

# Lookup Table Reduction and Its Error Analysis of Hall Sensor-Based Rotation Angle Measurement

Young-San Shin, Seongsso Lee

**Abstract**—Hall sensor is widely used to measure rotation angle. When the Hall voltage is measured for linear displacement, it is converted to angular displacement using arctangent function, which requires a large lookup table. In this paper, a lookup table reduction technique is presented for angle measurement. When the input of the lookup table is small within a certain threshold, the change of the outputs with respect to the change of the inputs is relatively small. Thus, several inputs can share same output, which significantly reduce the lookup table size. Its error analysis was also performed, and the threshold was determined so as to maintain the error less than  $1^\circ$ . When the Hall voltage has 11-bit resolution, the lookup table size is reduced from 1,024 samples to 279 samples.

**Keywords**—Hall sensor, angle measurement, lookup table, arctangent.

## I. INTRODUCTION

HALL sensor is used to measure linear or angular displacement. It is exploited in many types of equipment such as joysticks, cameras, smartphones, motors and cars. Especially, it is widely used in automotive applications [1], as shown in Fig. 1. When a Hall sensor is displaced in the magnetic field, Hall voltage is induced in proportional to the linear displacement. To measure rotation angle, a 3-axis Hall sensor is used, as shown in Figs. 2 and 3. In the mechanical movements, either a magnet or Hall sensor rotates in two displacements ( $\theta$ ,  $\phi$  of angular coordinates), as shown in Fig. 1. In the electronic measurement, three displacements ( $x$ ,  $y$ ,  $z$  of rectangular coordinates) are measured by Hall voltages ( $V_x$ ,  $V_y$ ,  $V_z$ ), and are converted into two rotation angles ( $\theta$ ,  $\phi$ ) using an arctangent function, as shown in Fig. 3.

Usually, small microcontroller unit (MCU) is embedded in the Hall sensor for sensor signal processing, as shown in Figs. 3 and 4. However, in many cases, the small embedded MCU have no floating units and it is difficult to calculate the arctangent function [2], [3]. Therefore, a lookup table (LUT) is often used to calculate the arctangent function, but still it suffers from large table size [4], [5]. Small embedded MCUs have no external memory and it has only a small embedded memory shared with a program code, used data, and parameter tables. Therefore, LUT should be optimized to reduce memory occupancy.

In this paper, a two-stage LUT reduction technique is presented as an angle measurement in a Hall sensor-based rotation angle measurement, where several addresses of the

LUT shares the same sample. Also, its error analysis was performed.

## II. TWO-STAGE LUT

Fig. 5 shows the proposed Hall sensor-based rotation angle measurement system with two-stage LUT. Rotation angle is measured as two Hall voltages, i.e.  $V_x$  and  $V_y$ . Digitized Hall voltages by 11-bit analog-to-digital converter (ADC) are  $X$  and  $Y$ . Note that  $X$  and  $Y$  are 11-bit signed integers. Angle calculation is to find  $\text{Angle} = \text{LUT}[X]$ , where  $X$  is the digitized Hall voltage and  $\text{LUT}[\cdot]$  is the LUT. There are two digitized Hall voltages,  $X$  and  $Y$ , but  $X$  and  $Y$  can share the same LUT. Furthermore, positive  $X$  ( $\geq 0$ ) and negative  $X$  ( $\leq 0$ ) can also share the same LUT with opposite signs. Nevertheless, it requires 1,024 samples in the LUT.

Fig. 6 shows a curve between  $X$  and  $\text{Angle} = \text{LUT}[X]$ . When the magnitude of  $X$  is lower than a threshold  $X_T$ ,  $\Delta(\text{LUT}[X])/\Delta(X)$  is small. This means that several  $X$ 's can share same  $\text{LUT}[X]$  in the range of  $X \leq X_T$ . When  $X > X_T$ , conventional LUT is used. This two-stage LUT can significantly reduce the LUT size.

In this paper, when the Hall voltage has 11-bit resolution, threshold  $X_T$  is set to 851, where approximation error should be kept under  $1^\circ$ . As a result, the LUT size is reduced from 1,024 samples to 279 samples. The detailed process of the proposed two-stage LUT is described in Fig. 5.

## III. ERROR ANALYSIS

In the proposed two-stage LUT, several digitized Hall voltages share a common value. So there is some error between the measured and approximated rotation angles, as shown in Fig. 7.

When  $X$  increases,  $\Delta(\text{LUT}[X])/\Delta(X)$  also increases; this is due to the fact that  $\text{LUT}[X]$  is a monotonically decreasing function of  $X$ , as shown in Fig. 6. Thus, total error increases until  $X$  reaches to  $X_T$ , as shown in Fig. 8. When  $X$  exceeds  $X_T$ , the number of  $\text{LUT}[X]$  samples is not reduced, and error becomes negligible.

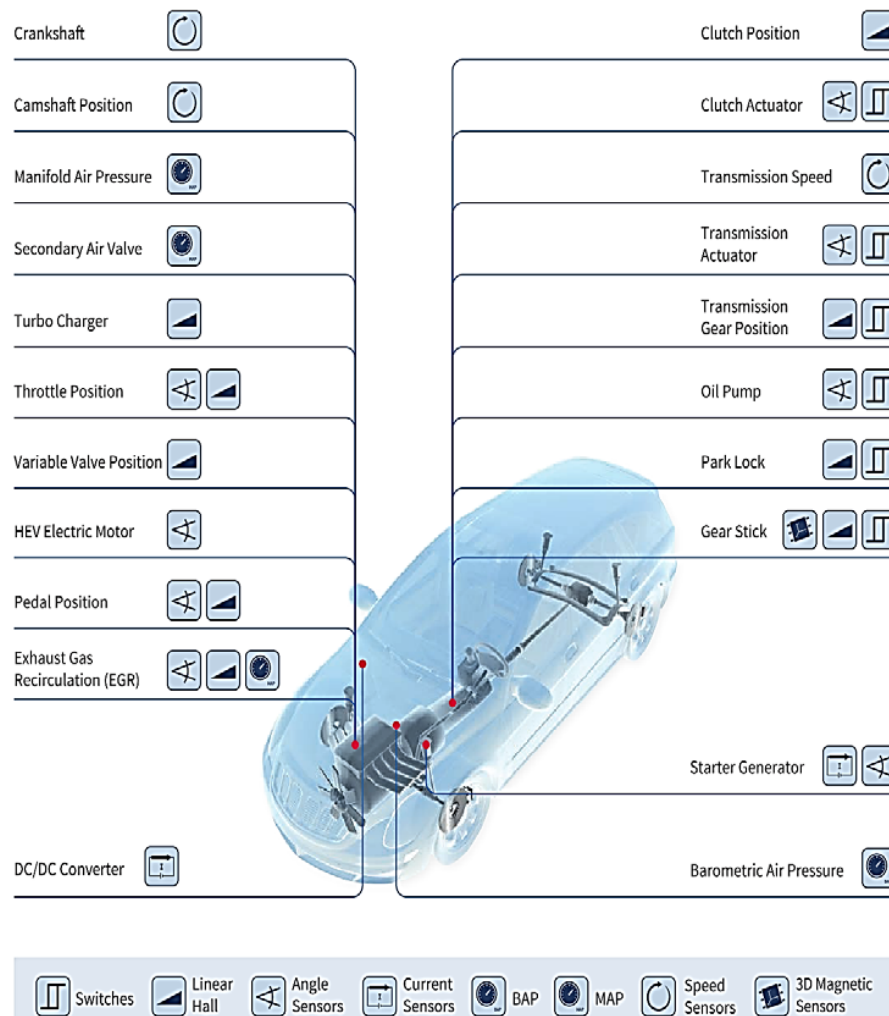


Fig. 1 Hall sensor in the automotive applications [1]

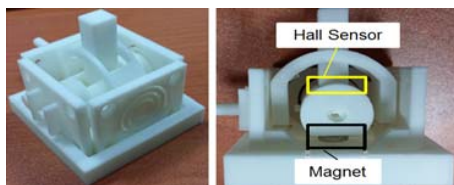


Fig. 2 Rotation angle measurement with Hall sensors

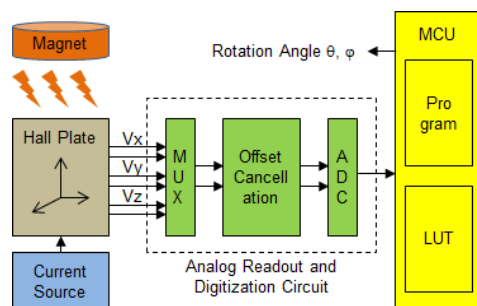


Fig. 3 Measurements of Hall voltages and calculation of rotation angle in Hall sensor

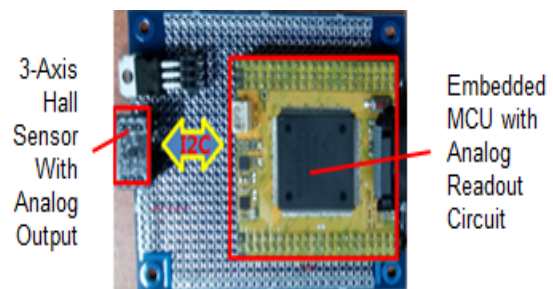


Fig. 4 3-axis Hall sensor system

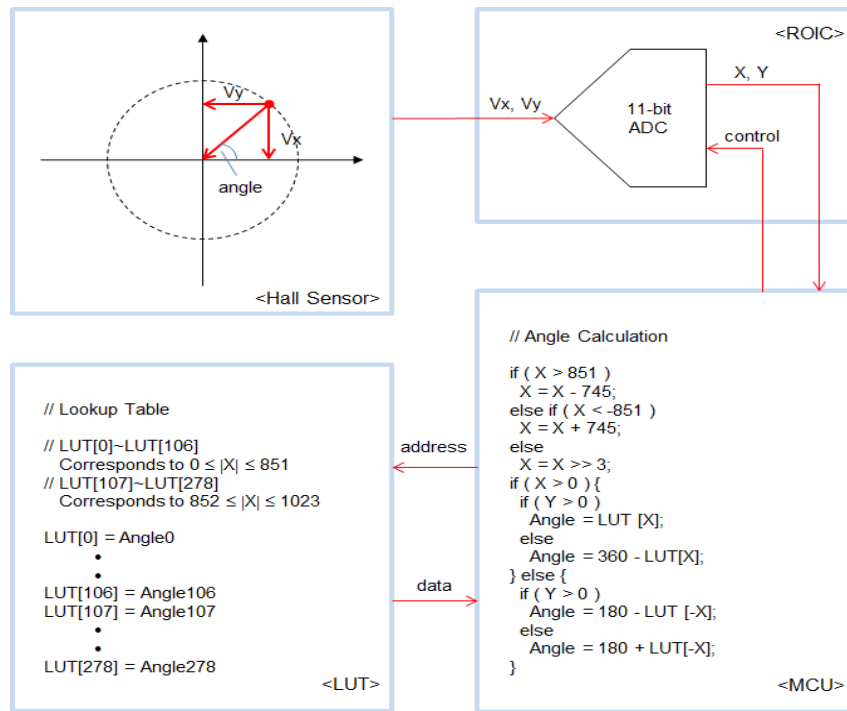


Fig. 5 The proposed Hall sensor-based rotation angle measurement system with two-stage LUT

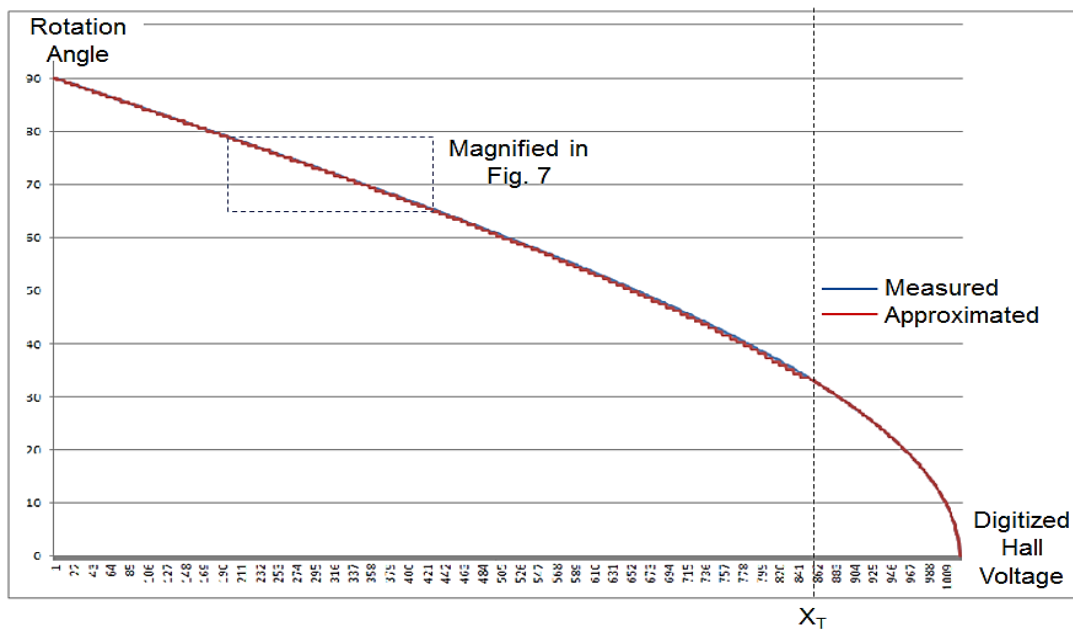


Fig. 6 Curve between digitized Hall voltage (X-axis) and rotation angle (Y-axis)

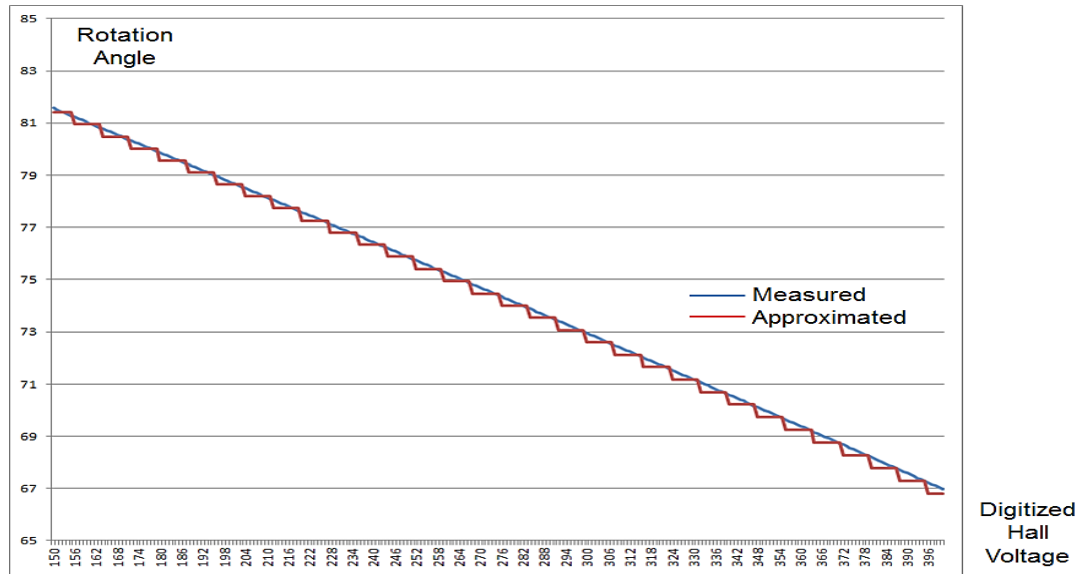


Fig. 7 Magnified curve between digitized Hall voltage (X-axis) and rotation angle (Y-axis)

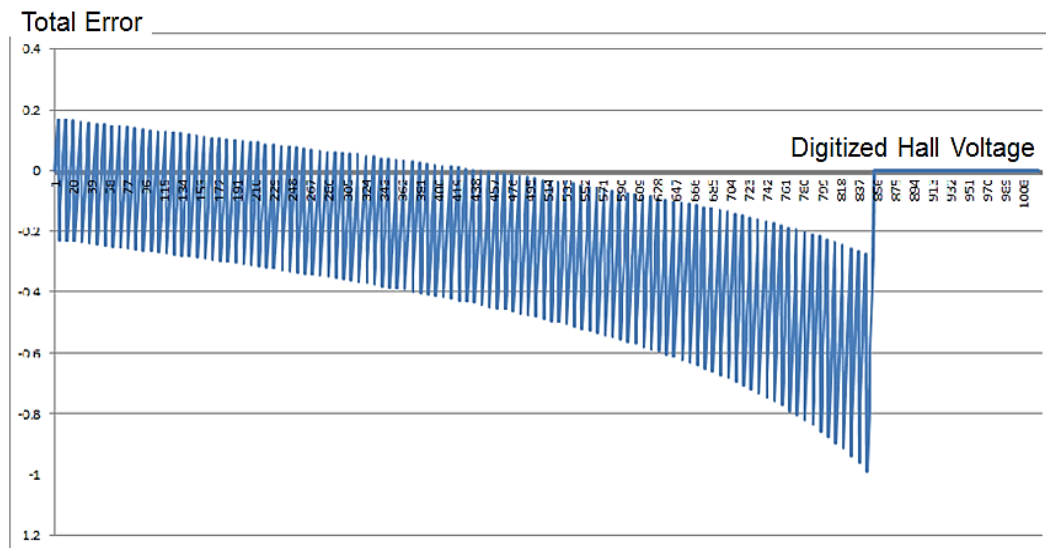


Fig. 8 Total error in the proposed two-stage LUT

#### ACKNOWLEDGMENT

This research was supported by the Ministry of Science, ICT and Future Planning, supervised by the Institute for Information & communications Technology Promotion (R7117-16-0165). This research was also supported by Industrial Core Technology Development Program (10049095) funded by the Ministry of Trade, industry & Energy, Korea."

#### REFERENCES

- [1] Infineon, "Sensor Solutions for Automotive, Industrial and Consumer Applications", [http://www.infineon.com/dgdl/Infineon-Sensor\\_Solutions\\_for\\_Automotive\\_Industrial\\_and+Customer\\_Appl\\_BR-2015.pdf?fileId=5546d4614937379a01495212845c039f](http://www.infineon.com/dgdl/Infineon-Sensor_Solutions_for_Automotive_Industrial_and+Customer_Appl_BR-2015.pdf?fileId=5546d4614937379a01495212845c039f), (Accessed on 31 Oct. 2016).
- [2] F. Dinechin and M. Istvan, "Hardware Implementations of Fixed-Point Atan2", in Proc. of IEEE Symposium on Computer Arithmetic, 2015, pp. 34-41.
- [3] A. Ukil, V. Shah and B. Deck, "Fast computation of arctangent functions for embedded applications: A comparative analysis", in Proc. of IEEE International Symposium on Industrial Electronics, 2011, pp. 1206-1211.
- [4] R. Lyons, "Another contender in the arctangent race," IEEE Signal Processing Magazine, vol. 20, no. 1, pp. 109-111, 2004.
- [5] S. Rajan, S. Wang, R. Inkol, A. Joyal, "Efficient approximations for the arctangent function," IEEE Signal Processing Magazine, vol. 23, no. 3, pp. 108-111, 2006.

**Young-San Shin** received B.S. and M.S. degrees in E.E from Soongsil University, Korea, in 2009 and 2011, respectively. He is currently pursuing Ph.D. degree in E.E. Soongsil University, Korea. His research interests include analog-digital converter, electromagnetic compatibility, readout IC, phase-locked loop, spread spectrum clock generator, microprocessor, sensor signal processing, and automotive SoC.

**Seongsoo Lee** received B.S, M.S, and Ph.D. degrees in E.E. from Seoul National University, Korea in 1991, 1993, and 1998, respectively. In 1998-2000, he was a research associate in Institute of Industrial Science, University of Tokyo, Japan. In 2000-2002, he was a research professor in Department of Electronic Engineering, Ewha Womans University, Korea. He joined School of Electronic Engineering at Soongsil University, Korea in 2002, where he is currently a professor. His research interests include high efficiency video coding, low-power SoC, multimedia SoC, power management SoC, battery management SoC, sensor SoC, and automotive SoC.