of the intended design.

As a consequence of the sixteenth-century dissolution of the monasteries in particular, Britain has a wide range of ruined buildings which include springer stones and wall arches of lost vaults, meaning that any research into the possibility of their digital construction is of value for heritage interpretation. The vault designs of fourteenth-century Britain are of a hugely inventive character and exhibit a wide range of designs including simple quadripartite vaults, sometimes with four centred arches, additional ribs including tiercerons, liernes and ridge ribs and the invention of fan vaulting, all of which can exhibit a variety of two-and-three-dimensional geometries. This leads to the possibility of multiple hypothetical design solutions, which will be explored in the north transept vault at Nantwich, along with comparisons to other contemporary vaults to support each hypothesis. The findings can also be used to assess the legitimacy of vaults added to medieval churches in the nineteenth century, such as the Scott crossing vault also at Nantwich.

The use of digital techniques is particularly relevant to unfinished, damaged or destroyed heritage assets as they offer the opportunity to rapidly present these multiple representations rather than a single, static, interpretation. This consequently results in a richer and more thorough investigative process. One objective is to offer transparency of methodology, ensuring that any viewer or reader of the research has a clear understanding of the steps taken to create the appropriate representations, providing rigour as set out by the London Charter as a benchmark in heritage visualisation projects [3]. Additionally, showing multiple hypothetical solutions can encourage more active engagement on the part of visitors to the site or viewers of such digital models. The paper will offer examples of how simulations can be displayed according to levels of certainty based on tracing the rib curvatures of the existing springer stones, with different results of various certainties according to the hypotheses employed.

II. Precedents and Parallels

This case study forms part of a larger research project investigating medieval vaults in the British Isles using digital techniques [4]. Here laser scanning and photogrammetry form the starting point of a detailed exploration of vault design and construction, for example, point cloud and mesh models of vault ribs converted from the survey scans can be traced digitally to attain their underlying geometries. This data can then be analysed to hypothesise the two-and-three-dimensional design processes used by the medieval masons. Comparable studies using similar tools can be seen internationally such as Tallon's investigations of proportions in French churches and cathedrals including Amiens and Beauvais [5], López-Mozo et al. in Spain and central Europe, such as Basel and Bebenhausen [6] as well as Ibarra Sevilla in the Mixteca region of Mexico [7], all of which have added to

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the corpus of knowledge on Gothic vault design and construction. However, this paper is unique as it is primarily investigating a vault that was most likely never completed, rather than built and in-situ or built and then destroyed.

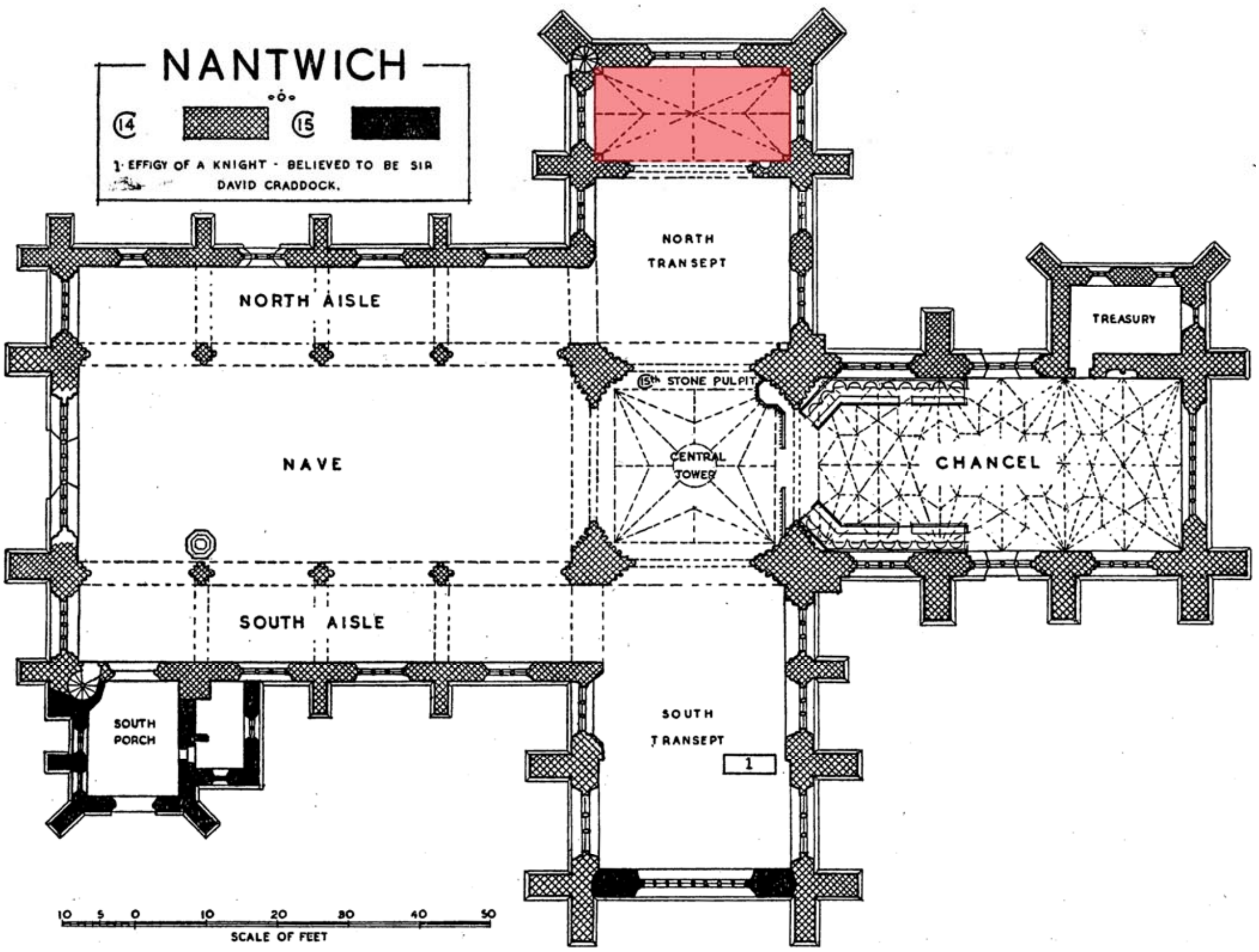

Fig. 1 Plan of St Mary's church, Nantwich, showing the unfinished vaults of the north transept highlighted in red [2]

Fig. 2 The north transept at Nantwich in which the existing springer stones and the built transverse wall arch can be seen. Photograph by J. R. Peterson

Wider research into the use of digital techniques to investigate unfinished architecture can be seen at the Sagrada Familia church in Barcelona, where Gaudi's original forms are rationalised using ruled surface geometries and hyperboloids and then interpreted and enhanced using computer-aided design and construction [8]. Such explorations demonstrate the role of digital techniques in enhancing our understanding of historic architecture, particularly the opportunities available in Gothic vault construction which can be difficult to access due to their height above ground, as well as damaged, destroyed or unfinished architecture and the unique possibilities to explore several hypotheses on their built form.

III. Methodology

The vaults at Nantwich were digitally documented using laser scanning. A Faro Focus x330 scanner was positioned on a tripod on the church floor, enabling a detailed and accurate point cloud model to be produced with points recorded at three millimetre intervals across their surfaces. Three areas were scanned in detail; the chancel, north transept and crossing, with the north transept forming the main discussion of this paper. In the north transept, although a relatively small space, three scans were taken to ensure as much detail as possible was recorded for the unfinished vaults. In the chancel, which formed a potential comparison with the north transept, one scan was taken for each of the three vault bays. This was deemed sufficient, as the shallow curvatures of the vaults meant that enough detail could be seen from the centre of each bay. Once digitally documented, the point clouds were processed using Faro Scene, the laser scanner's proprietary software, where they were registered to form a single point cloud model of the areas studied. The combined point cloud models could then be converted into additional formats to assist with analysing the existing and unfinished vaults; notably orthophotos and mesh models. Orthophotos were used to study the two-dimensional geometry of the vaults, as perspective is removed with point clouds presented orthographically in plan and section. Mesh models offered an alternative method of studying the three-dimensional geometry of vault ribs besides the use of the original point clouds (Fig. 3).

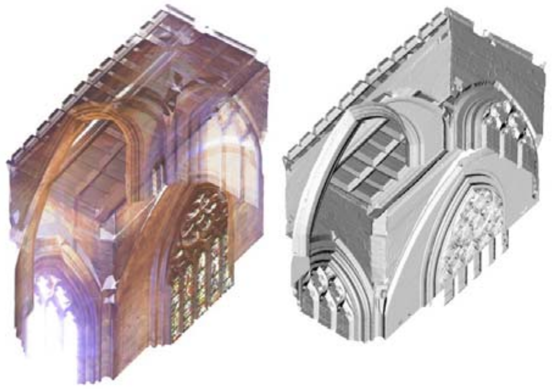

Fig. 3 Point cloud model and mesh model of the north transept

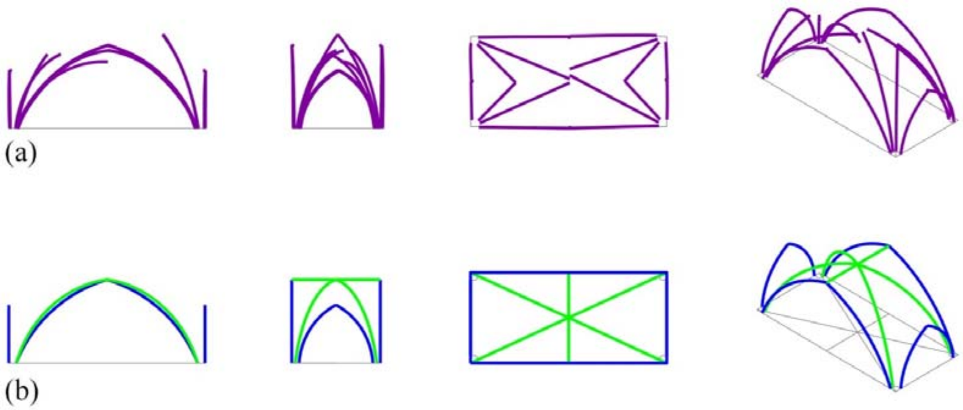

Fig. 4 Sections, plan and isometric drawings of the north transept showing (a) the traced intrados lines (purple) of the in situ wall ribs and projections based on the unfinished diagonal and tierceron ribs and (b) the known elements for the hypothesised three-dimensional design based on the existing built fabric, with the wall ribs (blue) and diagonal and longitudinal ridge which were deduced using the in-situ evidence (green)

Once the required digital representations were produced, the process of understanding the vaults in the north transept could begin. Firstly, a mesh model of the survey data was exported to Rhinoceros 5, a 3D digital modelling program. Using the mesh model, the intrados lines of the built transverse and longitudinal wall ribs were traced in order to ascertain their geometries. These were the only ribs of the unfinished vault that were completed, therefore they provided vital clues of the design, for example apex heights, rib curvatures, radii and centre points. Following this, the mesh model was used to establish the likely direction of the unfinished ribs that were evident on the in-situ springing stones. A vertical section plane was cut along the central intrados line of each unfinished rib, which was then viewed in plan to begin to understand the geometry of the missing ribs. Once this was established, the section planes were overlaid with the detailed point cloud models to give the most accurate indication available of the likely rib curvatures. These were traced and, along with the existing longitudinal and transverse wall ribs, compiled into Table I to document the geometries of all ribs along with a set of orthographic drawings to visually describe the tracings (Fig. 4 (a)). The starts of longitudinal and transverse ridge ribs were also evident in the north transept, all four of which suggest a horizontal, or near horizontal, ridgeline at the point of junction with the wall. With all of these clues, we could then begin to hypothesise the possible design of the unfinished vault elements.

IV. Simulations

The first stage of the simulation process was to identify the two-dimensional tracing floor plan that would have been produced as part of the design process. In the following figures, the data is presented using a colour coding system in terms of the level of certainty for the placing of ribs. This is to

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ensure that the viewer of the research is aware that there are several alternative simulations, rather than assuming there is one plausible scenario or one definitive solution. Previous research into unbuilt, damaged, destroyed or unfinished architecture has suggested methods of providing clarity in terms of the methods used, for example Kensek, Swartz Dodd and Cipolla suggest a colour coding system or varying transparencies in digital models to indicate levels of certainty in the evidence used to create them [9]. More recently Vitale proposes an ontology that provides the metadata behind digital models in order to make the viewer aware of the research process that has been conducted for a specific digital heritage project [10]. Masuch, Freudenberg, Ludowici, Kreiker and Strothotte provided a framework to facilitate the digital reconstruction of archaeological remains [11], which was adapted for this research and combined with a 'traffic light' colour coding system inspired by Kensek, Swartz Dodd and Cipolla [9]. Deductions, which can be understood directly from the evidence available in the existing church are coloured in green whereas analogies, which can be derived from secondary information such as contemporary architecture, are coloured in orange. Assumptions, where we have no substantial evidence for a design but know something had to be there, are coloured in red. Additionally, primary evidence that exists in situ and is directly available for representation is coloured in blue.

TABLE I
ARCH GEOMETRY IN THE NORTH TRANSEPT OF NANTWICH ST MARY'S CHURCH

Rib reference	Rib radius (mm)	Apex height (mm)
Transverse 1 (NW)	5850	3514
Transverse 2 (NE)	5236	3514
Transverse 3 (SE)	6593	3537
Transverse 4 (SW)	6423	3537
Average	**6026**	**3526**
Longitudinal 1 (NW)	2939	2465
Longitudinal 2 (NE)	2862	2454
Longitudinal 3 (SE)	3217	2454
Longitudinal 4 (SW)	3082	2465
Average	**3025**	**2460**
Diagonal 1 (NW)	4316	2822
Diagonal 2 (NE)	5136	3312
Diagonal 3 (SE)	5284	3301
Diagonal 4 (SW)	5237	3300
Average	**5219***	**3304***
Tierceron 1 (NW)	3966	3145
Tierceron 2 (NE)	6963	3986
Tierceron 3 (SE)	7927	3986
Tierceron 4 (SW)	6210	3145
Average	**7033****	**3566**

*Data for diagonal 1 excluded from rib radius and apex height average
**Data for tierceron 1 excluded from rib radius average

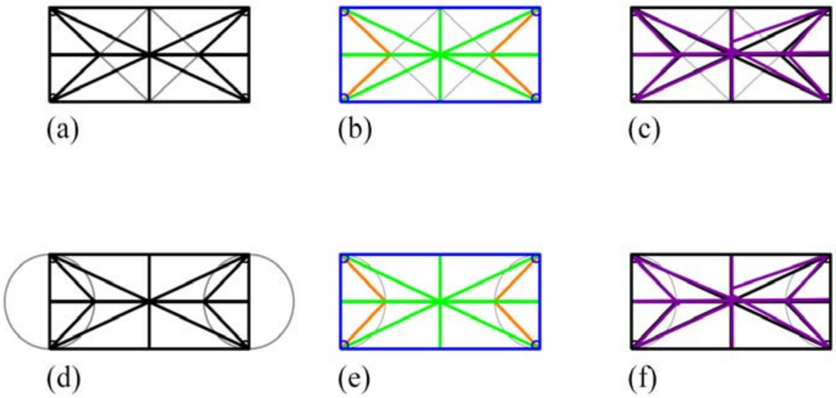

Fig. 5 (a)-(c) Option one and (d)-(f) option 2 demonstrating the two plausible tracing floor plans for the north transept. (a) and (d) indicate the simulated tracing floor plans; (b) and (e) indicate the levels of certainty of each of the ribs and (c) and (f) overlay the tracing floor plans with the directional lines created using the vertical section planes cut through the springing stones in purple

The bounding transverse and longitudinal wall ribs mentioned, along with the walls themselves, gave us an accurate account of the vault's dimensions in plan. Additionally, the starts of the transverse and longitudinal ridge ribs gave a high degree of certainty to deduce their locations crossing the vault bay. Finally, the vertical section planes cut through the springing stones at each corner gave an indication of the likely direction of the additional ribs in the vault, which confirmed the inclusion of diagonal and tierceron (or intermediate) ribs (Fig. 4 (a)). The section planes gave a good suggestion of the directions of the ribs, however, some of the directions were less convincing in plan than others, notably the north east diagonal rib not meeting the other diagonal ribs in the centre of the bay. This could be attributed to two possible factors: firstly, the digital data and its subsequent interpretation, such as the sections taken through the springing stones, may not be reliable. Secondly, the direction of the rib on the springing stone itself could be slightly out. Either way, the evidence of the other three diagonal ribs suggests that this inconsistent fourth rib would have met them in the centre of the bay if built. As such, in the second scenario, it is likely that the masons would have adjusted the rib direction by removing a small amount of stone from the in-situ springer and amending the curve slightly for the main rib itself.

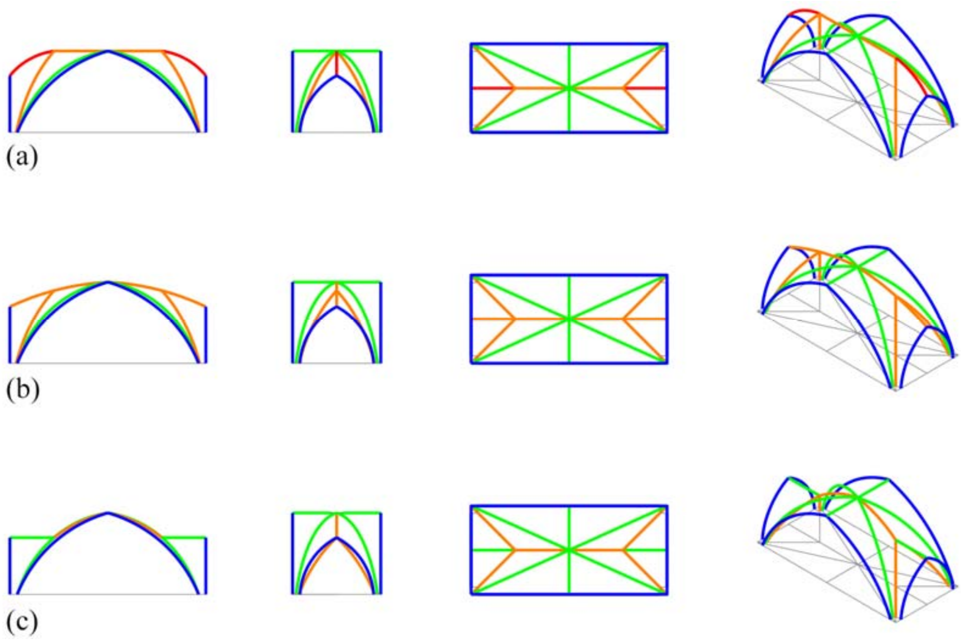

Fig. 6 Sections, plan and isometric drawings simulating (a) option one, (b) option two and (c) option three of the possible vault design in the north transept

The information based on the digital survey of the north transept was then used to hypothesise the tracing floor plan process, for which two options are suggested. The first, and simplest option, was to draw a cross from the mid-point of each wall rib to locate the ridge ribs, and then add a diagonal line from each bay corner to the opposite bay corner to locate the diagonal ribs, followed by an additional two lines this time from each bay corner to the opposite wall rib mid-point to locate the tiercerons. This geometry provides all of the points required to assist with the design and construction of the three-dimensional vault ribs (Figs. 5 (a)-(c)). Comparisons with the vault bays in the chancel at Nantwich revealed that the same method was used as described in this first option for the north transept, this time with additional processes required afterwards to increase the complexity of the vaults. However, applying this process to the north transept, it appeared that the meeting points of the tiercerons were not close enough: the position of the junction of the tiercerons in the hypothetical model compared to the junction of the suggested actual tiercerons derived from the section planes taken were 175 mm away from each other in the western half of the bay, and 300 mm away from each other in the eastern half. Therefore, a second option was considered. Again, a cross was drawn from the mid-point of each wall rib to locate the ridge ribs with a saltire drawn by connecting each bay corner to the opposite bay corner to locate the diagonal ribs. At this stage, rather than using the secondary saltires as for option one, circles were drawn with their centre points at the mid-point of each longitudinal wall rib, with their edge at the corner of the bay to give the radius. Where the circle crossed with the transverse ridge rib gave the location of the end of each tierceron in plan (Figs. 5 (d)-(f)). This second option was much closer to the directional lines created using the vertical section planes cut through the springing stones at each corner, with the distance between each 40 mm in the west of the bay and 85mm in the east of the bay. As such, this second option was chosen for the full three-dimensional simulations of the vault bay.

For the three-dimensional simulation, the averages from each data category in Table I were used to suggest the likely height of the apex of each rib as well as rib radii. However, the data for the tierceron and diagonal rib in the north west corner of the bay was ignored, as this was inconsistent with the other three corners, which is visually evident in Fig. 4 (a). Again, the data for the transverse and longitudinal wall ribs was conclusive in terms of accuracy, as these were built into the bay in preparation for the missing vault elements. The apex of the transverse wall arches was 3526 mm above the impost (the point where the ribs diverge from the springer stones) and the apex of the longitudinal wall arches was 2460 mm above the impost. The start of the diagonal ribs, when projected upwards, suggested an average apex height of 3304 mm above the impost. Taking into account that there could be some leeway in this figure, it was therefore highly likely that the apex of the diagonal rib, which is also the central point of the vault, was intended to be at the same height as the apex of the transverse wall arches. Consequently, this suggests that the longitudinal ridge ribs, which run between the apex of each transverse wall arch and the centre point of the vault, would have been laid horizontally. This is also supported by the in-situ construction of the start of horizontally placed ridge ribs. In the simulations, the transverse and longitudinal wall ribs were therefore coloured blue as known built elements and the diagonal ribs as well as the longitudinal ridge ribs were coloured green, as their geometry was reasonably deduced based on our study of the existing built geometry (Fig. 4 (b)). This left the tiercerons and transverse ridge ribs, as unknown elements with varying levels of certainty, to be investigated further.

Several options were proposed for the three-dimensional simulations, all of which draw on the evidence available in different ways. The first option (Fig. 6 (a)) is based on the average apex height of the tiercerons when projected upwards being 3566 mm, which is very close to the diagonal and transverse wall arch apex heights of 3304 mm and 3526 mm respectively. As such, option one presumes that all three of these apexes are at the same height, resulting in the transverse ridge rib between the centre of the bay and junction with the tierceron being horizontal. The transverse ridge rib then has to span between the tierceron apex and longitudinal wall arch apex, which is 2460mm above the impost. As such, this missing rib needs to curve downwards considerably to complete the vault. The main problem with this arrangement is that the curvature downwards from the tierceron apex to the longitudinal wall arch apex results in a steep decline, whereas the start of the transverse ridge rib that was built in-situ is close to horizontal. Additionally, evidence of similar contemporary vault design and construction could not be found. Therefore, the first simulation was discounted and additional options were needed to investigate the three-dimensional design.

The second option (Fig. 6 (b)) takes into account this presence of the start of the transverse ridge rib that appears to be close to horizontal. Additional support for this option is offered by the observation that the projected apex height of the two western tiercerons is lower than the apexes of the diagonal ribs and the transverse wall ribs (Fig. 4 (a)). In the first option, the average apex height of the tiercerons was higher, based on the additional data derived from the eastern tiercerons. Using only the data from the projected tiercerons in the western half of the bay supports the evidence of a horizontal transverse ridge rib continuing either horizontally or upwards with a shallow curvature. As discussed previously, the projected tiercerons may be unreliable, as the only evidence we have of them is the short curvatures provided from the springer stones, which is why we have tested different options. Consequently, option two simulates a transverse ridge rib with a shallow curvature between the apex of the longitudinal wall arch and the central point. The curvature is defined by the height of the apex of the projected tiercerons in the western half of the bay which is 3145 mm above the impost. Finally, using this information, the hypothesised tiercerons could then be added to complete the simulation of option two. Evidence of a similar design can be found in the chancel of the church. As this area of vaults was also digitally surveyed, we have measured evidence that it contains a similar curved ridge rib, visualised using an orthorectified short section (Fig. 7). In addition to the chancel at Nantwich, studying other vaults visually suggests further comparative designs with a curved ridge, for example the Wells cathedral choir aisles (probably 1330s [12]), and the nave at Worcester cathedral (1377 with additional western bays added later [13]), the former of which according to Maddison [14] is roughly contemporary with the Nantwich vault, a date that he later revises to be contemporary with the latter at Worcester [15]. The wooden cloister vaults at Lincoln cathedral (1230-1300), the Lady Chapel at Ely cathedral (normally dated 1321-1345) as well as the Norwich cathedral nave (1464-1472) and south transept (after 1509) also share similar curved ridges to the second option for the north transept at Nantwich [16], [17].

Fig. 7 Orthophoto short section created using a point cloud to demonstrate the rib curvature in the chancel of St Mary's, Nantwich

The third and final option for the three-dimensional simulation (Fig. 6 (c)) again refers to the presence of the start of the transverse ridge rib, which appears to be close to horizontal. This time the transverse ridge rib is defined as horizontal, rather than having a slight upward curvature as seen in option two. Consequently, the second section of the rib has to curve upwards to meet the ribs at the central apex. Looking at the existing geometry, the diagonal ribs and transverse wall ribs suggest that the vault may have taken a tunnel form. Therefore, the curvature of the second section of the missing transverse ridge rib is designed to follow the same tunnel arch form. Additionally, the tiercerons, which have a lower apex than in either option one or two, follow this same curvature. The final design simulation is therefore made of two intersecting tunnel vaults. This third option adheres most strictly to the known elements, particularly the starts of the near-horizontal ridge ribs, but the least convincing element is the lower tierceron apex based on the known curvature from the existing springer stones. However, the curvature of the springer stones could have been adjusted slightly by removing a small amount of material from their faces to compensate for this factor. Evidence, observed visually, of a similar design can be found in the choir vaults at Wells cathedral, of which construction probably commenced in the 1330s [12], Gloucester cathedral choir dated between 1337-1367 and Lady Chapel dated 1468-1482 [18], the nave at Ottery St Mary church which began as early as 1337 [19], as well as the Tewkesbury abbey nave which is dated to the 1340s [20].

Comparing all three simulations, the third option seemed the most likely scenario based on its adhering most rigorously to the existing built fabric of the vaults. Option two was also plausible, as seen in other contemporary vaults; however the slight upward curvature of the transverse ridge rib compared to the horizontal in situ start of the rib made it less likely than option three. The first option, although taking into account the potential inclusion of a higher tierceron apex based on the existing fabric, seemed the least probable option of the three based on the steep downward curvature of the transverse ridge rib compared to the horizontal in situ start of the rib. The inclusion of additional ribs, or liernes, which can be observed in the chancel of the church, was also investigated. However, there is no evidence of additional ribs on the wall arches, and including additional ribs that do not connect to the walls seems highly unlikely based on a lack of comparative examples elsewhere, particular based on the rectangular shape of the bay which leaves less space for additional ribs in the longitudinal direction.

V. Dissemination

Once the simulation options were finalised, dissemination methods were investigated using both digital and physical modelling. The first step was to create a digital solid model of the existing north transept at Nantwich using Rhinoceros 5, with the digital mesh and point cloud survey models acting as guides to produce accurate results. Following on from this, the wireframe models created to investigate the different options for the design and construction of the missing vault were used to form the full geometry of the hypothesised ribs. Because the wireframe curves represented the intrados lines of the missing ribs, their profiles could be extruded along these curves to form the solid geometry of the stonework. Finally, boss stones were added to obscure the joints between the various ribs, as seen in most contemporary vault examples such as the chancel also at Nantwich. A bossless vault, as in the south transept of Gloucester cathedral, was also possible but was rejected for present purposes on the basis of comparative rarity.

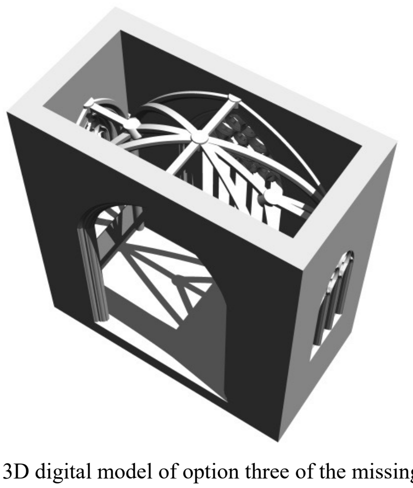

Fig. 8 3D digital model of option three of the missing vault

The digital models of the different options were then uploaded to Sketchfab, a platform comparable to YouTube that is based on interactive models rather than video files. This consequently offered the opportunity for viewers to access the models through the project website [4], where they could be embedded, to examine the vaults in greater detail compared with static images. An example screenshot of the digital model for option three is shown in Fig. 8. The next step was to produce 3D prints as a method of creating physical representations of the different simulation options. A Formlabs Form 3D printer, which uses guided laser light to cure liquid resin into solid pieces, created 1:100 scale models of the digital models already produced. These could then be used as a further dissemination method to highlight the different possible vaulting options to the wider public, with the intention of exhibiting them in Nantwich St Mary's church itself, for visitors to study (Fig. 9).

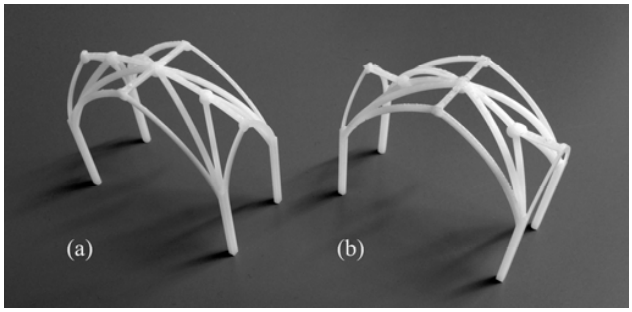

Fig. 9 3D print of (a) option two and (b) option three of the missing vault

The next stage of dissemination will be to investigate augmented reality techniques. Here we will look at overlaying the digital models of the different options for the missing vault design with the existing sprinter blocks and wall ribs in the church itself. This will be achieved using either GIS to locate the model, or most likely using QI codes on an interpretation panel in the church. Therefore, using a smartphone or tablet screen, viewers will be able to visualise the unbuilt vaults in situ as if they were completed.

VI. Conclusion

This paper demonstrates how digital surveying and analysis tools can be applied to medieval vaults, and the particular example of an unfinished design at St Mary's church in Nantwich. The process highlights the subsequent simulation options that can be hypothesised, both two-and-three-dimensionally, to gain a better understanding of the design. Additional rigour and context is provided through the study of comparative vaults to ensure the different simulated options are backed up by evidence, or not, in the case of the first three-dimensional simulation which was consequently disregarded. Based on all of the evidence available option three of the three-dimensional simulation process, which uses option two of the two-dimensional simulation process as a starting point, was deemed the most likely scenario had the vault been built. Dissemination techniques were investigated to demonstrate the research to a wider audience, which we hope to expand in future using augmented reality techniques and explore more widely looking at other medieval vaults in the British Isles.

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